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Optimal Mixing of Hedge Funds with Traditional Investment Vehicles

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Abstract:

In this paper, we discuss the state-of-the art techniques for optimal asset allocation to traditional and alternative investment vehicles, and we specifically account for the difficulties in estimating risk/return parameters from hedge fund return data. We first present various techniques allowing an investor to better assess the contrasted diversification properties of hedge funds. In particular, we introduce a multi-factor framework for the assessment of which funds should be included for which portfolio. We also present various competing models allowing investors to get a quantitative estimate of the optimal fraction of a given portfolio that should be allocated to hedge funds, in a context where only imperfect estimates of hedge fund expected returns are available. We not only discuss optimal strategic asset allocation decisions; we also explain how tactical asset decisions can also be made in a portfolio mixing traditional and alternative investment vehicles. Finally, we show how hedge fund can be used as portable alpha vehicles in a core/satellite portfolio approach.

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One of the by-products of the bull market of the 90's has been the consolidation of hedge funds as an important segment of financial markets. The value of the hedge fund industry worldwide is estimated at more than 500 billion dollars distributed among over 5,000 hedge funds. There seem to be two main reasons behind the success of hedge funds (see Amenc, Curtis and Martellini (2001) and Schneeweis and Spurgin (1999) for a detailed study). On the one hand, hedge funds seem to provide diversification with respect to other existing investment possibilities (beta benefit). On the other hand, it is argued that hedge funds provide an abnormal return adjusted by risk (alpha benefit).

In a nutshell, the diversification argument states that most hedge funds have a low beta with respect to traditional stock and bond indexes. This is reported for example in Schneeweis and Spurgin (1999, 2000) and Agarwal and Naik (2001), at least for some hedge fund strategies. The reason is that hedge funds can take advantage of shortselling and include derivatives and other non-traditional asset classes in a way that is not allowed to mutual funds. The second argument says that the alpha of hedge funds is positive. There is a growing literature on the measurement of hedge fund risk-adjusted performance. Although these studies differ in the data and models they use and their results are not completely homogeneous, most of them conclude that there is some evidence of abnormal performance, at least in some segments of the hedge fund industry.

A standard way to present the benefits of hedge fund investing is to show the improvement they allow for in a mean-variance analysis. For example, Schneeweis and Spurgin (1999) construct a mean-variance frontier with the S&P500, the Lehman Brothers Bond Index and a hedge fund index, the EACM 100. The conclusion of this study is that the inclusion of hedge funds greatly improves the mean-variance frontier of investment possibilities. In fact, the in-sample Sharpe ratio of hedge funds seems to be so superior to the Sharpe ratios of the two other funds considered in this paper that the optimal investment strategy of an investor that uses mean-variance analysis should be to invest almost all the risky part of the portfolio in hedge funds. We view this normative prescription of an almost exclusive investment in hedge funds as evidence of the failure of in-sample static mean-variance analysis to generate a reasonable asset allocation including both traditional and alternative investment vehicles.

Although existing literature seems to grant the interest of hedge funds as valuable investment alternatives, there seems to be several other shortcomings in the presentation of the advantages of including hedge funds in an investor's asset allocation. On one hand, as it is clear from the arguments presented above, the analysis is in general cast in a mean-variance setting. This is very restrictive because of all the well known assumptions on the preferences of the investors and/or returns of the securities that are necessary in order to make this setting appropriate. Furthermore, the analysis is static and rules out the possibility of non-myopic behavior. On the other hand, this analysis relies heavily on good estimates of expected returns. This problem seems to be far from solved (see Britten-Jones (1999)). A related issue is the difficulty of measuring the alphas of hedge funds. Amenc, Curtis and Martellini (2001) estimate alphas across several models and conclude that their quantification varies greatly with the different models.

What percentage of their portfolio should investors allocate to alternative investment vehicles given their low betas and potential for positive alphas? In this paper, we discuss the state-of-the art techniques for optimal asset allocation to traditional and alternative investment vehicles, and we specifically account for the difficulties in estimating risk/return parameters from hedge fund return data.

We first present various techniques allowing an investor to better assess the contrasted diversification properties of hedge funds. In particular, we introduce a multi-factor framework for the assessment of which funds should be included for which portfolio. We also present various competing models allowing investors to get a quantitative estimate of the optimal fraction of a given portfolio that should be allocated to hedge funds, in a context where only imperfect estimates of hedge fund expected returns are available. We not only discuss optimal strategic asset allocation decisions; we also explain how tactical asset decisions can also be made in a portfolio mixing traditional and alternative investment vehicles. Finally, we show how hedge fund can be used as portable alpha vehicles in a core/satellite portfolio approach.

1. Advanced Techniques for Hedge Fund Style Selection

Investors' interest in hedge funds can be explained in particular through the fact that alternative funds actually present real diversification strengths through their exposure to risks other than market risks. In this section, we discuss advanced techniques allowing an investor to better assess the contrasted diversification properties of hedge funds.

1.1 Diversification Properties of Hedge Funds

A detailed analysis of the correlation of hedge fund returns with those of traditional markets tends to prove that it is simplistic to consider those funds as being part of a homogenous asset class. There are actually a large number of alternative strategies, with each having different diversification capabilities. In this section, we explain how to sort different hedge fund style into different categories in terms of the diversification benefits they allow for.

Investors' interest in hedge funds can be explained in particular through the fact that alternative funds actually present real diversification benefits through their exposure to risks other than market risks.

These benefits are all the more attractive in a context of relative decline of investment opportunities in traditional asset classes due to the low degree of diversification offered by a purely geographic or sector asset distribution. It is actually well known that the limitations of international diversification tend to take effect at the exact moment that the investor has a need for it, namely in periods when the markets drop significantly (see for example Longin and Solnik (1995)). In short, the correlation between the stock markets in different countries converges towards 1 when there is a sharp drop in US stock markets.

In contrast, it seems that the diversification offered by hedge funds, or to be more precise, some hedge funds, is relatively stable: the conditional correlations (calculated from a sample that only contains periods corresponding to the most significant decreases or increases in a traditional reference index) between the returns on hedge funds and those of stock and bond market indices are relatively similar to the unconditional correlations (on this point, see also Schneeweis and Spurgin (1999)). For example, the Market Neutral and Macro strategies retain a stable market risk exposure whatever the market conditions, as can be seen from table 1.

Table 1: Conditional correlations of hedge fund styles (HFR) with the stock market (S&P 500) (02/1990 – 10/2001)

Correlation coefficients	Market falling significantly (1)	Market stable (2)	Market rising significantly (3)	(1) - (3)	Type of correlation
Convertible Arbitrage	0,49	0,30	0,07	***0,42***	**Unfavourable***
Distressed Securities	0,64	0,32	-0,18	***0,82***	
Emerging Markets	0,75	0,32	0,32	***0,43***	
Equity Hedge	0,60	0,46	0,15	***0,45***	
Relative Value	0,58	0,24	-0,28	***0,86***	
Equity Non-Hedge	0,74	0,59	0,36	***0,38***	
Event Driven	0,79	0,54	-0,12	***0,91***	
Fixed Income Arbitrage	0,68	0,47	-0,13	***0,82***	
Market Neutral	0,03	0,05	-0,06	***0,09***	**Stable****
Macro	0,29	0,18	0,21	***0,08***	
Short Selling	-0,48	-0,59	-0,34	***-0,14***	**Favourable*****
Market Timing	0,15	0,53	0,29	***-0,14***	

** (1) - (3) > 0,10   ** - 0,10 < (1) - (3) < 0,10   *** (1) - (3) < - 0,10*

On the other hand, for some strategies (Convertible Arbitrage, Emerging Markets, Distressed Securities, Relative Value and Event Driven), we observe an increase in the correlation coefficient when market conditions deteriorate. This development is obviously not favourable for investors, since their diversification strategy will lose its effectiveness precisely when they need it the most. Other strategies, conversely, will see their correlation coefficient increase as the market performance improves. The investor will therefore be exposed to a rise in the market and hedged against a fall in the market! These particularly favourable strategies for the investor are the following: Market Timing and Short Selling.[1]

It is nevertheless appropriate, at this stage, to recall the limitations of evaluating conditional correlations. More often than not, they are calculated on the basis of hedge fund performance compared to that of a sample of the market indices' best or worst months or days for a given period. The conditional correlations thereby obtained correspond not to extreme values, but to a mean that is itself sensitive to the sample chosen.

The integration of the risk of an increase in the correlation coefficients in extreme market conditions has been the subject of recent research, notably after the crisis in the summer of 1998. On this topic, Anson (2000) notes that the returns on composite indices of hedge fund styles, whether equally weighted (HFR) or proportional to the assets managed (CFSB/Tremont), were significantly affected by the debt market crisis (August 1998) and not by the near-bankruptcy of LTCM (September and October 1998). Therefore, the sudden and dramatic increase in the correlation coefficients, and not the systemic crisis in the alternative universe, was the main source of risk during that period.

A detailed analysis of the correlation of hedge fund returns with those of traditional markets tends to prove that it is simplistic to consider those funds as being part of a homogenous asset class.

[1] The difference between conditional and unconditional correlations is actually one of the consequences of the non-linear exposure of hedge fund returns with respect to standard asset classes.

There are actually a large number of alternative strategies, with each having different diversification properties. Some strategies such as Market Neutral, Relative Value, or Convertible Arbitrage generally have a low level of correlation with the performances of the S&P 500, a popular stock market index (correlations typically less than 0.5 as an absolute value), and with those of the Lehman Brothers US Aggregate Index, the reference bond index. However, for other strategies such as Equity Non Hedge or Short Selling, this is absolutely not the case (see table 2).

Table 2: Correlation with stock (S&P 500) and bond (LBGBI) indices (02/1990 – 10/2001)

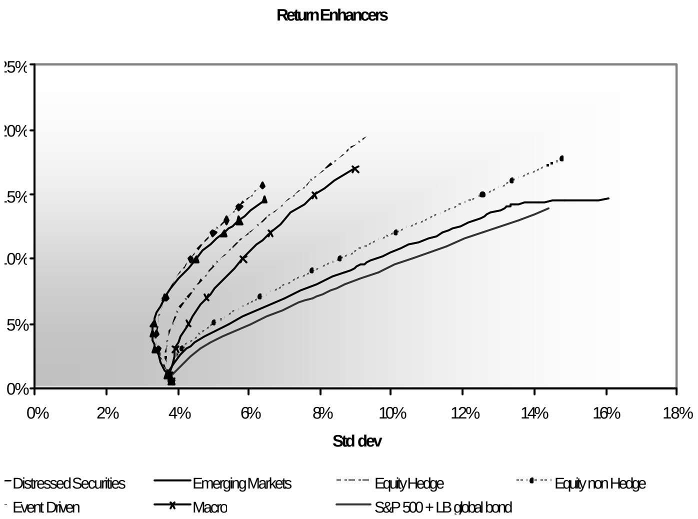
Correlation Coefficients	**S&P 500**	**Lehman US**
Convertible Arbitrage	0.31	0.18
Distressed Securities	0.37	0.01
Emerging Markets	0.57	0.03
Equity Hedge	0.63	0.12
Market Neutral	0.12	0.23
Equity non-Hedge	0.77	0.13
Event Driven	0.59	0.10
Fixed Income Arbitrage	0.42	0.13
Macro	0.42	0.37
Relative Value	0.34	0.04
Short Selling	-0.69	-0.07
Market Timing	0.68	0.19

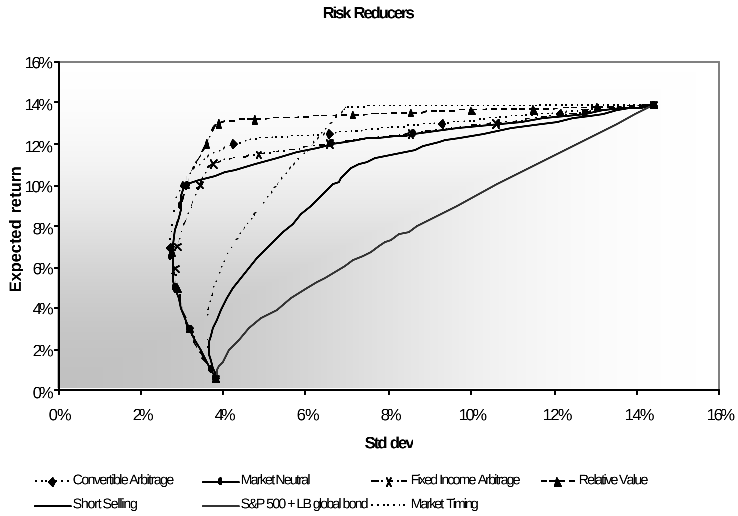
1.2 Improvement of the Risk-Return Profile of a Bond/Stock Portfolio

Some hedge funds have a high level of correlation with the market, and offer returns that are particularly high. Adding this type of funds to a portfolio made up of stocks and bonds would result in an increase in the expected return while retaining a high degree of volatility. This is a return enhancement property. Conversely, integrating certain alternative strategies with low exposure to market risk, or indeed negative exposure, will result in a lowering of the portfolio's volatility. This is a risk reduction property.

That hedge funds exhibit contrasted exposure with respect to the returns on traditional assets allows us to envisage very diverse forms of alternative diversification, as is shown in the figures 1 and 2 below.

Figures 1 and 2: Diversification profiles: inclusion of hedge fund styles (HFR) with stock (S&P 500) and bond (LBGBI) portfolios (02/1990 – 10/2001)





Some hedge funds exhibit a high level of correlation with the market, and offer returns that are particularly high. Adding this type of fund to a portfolio made up of stocks and bonds would result in an increase in the expected return while retaining a high degree of volatility. Distressed Securities, Emerging Markets, Event Driven, or Global Macro present these characteristics; these strategies can therefore be seen as "Return Enhancers." Conversely, integrating certain alternative strategies with low exposure to market risk, or indeed negative exposure, will result in a decrease in the portfolio's volatility. Convertible Arbitrage, Fixed Income Arbitrage, Market Neutral or Short Selling (negative correlation) strategies correspond to this profile. These strategies can therefore be seen as Risk Reducers, or even as Pure Diversifiers (Short Selling). Efficient frontier analysis, however, offers very little robustness with respect to small changes in estimates of the variance-covariance matrix and expected returns. For that reason, it is necessary to pay particular attention to the potential impact of several biases in hedge fund index returns (and in particular the survivorship bias) when evaluating the performance and volatility of alternative strategies. We shall come back to this point in more detail later on in this article.

These differences in correlation with stock and bond markets can be explained through a difference in exposure to a number of risk factors that explain the returns of the alternative and traditional asset classes. Recent research on the analysis of hedge fund performance (Fung and Hsieh (1997a), Schneeweis and Spurgin (1999), Amenc, Curtis and Martellini (2002)) has highlighted the fact that hedge funds are not only exposed to market risk (unforeseeable variations in the prices of basic assets, stocks, bonds, etc.), measured by the traditional "beta," but also, as a result of the very nature of the strategies implemented, to volatility risks (unforeseeable variations in the variability of the prices), default risks (unforeseeable variations in the propensity of certain counterparties to no longer be able to respect their commitments) and liquidity risks (unforeseeable variations in the capacity to move quantities of assets in a "reasonable" time scale at market prices).

In table 3 below we present the exposure of hedge fund strategies with respect to the main sources of risk (apart from market risk) that affect the returns of financial instruments. This study was carried out using the performances of the different HFR indices over the period February 1993/October 2001. (It should be pointed out that monthly calculation of the correlations tends to smooth the results and therefore smoothes the impact of changes in the factor values within the months.)

Table 3: Correlations between different Hedge Fund strategies (HFR) and risk factors (02/1993 – 10/2001)

Correlation coefficients	Volatility	Exchange rate	Raw materials	Liquidity	Default	3 month US Treasury Bill	Slope of the yield curve
Convertible Arbitrage	-0.33	0.14	0.04	-0.05	0.10	0.08	-0.17
Distressed Securities	-0.50	0.06	0.13	-0.12	-0.06	-0.16	0.25
Emerging Markets	-0.48	0.03	0.07	-0.01	0.07	-0.21	0.27
Equity Hedge	-0.43	-0.14	0.20	-0.01	-0.04	0.04	-0.02
Market Neutral	-0.02	-0.11	-0.12	0.00	-0.08	0.16	-0.15
Equity non-Hedge	-0.50	-0.17	0.17	-0.04	-0.04	-0.01	0.07
Event Driven	-0.57	-0.03	0.15	-0.03	0.00	-0.06	0.09
Fixed Income Arbitrage	-0.41	0.15	0.09	-0.07	-0.02	-0.09	0.21
Macro	-0.35	0.19	-0.03	0.09	-0.07	-0.16	0.16
Relative Value	-0.41	0.03	0.10	-0.13	-0.01	-0.12	0.12
Short Selling	0.37	0.19	-0.15	0.05	0.03	0.02	-0.05
Market Timing	-0.31	-0.15	0.05	0.13	0.07	-0.07	0.01
S&P 500	-0.42	-0.14	0.01	0.08	-0.07	0.10	-0.03
Lehman US Aggregate	-0.14	0.02	-0.01	0.19	-0.07	0.11	-0.18

The data used to characterise the different sources of risk is as follows:

The volatility risk is measured by the relative price variations of the VIX contract, the underlying of which is the implicit volatility of the S&P 100
The currency risk is measured by the evolution of the exchange rate of the US dollar compared to a basket of foreign currencies
The raw material risk is measured by the relative price variations of a barrel of crude oil
The liquidity risk is measured by the evolution of the volume of securities exchanged on the NYSE
The default risk is measured by the relative variations of the differential between the returns on bonds rated Baa and Aaa by Moody's
The slope of the yield curve is obtained by calculating the difference between the rate of return of a bond with a 30-year maturity and that of a 3-month Treasury bill.

On the other hand, it should be noted that part of hedge fund returns can be explained by their exposure to different risk factors. For instance, it has often been observed that a number of hedge funds pursuing "fixed-income arbitrage" strategies act as liquidity providers on defaultable bond markets, a role typically taken on by the trading desks of the major investment banks. As a result, they collect a liquidity premium, which explains part of their performance.

It is in fact natural for hedge fund managers to seek to use the multiple facets of risk, and therefore of return, as it increases the degree of liberty in investment decisions.

1.3 From a Single to a Multi-Fund Approach

Since choosing a bad manager may easily wipe out all the benefits of a hedge fund allocation, investing in only one hedge fund is likely to be sub-optimal. The reasons are threefold.

- Firstly, dramatic performance differentials between competing funds raise the issue of whether a single investment instrument can deliver consistent returns close to those of the broad hedge-fund indices that are used at the strategic asset-allocation level.
- Secondly, a number of individual hedge funds have collapsed under the weight of spectacular frauds or investment debacles (Manhattan Capital Management, Maricopa Investment Corporation, Lipper Convertible Arbitrage, etc.). This has raised concerns among investors, who often lack sufficient information to evaluate comparative hedge fund performance and to perform the necessary exhaustive on-site due diligence checks.
- Finally, investing only with managers who have a good reputation and an established track record does not provide a complete hedge, as illustrated by the debacle of the brain trust that was Long Term Capital Management LP.

Consequently, risk-conscious investors are coming back to the central tenet of modern portfolio theory, namely, diversification. By combining several hedge funds with differing return distributions and risk profiles in a portfolio, investors are able to diversify specific risk away and ensure a more disciplined exposure to the overall hedge fund asset class. This is likely to result in better long-term risk-adjusted returns. Those willing to avoid the logistical problems and record-keeping headaches of tracking several hedge funds may even delegate the portfolio construction and monitoring activities to a fund of hedge funds. This is the preferred investment structure for most institutional investors, since it gives them instant diversification and frees them from the responsibility of monitoring managers

Over the recent years, multi-strategy funds of hedge funds have flourished and are now the favorite investment vehicles of institutional investors to discover the world of alternative investments. More recently, funds of hedge funds that specialize within an investment style have also emerged. Both types of funds put forward their ability to diversify risks by spreading them over several managers. However, diversifying a hedge fund portfolio also raises a number of issues, such as the optimal number of hedge funds to really benefit from diversification, and the influence of diversification on the various statistics of the return distribution (e.g., expected return, skewness, kurtosis, correlation with traditional asset classes, value at risk and other tail statistics).

In this section, we summarize some of the recent findings in the literature on these issues. While the benefits of diversification have been widely studied and documented for traditional assets, the research on hedge fund diversification has actually been rather scarce. It is only recently that a few papers started investing in the issue of how many hedge funds were required in a hedge fund portfolio to efficiently reduce volatility. The answers vary greatly depending on the sample considered and the time-period investigated.

Intuitively, the existence of hedge fund diversification benefits will depend upon the number of hedge funds in a portfolio. Beyond the agreement that holding only a few funds may imply under-diversification, exposure concentration, and, therefore, too much risk, while holding too many funds may result in over-diversification, the dilution of each fund's contribution and the neutralization of most diversification benefits, there seems to be no consensus on the optimal number of funds. On the academic side, the literature suggests that approximately eight to ten managers should be sufficient to reduce significantly the overall risk of the portfolio (see

Billingsley and Chance (1996) for managed futures, Henker and Martin (1998), for CTAs and Henker (1998) for hedge funds). However, Amin and Kat (2000) show that one has to hold at least twenty funds to fully realize the diversification potential in hedge funds. From the practitioner's perspective, the consensus seems to be that at least twenty to thirty managers are necessary to diversify effectively, as shown by the information released by funds of hedge funds. The short note by Ruddick (2002) evidences that the maximum benefits of diversification are reached with around 20 funds, and that it is still possible to have them at around 40 funds if the quality of new additions can be maintained.

Most of the attention is dedicated to the diversification benefits accrued by adding hedge funds to traditional asset portfolios. There has thus far been very little large-scale research done on the topic of pure alternative assets diversification. For example: do the diversification benefits, if any, go beyond volatility and also affect other important statistics (e.g., average returns, skewness, kurtosis, correlation with other asset classes, value at risk, maximum drawdown, etc.)? Do they differ within styles and across styles?

In a recent paper, Learned and Lhabitant (2002) address these questions by studying the benefits of diversification in the contect of a fund of hedge funds. They build equally weighted portfolios of randomly selected hedge funds. By repeating the process several times and studying the characteristics of the resulting portfolios (50,000 in total), they are able to study the impact of naively increasing the number of hedge funds in a portfolio. Their main empirical findings can be summarized in the following two points.

First, they tend to demonstrate that diversification works well in a mean-variance space. That is, increasing the number of hedge funds in a hedge fund portfolio decreases the portfolio's volatility, while maintaining its average return level. Downside risk statistics (such as maximum monthly loss, maximum drawdown or value at risk) are also reduced in larger-size hedge fund portfolios. This seems to validate the existence of funds of funds as useful investment vehicles. However, when one goes beyond the mean-variance framework and considers additional factors such as skewness and kurtosis, diversification is far from being a free lunch. For several strategies, diversification reduces positive skewness, may even generate negative skewness, and increases kurtosis, i.e., there is a trade-off between profit potential and reduced probability of loss. In addition, the correlation with the S&P 500 of large-sized hedge fund portfolios increases, which clearly evidences the dangers of diversification overkill, that is, the attempt of advisors to incorporate an unwieldy number of hedge funds in their portfolio construction process. Since most of the diversification benefits are reached for small-sized portfolios (typically 5 to 10 hedge funds), it therefore seems that hedge fund portfolios should rather be cautious on their allocations past this number of funds.

They also emphasize the difference between diversification by investment style and diversification by judgement. Clearly, the benefit of increasing managers within a strategy is a function of the homogeneity or heterogeneity of the sample from which the managers are drawn. Style diversification (diversification by investment style) obviously provides better opportunities for diversification than diversification by judgment. In that respect, it also seems that information about investment style reported by fund managers should be used in the portfolio construction process, albeit naive, to increase the diversification benefits.

2. Advanced Techniques for Hedge Fund Style Allocation

We have just discussed the benefits of naïve diversification strategies, based on the addition of equally-weighted portfolio of hedge funds in an investor's portfolio. In this section, we discuss advanced techniques allowing an investor to get a quantitative estimate of the *optimal* fraction of a given portfolio that should be allocated to hedge funds, in a context where only imperfect estimates of hedge fund expected returns are available. We also discuss the relative *optimal* weightings of several hedge funds within a portfolio.

2.1 Strategic Asset Allocation with Hedge Funds

In this section, we explain how investors can estimate what percentage of their portfolio should investors allocate to alternative investment vehicles given their low betas and uncertain potential for positive alphas. We specifically account for the fact that hedge fund data is scarce (mostly monthly returns) and not always reliable (biases in hedge fund returns). There are two main challenges involved in the application of standard asset allocation methods (e.g., efficient frontier analysis) to the design of optimal portfolios including hedge funds. One challenge is that it is extremely difficult to obtain a forward-looking estimate of a hedge fund expected return. Therefore, we will review advanced techniques consistent with the presence of significant parameter uncertainty in the asset allocation process. Another challenge comes from the fact that hedge fund returns are not in general normally distributed (see for example Amin and Kat (2001) or Lo (2001)), which makes the use of any asset allocation model based on sole estimates on expected return and volatility somewhat problematic. In what follows, we provide a review of optimal asset allocation models that account for more than the first two moments of hedge fund return distributions.

2.1.1. When Estimates of Expected Returns are not Available

A classic way to analyze and formalize the benefits of investing in hedge funds is to note the improvement in the risk-return trade-off they allow when included in a traditional long-only stock and bond portfolio. Since seminal work by Markowitz (1952), it is well-known that this trade-off can be expressed in terms of mean-variance analysis under suitable assumptions on investor preferences (quadratic preferences) or asset return distribution (normal returns).[2] In the academic and practitioner literature on the benefits of alternative investment strategies, examples of enhancement of long-only efficient frontiers through optimal investments in hedge fund portfolios abound (see for example Schneeweis and Spurgin (1999) or Karavas (2000)).

One problem is that, to the best of our knowledge, all these papers only focus on *in-sample* diversification results of standard sample estimates of covariance matrix. In sharp contrast with

[2] There is clear evidence that hedge fund returns may not be normally distributed (see for example Amin and Kat (2001) or Lo (2001)). Hedge funds typically exhibit non-linear option-like exposures to standard asset classes (Fung and Hsieh (1997a, 2000), Agarwal and Naik (2000)) because they can use derivatives, follow all kinds of dynamic trading strategies, and also because of the explicit sharing of the upside profits (post-fee returns have option-like element even if pre-fee returns do not). As a result, hedge fund returns may not be normally distributed even if traditional asset returns were. Fung and Hsieh (1997b) argue, however, that mean-variance analysis may still be applicable to hedge funds as a second-order approximation as it essentially preserves the ranking of preferences in standard utility functions. An alternative is mean-VaR analysis (see for example Favre et Galinao (2000) or Amenc, Martellini and Vaissie (2002)).

the large amount of literature on asset return covariance matrix estimation in the traditional investment area, there has actually been very little scientific evidence evaluating the performance of different portfolio optimization methods in the context of alternative investment strategies. This is perhaps surprising given that the benefits promised by portfolio optimization critically depend on how accurately the first and second moments of hedge fund return distribution can be estimated.

There are actually many reasons to believe that the main challenge is in estimating expected returns, as opposed to covariances, of hedge fund returns. First, there is a general consensus that expected returns are difficult to obtain with a reasonable estimation error. What makes the problem worse is that optimization techniques are very sensitive to differences in expected returns, so that portfolio optimizers typically allocate the largest fraction of capital to the asset class for which estimation error in the expected returns is the largest (e.g., Britten-Jones (1999) or Michaud (1998)).

On the other hand, there is a common impression that return variances and covariances are much easier to estimate from historical data. Since early work by Merton (1980) or Jorion (1985, 1986), it has been argued that the optimal estimator of the expected return is noisy with a finite sample size, while the estimator of the variance converges to the true value as the data sampling frequency is increased.[3]

2.1.1.1. Minimum Variance Approach

We first approach the question of optimal strategic asset allocation in the alternative investment universe in a pragmatic manner. Because of the presence of large estimation risk in the estimated expected returns, we evaluate the performance of an improved estimator for the covariance structure of hedge fund returns, focusing on its use for selecting the one portfolio on the efficient frontier for which no information on expected returns is required, the minimum variance portfolio.[4] In other words, because of the presence of large estimation risk in the estimated expected returns, we choose to evaluate the performance of the improved estimator for the covariance structure of hedge fund returns by focusing on its use for selecting the minimum variance portfolio, the only portfolio on the efficient frontier for which no estimation of expected returns is needed. Thus, we explain how an efficient allocation can be implemented by an investor who does not feel confident in his/her ability to generate a reliable looking-forward estimate of hedge fund expected returns.

2.1.1.1.1. Covariance Matrix Estimation

It has since long been recognized that the sample covariance matrix of historical returns is likely to generate high sampling error in the presence of many assets. However, contrary to the case of

[3] Singer, Stab and Terhaar (2002) suggest to use an asset pricing model such as the CAPM to generate forward-looking estimates of hedge fund expected returns. This is somewhat similar to Black and Litterman (1991, 1992) who show how to use the CAPM to generate a consistent set of reference estimates of risk premia.

[4] Alternatively, one motivation in focusing on the minimum variance portfolio is to note that it is the efficient portfolio obtained under the null hypothesis of no informative content in the cross-section of expected returns.

expected returns, several methods have been introduced to improve asset return covariance matrix estimation.

Several solutions to the problem of asset return covariance matrix estimation have been suggested in the traditional investment literature. The most common estimator of return covariance matrix is the sample covariance matrix of historical returns.

$$S = \frac{1}{T-1}\sum_{t=1}^{T}\left(H_t - \overline{H}\right)\left(H_t - \overline{H}\right)'$$

where T is the sample size, H_t is a $Nx1$ vector of hedge fund returns in period t, N is the number of assets in the portfolio, and H

$$\overline{H} = \frac{1}{T}\sum_{t=1}^{T} H_t$$

is the average of these return vectors. We denote by S_{ij} the (i,j) entry of S.

A problem with this estimator is typically that a covariance matrix may have too many parameters compared to the available data. If the number of assets in the portfolio is N, there are indeed $N(N-1)/2$ different covariance terms to be estimated. The problem is particularly acute in the context of alternative investment strategies, even when a limited set of funds or indexes are considered, because data is scarce given that hedge fund returns are only available on a monthly basis.

One possible cure to the curse of dimensionality in covariance matrix estimation is to impose some structure on the covariance matrix to reduce the number of parameters to be estimated. In the case of asset returns, a low-dimensional linear factor structure seems natural and consistent with standard asset pricing theory, as linear multi-factor models can be economically justified through equilibrium arguments (cf. Merton's Intertemporal Capital Asset Pricing Model (1973)) or arbitrage arguments (cf. Ross's Arbitrage Pricing Theory (1976)). Therefore, in what follows, we shall focus on K-factor models with uncorrelated residuals. Of course, this leaves two very important questions: *how much structure should we impose?* (the fewer the factors, the stronger the structure) and *what factors should we use?* A standard trade-off exists between model risk and estimation risk. The following options are available:

- Impose no structure. This choice involves low specification error and high sampling error, and led to the use of the sample covariance matrix.[5]

- Impose some structure. This choice involves high specification error and low sampling error. Several models fall within that category, including the constant correlation approach (Elton and Gruber (1973)), the single factor forecast (Sharpe (1963)) and the multi-factor forecast (e.g., Chan, Karceski and Lakonishok (1999)).

[5] One possible generalization/improvement to this sample covariance matrix estimation is to allow for declining weights assigned to observations as they go further back in time (Litterman and Winkelmann (1998)).

- Impose optimal structure. This choice involves medium specification error and medium sampling error. The optimal trade-off between specification error and sampling error has led either to an optimal shrinkage towards the grand mean (Jorion (1985, 1986)) or an optimal shrinkage towards the single-factor model (Ledoit (1999)), or to the introduction of portfolio constraints (Jagannathan and Ma (2000)).

In this section, following Amenc and Martellini (2002), we consider an implicit factor model in an attempt to mitigate model risk and impose *endogenous* structure. The advantage of that option is that it involves low specification error (because of the "let the data talk" type of approach) and low sampling error (because some structure is imposed). Implicit multi-factor forecasts of asset return covariance matrix can be further improved by noise dressing techniques and optimal selection of the relevant number of factors (see below).

More specifically, we explain how to use Principle Component Analysis (PCA) to extract a set of implicit factors. The PCA of a time-series involves studying the correlation matrix of successive shocks. Its purpose is to explain the behavior of observed variables using a smaller set of unobserved implied variables. Since principal components are chosen solely for their ability to explain risk, a given number of implicit factors always capture a larger part of asset return variance-covariance than the same number of explicit factors. One drawback is that implicit factors do not have a direct economic interpretation (except for the first factor, which is typically highly correlated with the market index). Principal component analysis has been used in the empirical asset pricing literature (see for example Litterman and Scheinkman (1991), Connor and Korajczyk (1993) or Fedrigo, Marsh and Pfleiderer (1996), among many others).

From a mathematical standpoint, it involves transforming a set of N correlated variables into a set of orthogonal variables, or implicit factors, which reproduces the original information present in the correlation structure. Each implicit factor is defined as a linear combination of original variables. Define H as the following matrix

$$H = (h_{it})_{\substack{1 \le t \le T \\ 1 \le i \le N}}$$

We have N variables h_i, $i=1,...,N$, i.e., monthly returns for N different hedge fund indexes, and T observations of these variables.[6] PCA enables us to decompose h_{tk} as follows[7]

$$h_{tk} = \sum_{i=1}^{N} \sqrt{l_i} U_{ik} V_{ti} := \sum_{i=1}^{N} s_{ik} V_{ti}$$

where
U is the matrix of the N eigenvectors of H'H
V is the matrix of the N eigenvectors of HH'

Note that these N eigenvectors are orthonormal. $?_i$ is the eigenvalue (ordered by degree of magnitude) corresponding to the eigenvector U_i. Note that the N factors V_i are a set of orthogonal

[6] These reeturns have first been normalized to show a zero mean and unit variance.
[7] For an explanation of this decomposition in a financial context, see for example Barber and Copper (1996).

variables. The main challenge is to describe each variable as a linear function of a reduced number of factors. To that end, one needs to select a number of factors K such that the first K factors capture a large fraction of asset return variance, while the remaining part can be regarded as statistical noise

$$\eta_{tk} = \sum_{i=1}^{K} \sqrt{\lambda_i} U_{ik} V_{ti} + \varepsilon_{tk} := \sum_{i=1}^{K} s_{ik} V_{ti} + \varepsilon_{tk}$$

where some structure is imposed by assuming that the residuals ε_{tk} are uncorrelated one to another. The percentage of variance explained by the first K factors is given by

$$\sum_{i=1}^{K} \lambda_i \Big/ \sum_{i=1}^{N} \lambda_i$$

A sophisticated test by Connor and Corajczyk (1993) finds between 4 and 7 factors for the NYSE and AMEX over 1967-1991, which is roughly consistent with Roll and Ross (1980). Ledoit (1999) uses a 5-factor model. In this paper, we select the relevant number of factors by applying some explicit results from the theory of random matrices (see Marchenko and Pastur (1967)).[8] The idea is to compare the properties of an empirical covariance matrix (or equivalently correlation matrix since asset returns have been normalized to have zero mean and unit variance) to a null hypothesis purely random matrix as one could obtain from a finite time-series of strictly independent assets. It has been shown (see Johnstone (2001) for a recent reference and Laloux et al. (1999) for an application to finance) that the asymptotic density of eigenvalues λ of the correlation matrix of strictly independent asset reads

$$f(\lambda) = \frac{T}{2\pi N} \frac{\sqrt{(\lambda - \lambda_{\max})(\lambda - \lambda_{\min})}}{\lambda}$$

$$\lambda_{\max} = 1 + \frac{N}{T} + 2\sqrt{\frac{N}{T}}$$

$$\lambda_{\min} = 1 + \frac{N}{T} - 2\sqrt{\frac{N}{T}}$$

Theoretically speaking, this result can be exploited to provide formal testing of the assumption that a given factor represents information and not noise. However, the result is an asymptotic result that cannot be taken at face value for a finite sample size. One of the most important features here is the fact that the lower bound of the spectrum λ_{min} is strictly positive (except for $T=N$), and therefore, there are no eigenvalues between 0 and λ_{min}. We use a conservative interpretation of this result to design a systematic decision rule and decide to regard as statistical

[8] Another decision rule would be: keep sufficient factors to explain x% of the covariation in the portfolio.

noise all factors associated with an eigenvalue lower than $?_{max}$. In other words, we take K such that $?_K > ?_{max}$ and $?_{K+1} < ?_{max}$.[9]

[9] In case no factor is such that the associated eigenvalue is greater than lambda max, we take $K=1$, i.e., we retain the first component as the only factor.

2.1.1.1.2. Empirical Results

There is at least a dozen of competing hedge fund index providers, and they provide a somewhat contrasted picture of hedge fund returns (see Amenc and Martellini (2001) or Fung and Hsieh (2001)). To represent the alternative investment universe, we choose in this section to use index returns from Credit Swiss First Boston - Tremont (CSFB-Tremont). We have obtained very similar results with HFR and EACM indices.

Our methodology for testing minimum variance portfolios is similar to the one used in Chan et al. (1999) and Jagannathan and Ma (2000). We use the previous 48 months of observations (beginning of 1994 to end of 1998) to estimate the covariance matrix of the returns of the 9 hedge fund sub-indexes. We then form two versions of global minimum variance portfolios: the nonnegativity constrained and the one with both nonnegativity constraint and a tracking error constraint. These portfolios are held for 6 months, their monthly returns are recorded, and the same process is repeated again. So, minimum variance portfolios have ex-post monthly returns from early 1999 to the end of 2000. The means and variances of these portfolios are used to assess the performance of optimal diversification.

More specifically, we consider the following two investment universes: a portfolio invested only in hedge funds (AI only) and an equity-oriented portfolio invested in traditional equity indexes and equity-related alternative indexes (AI/TI). The return on Credit Swiss First Boston/Tremont indexes (Convertible arbitrage, Dedicated short bias, Emerging markets, Event driven, Fixed-income arbitrage, Global macro, Long/short equity, Managed futures, Market neutral) and S&P indexes (S&P 500 growth, S&P 500 value, S&P 400 mid-cap, S&P 600 small cap) are used as proxies for the performance of alternative and traditional investment styles, respectively.[10]

In the AI only investment universe, we find that the ex-post volatility of the minimum variance portfolio generated using implicit factor based estimation techniques is almost 3 times lower than that of a naively diversified equally-weighted portfolio, and almost 7 times lower than that of the value-weighted Global Tremont Index, such differences being both economically and statistically significant (see table 4). This indicates that optimal variance minimization can achieve lower portfolio volatility. Differences in mean returns, on the other hand, are not statistically significant (t-stat = .11 and .16, respectively), suggesting that the improvement in terms of risk control does not necessarily come at the cost of lower expected returns.

Table 4: Multi-Style Multi-Class Strategic Allocation : AI Only Universe. Ex-post mean return and volatility of the minimum variance portfolio on the period 1999-2000. Mean and standard deviation are expressed in percentage per year, and obtained from monthly data though a multiplicative factor of 12 and square-root of twelve, respectively.

	Mean Return	Std Deviation
Minimum Variance Portfolio	12.16%	1.57%
Equally Weighted Portfolio	9.13%	4.79%
Global Tremont Index	12.50%	9.95%%

[10] The equity-oriented portfolio is invested in S&P 500 growth, S&P 500 value, S&P 400 mid-cap and S&P 600 small cap for the traditional part, and in Tremont dedicated short bias, Tremont market neutral and Tremont long/short for the alternative part.

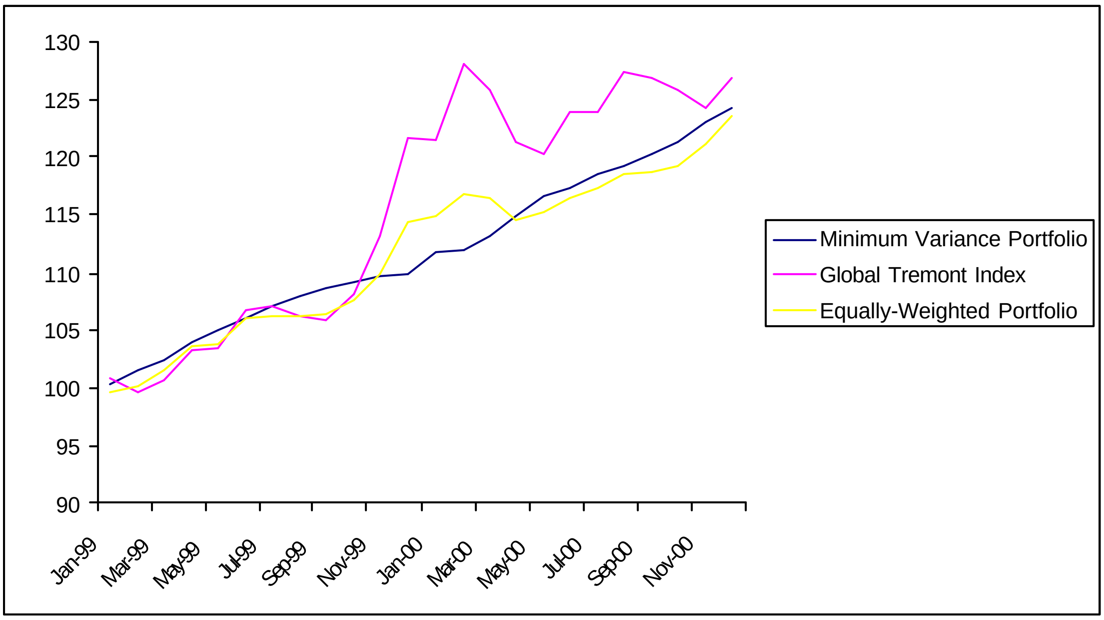
As an illustration, figure 3 displays the evolution of $100 invested in January 1999 in the Global Tremont Index, an equally-weighted portfolio of Tremont indexes and the minimum variance portfolio obtained from an implicit factor-based variance-covariance matrix estimator, where all factors with eigenvalues lower than $?_{max}$ are treated as noise. As can be seen from figure 3, the minimum variance portfolio has a much smoother path than its equally weighted and value-weighted counterparts.

Similar results are obtained in the AI/TI equity oriented universe. The ex-post volatility of the minimum variance portfolio generated using implicit factor based estimation techniques is almost 5 times lower than that of a naively diversified equally-weighted portfolio, and almost 9 times lower than that of the S&P 500 (table 5).

Table 5: Multi-Style Multi-Class Strategic Allocation : AI/TI Universe. Ex-post mean return and volatility of the minimum variance portfolio on the period 1999-2000. Mean and standard deviation are expressed in percentage per year, and obtained from monthly data though a multiplicative factor of 12 and square-root of twelve, respectively.

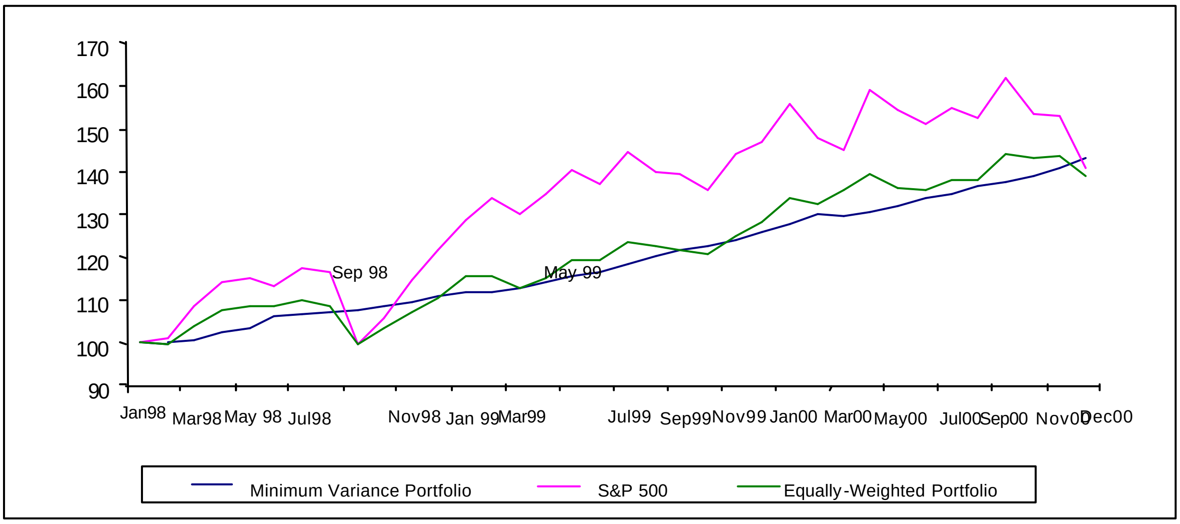
	Mean Return	**Std Deviation**
Minimum variance portfolio	12.46%	2.02%
Equally weighted portfolio	12.66%	9.62%
S&P 500	13.16%	17.67%

Figure 3. This graph displays the evolution of $100 invested in January 1999 in the Global Tremont Index, an equally-weighted portfolio of Tremont indexes and the minimum variance portfolio obtained from an implicit factor-based variance-covariance matrix estimator, where all factors with eigenvalues lower than $?_{max}$ are treated as noise.



A confirmation of these results can be found in the following figure, that displays the evolution of $100 invested in January 1998 in the S&P 500, an equally-weighted portfolio of traditional and alternative equity-oriented indexes, and the minimum variance portfolio.

Figure 4. Multi-Style Multi-Class Strategic Allocation : AI/TI Equity Universe. This graph displays the evolution of $1,000 invested in January 1997 in the S&P 500, an equally-weighted portfolio of S&P 500 growth, S&P 500 value, S&P 400 mid-cap and S&P 600 small-cap for the traditional part, and in Tremont dedicated short bias, Tremont market neutral and Tremont long/short, and the minimum variance portfolios obtained from an implicit factor-based variance-covariance matrix estimator,, where all factors with eigenvalues lower than $?_{max}$ are treated as noise.



2.1.1.2. Minimum VaR Approach

While it is a classic way to analyze and formalize the benefits of investing in hedge funds to note the improvement in the risk-return trade-off they allow when included in a traditional long-only stock and bond portfolio, we know that this trade-off can be expressed in terms of mean-variance analysis under suitable assumptions on investor preferences (quadratic preferences) or asset return distribution (normal returns). Recent research has emphasized the non-linear nature of hedge fund returns and suggested that attempts to reduce a portfolio volatility can lead to unfortunate increases in higher moments of the portfolio returns.

Most hedge fund managers follow dynamic investment strategies. This distinguishes them from the buy-and-hold type strategies often practised in traditional investment management. Moreover, the use of static or dynamic positions in derivatives and optional instruments reinforces the non-linear and dynamic character of alternative strategies (see Fung and Hsieh (1997a)). However, it is well-known that risk measures such as the beta or the Sharpe ratio do not allow the dynamic and non-linear dimensions of hedge fund risks to be accounted for (see for example Dybvig (1988a, 1988b), Leland (1999) or Lo (2001)).

Recent research has showed that the returns of alternative funds are clearly not Gaussian (see for example Brooks and Kat (2001)). In the case of portfolios that include derivative instruments, the assumption of Gaussian returns is not in fact tenable. Even if the return of the traditional asset classes were Gaussian, the return of funds using derivative instruments or dynamic strategies relating to those traditional classes would not be. Derivative instruments generally generate cash flows that are non-linear functions of the underlying asset value, and it is well known that a non-linear function of a Gaussian variable is not distributed in a Gaussian manner.

Investors generally display a non-trivial preference for the third and fourth order moments of return distribution (skewness and kurtosis), as is evidenced by the development of measures of extreme risk such as the Value-at-Risk (VaR).[11]

With that in mind, we present in what follows a pragmatic application of the VaR calculation in a fat tail distribution environment, along with its integration into a Mean-VaR optimisation process.[12] The Mean-VaR optimization method, such as introduced by Favre and Galinao (2000), first consists of calculating a VaR using a normal distribution formula and then a Cornish-Fisher expansion to take the skewness and kurtosis into account. Within the Gaussian framework, the VaR can be calculated explicitly by using the following formula:

$$P(dW \leq -VaR) = 1 - \alpha$$

$$VaR = n s W dt^{0.5}$$

where n = number of standard deviations at (1-α)
σ = annual standard deviation
W = current value of the portfolio
dt = fraction of the year

The analytical side of this normal VaR formula was then adjusted using the Cornish-Fisher extension (1937) as follows:

$$z = z_c + \frac{1}{6}(z_c^2 - 1)S + \frac{1}{24}(z_c^3 - 3z_c)K - \frac{1}{36}(2z_c^3 - 5z_c)S^2$$

where Zc = the critical value of the probability (1-α)
S = the skewness
K = the excess kurtosis (i.e. kurtosis minus 3)

The adjusted VaR is therefore equal to:

$$VaR = W(m - zs)$$

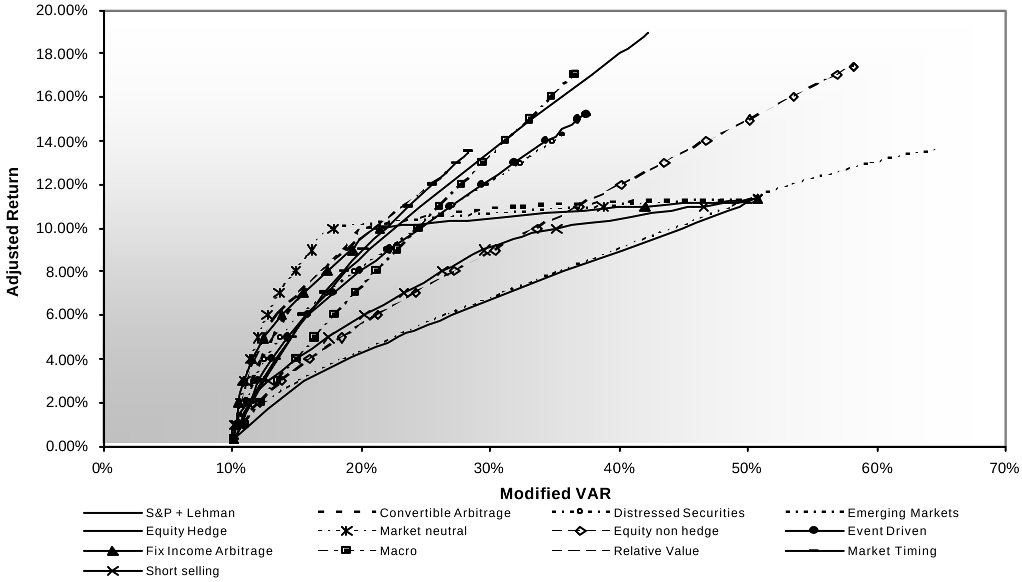
It should be noted that if the distribution is normal, S and K (represents the excess kurtosis in the formula) are equal to zero and consequently, z=Zc, and we come back to the Gaussian VaR.

We carried out an efficient frontier calculation in a mean-VaR space, where we use the VaR at a threshold of 99% integrating the Cornish-Fisher correction, allowing investors' aversion to the extreme risks related to alternative investment to be taken into account (see figure 5). These

[11] A significant amount of literature has actually commented widely on the limitations and necessary adaptations of the VaR for alternative investments (see for example Chung (2000)).
[12] Other authors have followed different avenues for extending the standard mean-variance analysis to a framework better suited for the case of hedge fund. For example, Indjic (2002) considers a strategic asset allocation problem where the objective is to minimise correlations between a hedge fund portfolio and a basket of equities, a problem solved by using a compact representation of the equity basket in terms of principal components analysis (PCA).

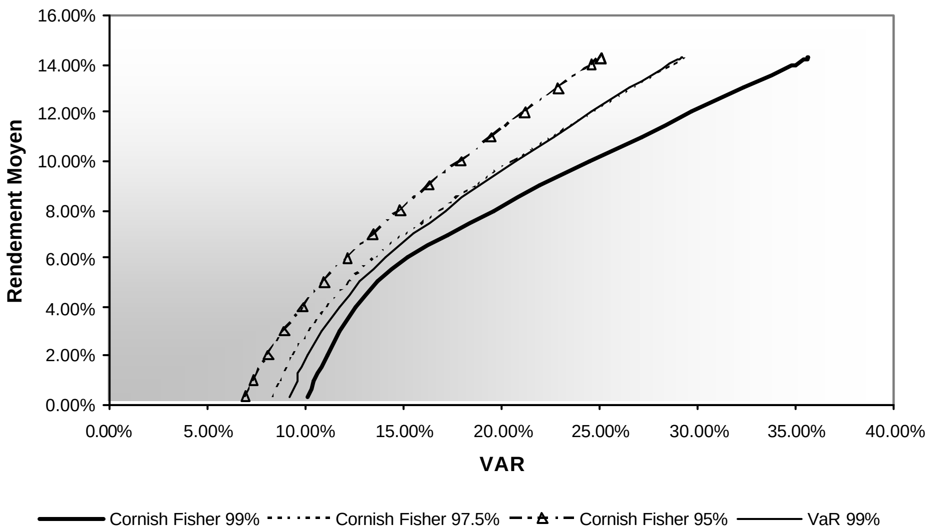
efficient frontiers were calculated from HFR style performances over the period February 1990 - March 2002. (Note that we could also have integrated the estimated survivorship bias for each alternative fund style, as explained in section 2.1.1.3.).

Figure 5. Modified (Cornish-Fisher) mean/VaR efficient frontiers using stocks (S&P 500), bonds (LBGBI) and hedge fund styles (HFR) (02/1990 – 03/2002)



Interestingly, we actually find that efficient frontiers obtained using a Gaussian parametric VaR without a Cornish-Fisher correction for a 99% threshold are very close to those obtained with a VaR adjusted according to the Cornish-Fisher extension, but at a 97.5% threshold. As an example, we consider in figure 6 below the case of "distressed securities" strategies. We can therefore consider that investors who only take first and second order moments into account greatly underestimate (a factor of 2.5) the extreme risk to which they are exposed.

Figure 6. Comparison of mean/VaR optimisations in the case of "distressed securities" type strategies (HFR) for the period (02/1990 – 03/2002)



2.1.1.3. Accounting for the Presence of Biases in Hedge Fund Databases

There is another specific problem when applying portfolio selection analysis to hedge funds, which is related to the need to account for the presence of many biases in hedge fund returns.

The decision to post the performance of hedge funds in one of the competing databases (TASS, MAR, HFR) is purely voluntary and only a certain number of funds decide to participate. This leads to a "self reporting bias". Since the funds that have refused to report to any of the databases are by definition unobservable, it is not possible to evaluate the impact of this bias. Some funds choose not to publish their performance because the performance does not appear satisfactory, others because they have already reached their critical size. It is therefore difficult to know whether this bias has a positive or negative impact on the performances announced.

Since hedge funds that have performed poorly leave the industry, the funds that are still present in a database tend to be funds that have performed better than the average of the population. In this case we speak of a "survivorship bias." Fung and Hsieh (2001) valued the average impact of this bias at 3.0%, compared to 2.6% for Park, Brown and Goetzmann (1999). The various databases are affected in different ways by this bias. For example, the TASS database has a higher survivorship bias than the HFR database because it has a default rate that is higher than HFR's.

Databases also have selection criteria that can be very different, and the data provided will not be representative of the same management universe. This is referred to as a "selection bias." For instance, HFR excludes managed futures from its databases while TASS and MAR take them into account. Most funds are present in one but not the other: of the 1,162 HFR funds and the 1,627 TASS funds, only 465 are common to both databases. 59% of the funds that are still in activity and 68% of the funds that no longer report to HFR are not part of the TASS database (cf. Liang (2001)). Fung and Hsieh (2001) valued the impact of this bias at 1.4 %, compared to 1.9 % for Park, Brown and Goetzmann (1999).

Out of the 465 funds in common between the HFR and TASS databases, only 154 (or 33.1%) have been included in both databases at the same time. However, when a fund is added to a database, all or part of its historical data is recorded ex-post in the database. Since it is in the funds' interest to display the most positive performance possible, it is probable that the mean performance displayed by the funds during their incubation period will be better than that of funds that have belonged to the corresponding database for a long time. In this case we talk about an "instant history bias." Fung and Hsieh (2001) valued the impact of this bias at 1.4 % per year. If the funds are not recorded at the same date in two different databases, it is probable that the two databases will not be exposed to "instant history bias" in the same way. This risk is heightened by the fact that only 47% of the performances recorded are strictly identical.

To test the impact of the biases, and in particular the survivorship bias, in an optimal selection approach for portfolios that include the alternative class, we have generated efficient frontiers that account for the presence of a survivorship bias.

We now describe the methodology we use to generate efficient frontiers in the presence of bias in performance reporting. We can firstly correct, as is classic in the literature, the estimation of mean returns to take account of survivorship bias. To do this, we initially estimate the historical mean return of the alternative class (sample mean). We then subtract a value that corresponds to the estimation of the survivorship bias carried out by Fung and Hsieh (2001), i.e. 3%, and Park, Brown and Goetzmann (1999), i.e. 2.6%. The variance-covariance matrix is also affected by the presence of survivorship bias. We must therefore, at a second stage, also correct the volatility of the performances of the alternative class. We propose to take this into account through the following model, blending a Poisson process with the return law.

To simplify, we consider a problem with 3 assets: a stock index, a bond index and an alternative fund. The presence of survivorship bias leads us to replace the return distribution of the alternative fund i R_i with $1_{\{t_i>T\}}R_i$, where t_i represents the uncertain eventual disappearance date of the fund and $1_{\{A\}}$ represents the indicator function of event A. Therefore, if the fund does not disappear before the investment's time horizon T, the investor has a return that we assume, for simplicity, to follow a classic normal distribution. However, if the fund disappears, the investment is suddenly reduced to 0. We complete the model by assuming that the disappearance date is distributed according to a Poisson process, independent from the return R_i, with an intensity denoted as l_i. Thus, the probability of survival of the fund over the period [0,T] is approximately given by $e^{-l_i T}$. This probability can be estimated historically. As shown by Liang (2001), the choice of database used to calculate this probability has a notable influence on the results obtained. Therefore, even though the HFR database has an annual default rate of only 2.17%, the default rate of the TASS database is 8.3%. Since for the moment there is no consensus in the literature on the default rate to be selected, it seems coherent to take the study carried out by Liang as a reference. We actually correct the volatility of the alternative class by deliberately making two relatively pessimistic assumptions on the default rate for hedge funds: 8.3% (i.e. estimation carried out by Liang (2001) with the TASS database) and 19% (i.e. the default rate observed by Liang (2001) for CTA).

We also assume that the investor has a quadratic utility. Therefore, while this distribution for the alternative fund is not Gaussian (convolution of a Gaussian and a Poisson process), the optimal decision rule only takes the variance of the assets' return distribution into account as a risk measure. In short, it involves calculating the variance of the return distribution for alternative funds exposed to survivorship risk, together with its covariance with the traditional asset classes.

To do that, we use the variance decomposition formula:

$$Var\left(1_{\{t_i>T\}}R_i\right)=E\left[Var\left(1_{\{t_i>T\}}R_i\right)\middle| t\right]+Var\left[E\left(1_{\{t_i>T\}}R_i\right)\middle| t\right]=e^{-l_i T}Var(R_i)+\left[e^{-l_i T}\left(E(R_i)\right)^2-e^{-2l_i T}\left(E(R_i)\right)^2\right]$$

and obtain $Var\left(1_{\{t_i>T\}}R_i\right)=e^{-l_i T}\left[E\left((R_i)^2\right)-\left(E(R_i)\right)^2\right]+\left[e^{-l_i T}\left(E(R_i)\right)^2-e^{-2l_i T}\left(E(R_i)\right)^2\right]$ or

$$Var\left(1_{\{t_i>T\}}R_i\right)=e^{-l_i T}E\left((R_i)^2\right)-e^{-2l_i T}\left(E(R_i)\right)^2 \qquad (1)$$

It is also necessary to consider terms of the following type:

$$Cov\left(1_{\{t_i>T\}}R_i, R_j\right)=E\left[Cov\left(1_{\{t_i>T\}}R_i, R_j\right)\middle| t_i\right]+Cov\left[E\left(1_{\{t_i>T\}}R_i\right)\middle| t_i, E(R_j)\middle| t_i\right]$$

where R_j represents the return of a traditional stock or bond class.

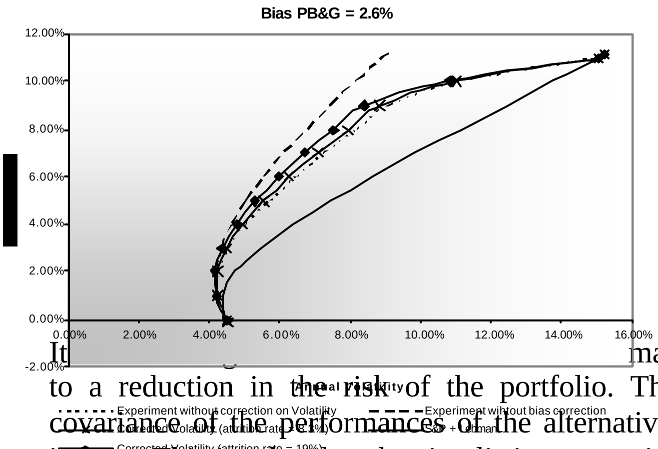
We finally obtain (since the second term of the decomposition is null)

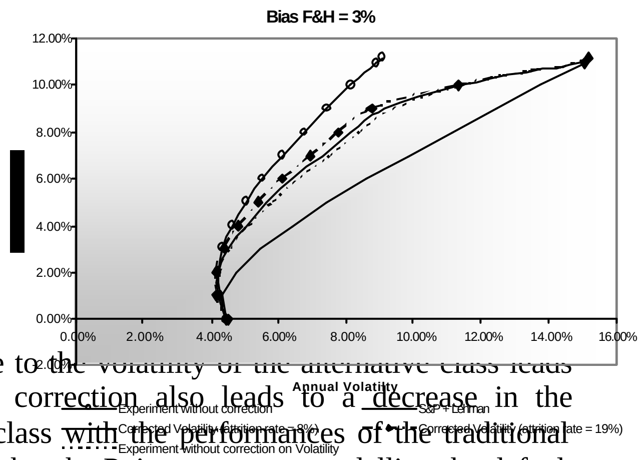
$$Cov\left(1_{\{t_i > T\}} R_i , R_j\right) = e^{-l_i T} Cov\left(R_i , R_j\right) \qquad (2)$$

We then consider a portfolio invested in 3 asset classes, 2 traditional (stocks and bonds) and 1 alternative, in such a way that only the alternative fund is exposed to the survivorship risk. We further assume that the variance-covariance matrix associated with the 3 asset classes, obtained by integrating the modifications brought about by the survivorship risk (equations (1) and (2)), denoted Ω, is non-singular. The efficient frontier associated with these 3 classes is defined as the location of the realisable portfolios with the smallest variance for a given expected return (or, in its dual formulation, as the set of realisable portfolios with the highest expected return for a given volatility).

The results that we obtain when we correct the return and the volatility of the alternative class for survivorship bias are presented in figures 7 and 8 below.

Figures 7 and 8. Efficient frontier analysis un the presence of survivorship bias.





It ... make to the volatility of the alternative class leads to a reduction in the risk of the portfolio. This correction also leads to a decrease in the covariance of the performances of the alternative class with the performances of the traditional indices. This is related to the simplistic assumption that the Poisson process modelling the default is independent from the return distributions of the traditional assets. Hence, adding a random variable that is independent from the returns of the asset prices leads to an increase in the diversification between the alternative class and the traditional class, and thus leads to a decrease in the total volatility of the portfolios. However, it is likely that the default process would not be independent from the returns on the stock and bond indices. We could imagine, for instance, that the risk of alternative funds defaulting is higher, at the very moment when stock markets are falling. In the absence of data on the dependence between the default risk and the return on traditional classes, it is nonetheless technically difficult to calibrate models taking that dependence into account. We therefore simply note that the model used tends to overestimate the diversification powers of the alternative class.

The consequence is therefore a slight improvement (shift towards the north-west) in the efficient frontier. This does not however compensate for the deterioration (i.e. shift towards the south-east) of the efficient frontier brought about by correcting the returns, regardless of the estimation chosen for the survivorship bias.

These results reveal lesser benefits from alternative fund diversification than those obtained in the framework of a classic mean-variance analysis without integrating the survivorship bias. The efficient frontiers that integrate the alternative class continue nonetheless to dominate the efficient frontier obtained from traditional classes only (stocks and bonds). Furthermore, it is particularly interesting to note that the alternative class allows the efficient frontier to be improved by reducing the global risk of the portfolio (i.e., "risk reducer" strategies).

2.1.2. When Estimates of Expected Returns are Available

In practice, investors and managers of funds of hedge funds generate estimates for hedge fund expected returns, or abnormal returns, from a mix of quantitative (improved estimators of expected returns) and qualitative analysis (due diligence). In this section, we describe a methodology that can be used by a sophisticated investor who has access to reliable, albeit imperfect, estimates of hedge fund alphas, We offer an explicit solution for the optimal allocation problem of a non-myopic investor with incomplete information who allocates wealth between a risk-free security, a passive portfolio and a set of hedge funds. This is based on the theory of stochastic control in a continuous -time setting with Baysian update (Kalman filter approach).

We also address an important question that investors in hedge funds often ask, which is where should they take the money they are planning to allocate to the hedge fund from. Optimization results often show that long-short funds tend to replace equity as these two investment styles are highly correlated. In this section, we formalize such intuition. In particular, we show that low beta hedge funds may serve as natural substitutes for a significant portion of an investor's risk-free asset holdings, while high betas hedge funds can be regarded as substitutes for a portion of equity holdings.

2.1.2.1. Optimal Allocation to Hedge Funds

We first recall some results obtained by Cvitanic et al. (2002), who consider a non-myopic investor with incomplete information allocates wealth between a risk-free security, a passive portfolio and a hedge fund.

Uncertainty about risky asset prices in the economy is represented by a standard filtered probability space on which is defined a 2-dimensional Brownian motion $W = \left(W^1, W^2\right)$. We assume that the investor can choose among three assets, a risk-free asset B_t, and two risky assets. The first of them has a price that we denote by P_t and we interpret it as a traditional long-only portfolio, e.g., the S&P500. The second security, whose price we denote by A_t, is a hedge fund. The dynamics of the price of these assets are given by

$$\begin{cases} \dfrac{dB_t}{B_t} = rdt \\ \dfrac{dP_t}{P_t} = \tilde{m}_p dt + s_p dW_t^1 \\ \dfrac{dA_t}{A_t} = \tilde{m}_A dt + s_1 dW_t^1 + s_2 dW_t^2 \end{cases}$$

where the invertible volatility matrix $s = \begin{pmatrix} s_p & 0 \\ s_1 & s_2 \end{pmatrix}$ as well as the interest rate r are assumed to be constant, for simplicity. Alternatively, we could rewrite the dynamics of the prices as depending in a single Brownian motion each, but the previous notation allows simpler representation and interpretation of the optimal investment strategy.[13]

In this setting, we consider a risk averse investor who has access to the three securities described above and who maximizes utility of final wealth, where preferences are assumed to be represented by a power utility with risk-aversion coefficient denoted by 1-a, where a<0 (a=0 corresponds to a logarithmic -myopic- utility).

We assume that the investor observes neither the constant mean returns vector nor the source of noise but observes the price processes.

Define the “risk premium” vector process $q = s^{-1} \cdot \begin{pmatrix} \tilde{m}_p - r \\ \tilde{m}_A - r \end{pmatrix} = \begin{pmatrix} \dfrac{\tilde{m}_p - r}{s_p} \\ \dfrac{-s_1}{s_p s_2}\left(\tilde{m}_p - r\right) + \dfrac{\tilde{m}_A - r}{s_2} \end{pmatrix}$.

Because investors do not have good estimates for expected returns, we assume that the vector of risk-premium has a normal prior distribution, independent of the Brownian motion W:

[13] We focus on lognormal processes, although the model can in principle be solved for more general processes with stochastically time-varying drift and volatilities.

$$q \xrightarrow{law} N\left(f = \begin{pmatrix} f_p = \dfrac{m_p - r}{s_p} \\ f_A = \dfrac{-s_1}{s_p s_2}\left(m_p - r\right) + \dfrac{m_A - r}{s_2} \end{pmatrix}, \Delta \begin{pmatrix} d_p & 0 \\ 0 & d_A \end{pmatrix} \right)$$

where m_p (respectively, m_A) is the mean estimate of the uncertain expected return on the traditional portfolio (respectively, on the hedge fund).

Here we assume that the priors are independent, as can be seen from the fact that the off-diagonal terms in the covariance matrix of priors on risk-premium vector are zero (see Cvitanic et al. (2003) for the general case of correlated priors).

One may further decompose the drift of the active portfolio into the sum of two elements, a normal return component and an abnormal return component, or $\tilde{m}_A = r + b\left(\tilde{m}_p - r\right) + \tilde{a}$, where $b = \dfrac{s_1 s_p}{s_p^2} = \dfrac{s_1}{s_p}$ is the covariance between the return on the traditional portfolio, and the hedge fund. Note also that $s_2 = \sqrt{s_A^2 - b s_p^2}$ is the residual, or specific, component in hedge fund volatility.

In this setup, Cvitanic et al. (2002) show that the optimal holdings in the traditional portfolio and the hedge fund can be expressed in the following form

$$p_A = \frac{a}{s_2^2\left(1 - a - adT\right)}$$

$$p_p = \frac{m_p}{s_p^2\left(1 - a - adT\right)} - b p_A$$

where T is the investor' time-horizon, and where $a = m_A - r - b\left(m_p - r\right)$, is the expected value of the abnormal return alpha of the hedge fund for the investor with incomplete information, i.e., the best estimate that an investor has about the hedge fund abnormal return.

As expected, an increase in the expected alpha leads the investor to hold more of the active portfolio, everything else equal. On the other hand, an increase in the uncertainty around alpha leads the investor to hold less (or short less) of the active portfolio, everything else equal. An increase in the time-horizon also leads the investor to hold less (or short less) of the active portfolio. On the other hand, when there is no uncertainty around alpha, the solution is time-horizon independent. Finally, an increase in the specific risk of the active portfolio leads the investor to hold less (or short less) of it, everything else equal.

2.1.2.2. Where should the Funds come from and where should they go?

An important question that investors in hedge funds often ask is where should they take the money they are planning to allocate to the hedge fund from.

In the context of the model presented in the previous section, we can give a quantitative answer to that question. First define the optimal holdings in the traditional portfolio and risk-free asset in the absence of the hedge fund as p_p^0 and p_B^0, respectivel.. The changes in holdings due to the introduction of the active portfolio are

$$\Delta p_P := p_p^0 - p_P = bp_A$$
$$\Delta p_B := p_B^0 - p_B = (1-b)p_A$$

From this, it is easy to see that, when the optimal holding in the hedge fund p_A is positive (i.e., when the perceived hedge fund abnormal return a is positive), we have that $\Delta p_B \leq \Delta p_P \Leftrightarrow b \leq \frac{1}{2}$.

We find that the introduction of the active fund leads investors to optimally withdraw an amount from the money market account larger than that taken out of the passive fund when the active fund has a beta lower than 1/2. Intuitively, this is because the active portfolio becomes less (more) comparable to the passive fund as its beta decreases (increases). In other words, this result suggests that low beta hedge funds may actually serve as natural substitutes for a significant portion of an investor's risk-free asset holdings, while high beta hedge funds can be regarded as substitutes for a portion of equity holdings.

Neither the prior on the expected return of the passive fund asset, nor volatility of that fund, have any impact on that decision. It should be noted that the condition $b \leq 1/2$ holds for most non-directional hedge fund strategies. This, on the other hand, would be relatively unusual for traditional long-only active strategies.

2.1.2.3. Empirical Analysis

We use a proprietary database of individual hedge fund managers, the CISDM (previously known as the Managed Account Reports or MAR database), to calibrate and test the model. The MAR database contains monthly returns on more than 1,500 offshore and onshore hedge funds and managers usually select their own categories. There are 9 categories ("medians"), some of which are divided into sub-categories ("submedians"): Event- Driven Median (Distressed securities sub-median, Risk arbitrage sub-median), Global Emerging Median, Global International Median, Global Established Median (Global Established growth sub-median, Global Established small-cap sub-median, Global Established value sub-median), Global Macro Median, Zurich Market Neutral Median (Market Neutral arbitrage sub-median, Market Neutral long/short sub-median, Market Neutral mortgage-backed sub-median), Sector Median, Short-Sellers Median, Fund of Funds Median (Fund of Funds diversified sub-median, Fund of Funds niche sub-median).

In this study, we focus on 581 hedge funds + 10 indices and sub-indices in the MAR database that have performance data as early as 1996. It should be noted that using a specific sample from an unobservable universe of hedge funds introduces biases in performance measurement. There are three main sources of difference between the performance of hedge funds in the data base and the performance of hedge funds in the population (see Fung and Hsieh (2001)): survivorship bias, selection bias, instant history bias. Survivorship bias occurs when unsuccessful managers leave the industry, and their successful counterparts remain, leading to the counting of only the successful managers in the database. Selection bias occurs if the hedge funds in the database are not representative of those in the universe. Besides, when a hedge fund enters into a vendor database, the fund history is generally backfilled. This gives rise to an instant history bias. Overall, it is probably a safe assumption to consider that these biases account for a total approaching at least 4.5\% annual (see Park, Brown and Goetzmann (1999) and Fung and Hsiesh (2001)).

More specifically, the explicit solution derived in section 2 allows us to quantify the relationship between the optimal allocation to hedge funds and managerial skill with uncertainty around this managerial skill. There are actually at least three reasons why the abnormal return, or $\tilde{a}$, generated by managers can not be known with certainty by investors.

- Model risk: for a given fund and a given sample, estimates around alpha vary with the model under consideration
- Sample risk: for a given fund, and a given model, estimates of alpha vary with the sample under consideration
- Selection risk: for a given model and a given sample, estimates of alpha vary with the fund under consideration

The first source of uncertainty around managerial skill is due to the fact that investors do not have a dogmatic belief in one particular model but instead are uncertain about the true model they should use to measure risk-adjusted performance. In that sense, uncertainty around managerial skill may be calibrated from the variation of performance measurement across models. The second source of uncertainty around managerial skill is estimation risk that affects both the passive and the active funds. The third source of uncertainty arises from the fund picking problem. Even if investors are ready to believe that there are fund managers who are able to generate positive alphas, they do not necessarily know which ones, and past risk-adjusted performance, while providing some guidance, is not enough to ensure that fund picking risk can be hedged away (see Brown, Goetzmann and Ibbotson (1999) or Agarwal and Naik (2000) for results on the persistence of hedge fund performance).

In the absence of meaningful estimates of the magnitude of selection risk, we focus on model and sample risks in this paper. Sample risk is measured in terms of usual parameter uncertainty, using t-stat values as a measure of dispersion around point estimates. Obtaining an estimate of model uncertainty, on the other hand, is less straightforward. In this paper, we use 5 different asset pricing models to compute a fund abnormal return and use the dispersion in alphas across models as a measure of model uncertainty. In other words, we use as a prior for the unknown alpha of a given fund an equally-weighted average of posterior estimates for that alpha from different competing models that have been used in the literature on hedge fund performance. These models are listed below.

1. CAPM. This is a standard version of Sharpe (1964) CAPM. We use the S&P 500 as a proxy for the market portfolio.

2. CAPM with stale prices. We adjust standard CAPM market beta by running regressions of returns on both contemporaneous and lagged market returns given that, in the presence of stale or managed prices, simple market model types of linear regressions may produce estimates of beta that are biased downward (Scholes and Williams (1977), Dimson (1979), Asness, Krail and Liew (2001)).

3. CAPM with non-linearities. Because hedge fund portfolios typically involve nonlinear and/or dynamic positions in standard asset classes, we also apply Leland (1999) performance measurement for situations in which the portfolio returns are highly nonlinear in the market return.

4. Explicit single-index factor model. We test a single-factor model, where the return on an equally-weighted portfolio of hedge funds in the same style category is used as a factor (we perform objective cluster-based classification, as opposed to rely on managers' self-proclaimed styles).

5. Explicit multi-index factor model. Building on an approach initiated by Sharpe (1964, 1988, 1992), or Fama and French (1992), we use market indices as proxies for true unknown factors. Since hedge fund returns exhibit non-linear option-like exposures to standard asset classes, traditional style analysis offer limited help in evaluating the performance of hedge funds (Fung and Hsieh (1997a, 2000)). A possible remedy has been suggested to try and capture such non-linear dependence is to include new regressors with non-linear exposure to standard asset classes to proxy dynamic trading strategies in a linear regression. Natural candidates for new regressors are buy-and-hold positions in derivatives (Schneeweis and Spurgin (2000), Agarwal and Naik (2001)), or hedge fund indices (Lhabitant (2001)). Here, we follow the latter approach and use the CSFB/Tremont indices which are currently the industry's only asset-weighted hedge fund indices.[14]

We also introduce a "method 0" alpha for each fund, which is simply the excess mean return. This is the common practice for hedge fund managers who claim the risk-free rate should be used as a benchmark, and receive incentive fees based on performance of their fund over the risk-free rate. Note that, while commonly used in practice, the mean excess return is a meaningful definition of alpha only under the two restrictive assumptions that CAPM is the true model and the beta of the fund is zero.

For the purpose of illustrating the model of asset allocation between a passive and an active fund, we apply these 5 models to hedge fund indices and individual hedge funds on the period 1996-2000. Because we cannot present results on as many as 581 funds, we focus on 10 indices. These indices represent the return on an equally-weighted portfolio of hedge funds pursuing respectively event driven strategies (with sub-categories distressed and risk), market neutral strategies (with sub-categories arbitrage and long/short), short-sales strategies, and fund of funds strategies (with sub-categories niche and diversified).

[14] Amenc and Martellini (2001) have introduced a set of "pure style indices" and tested their superior power in the context of style analysis. We do not, however, use these pure style indices because data is not available before 1998.

In table 6, we present the summary statistics for these indices (beta with respect to the S&P500, mean return, total volatility, systematic volatility and specific volatility), as well as for an average fund, which is a hypothetical fund exhibiting the average characteristics of all funds in the data base. The reader should be cautioned that this average fund cannot be regarded as an equally-weighted index of all funds in the data base. For example the 6.63% volatility of the so-called average fund is the average of all funds volatility, and not the volatility of a fund posting performance the equally-weighted average of the performance on each fund.

Table 6: Summary Statistics. This table displays in percentage terms the mean return and total, specific and systematic volatility on each index obtained from monthly data on the period 1996-2000 and converted into annual numbers. It also displays the beta of these indices with respect to the S&P500.

Strategy	Beta	Mean Return	Volat.	Systematic Risk	Specific Risk
Ev. Dist.	0.23	10.94	6.56	3.71	5.41
Ev. Risk	0.14	13.14	3.98	2.21	3.31
Ev. Driven	0.16	12.28	4.71	2.63	3.91
FoF Div.	0.24	12.31	6.26	3.83	4.95
FoF Niche	0.15	11.87	4.36	2.41	3.63
FoF	0.22	11.22	5.60	3.53	4.35
Mkt Neutr. Arb	0.06	16.62	10.58	0.90	10.54
Mkt Neutr. L/S	0.04	12.01	2.08	0.61	1.99
Mkt Neutr.	0.02	11.02	1.42	0.39	1.36
Short Sale	-0.91	6.37	20.71	14.62	14.68
Average	0.03	11.78	6.63	0.56	6.60

We check, for example, that betas for market neutral indices are very close to zero, while the beta on the short-sale index is negative, as it should be. This suggests that significant diversification benefit might be generated from the inclusion of that asset class in an equity portfolio.

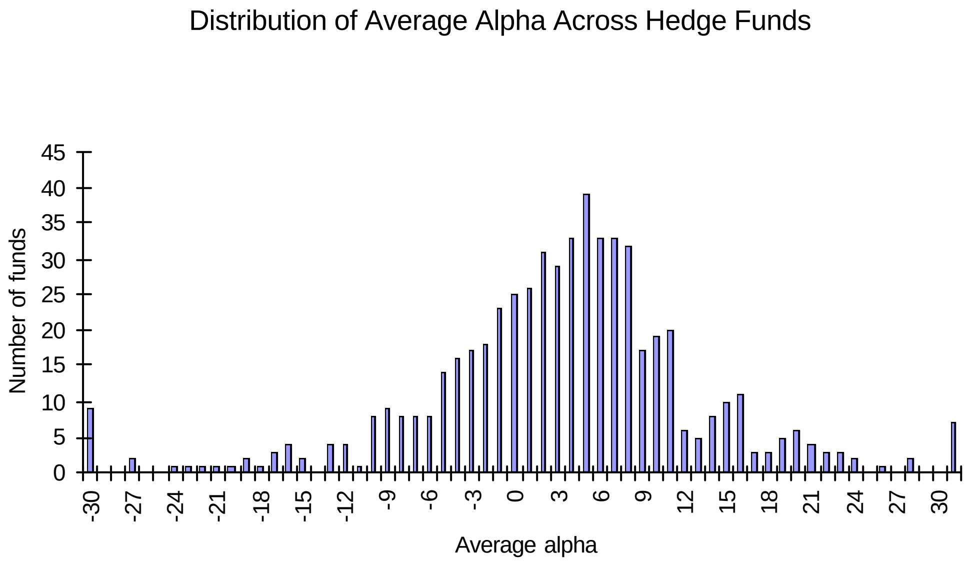
In table 7, we present the alpha on each index obtained using the various afore-mentioned models, as well as the mean and standard deviation of these alphas. The latter quantity can be regarded as a real-world empirical estimate of uncertainty about alpha driven by model risk. As can be seen from these numbers, model risk induces a significant amount of uncertainty around the estimate for alpha. That measure of uncertainty can be as high as 10% on this sample and for these indices, and higher than 30% for some funds in the database (see figure 9).

Table 7: Hedge Fund Abnormal Performances. This table displays in percentage terms the annual alpha on each index obtained using different asset pricing models. It also displays the average alpha across methods, as well as the dispersion (standard deviation). Method 0 is the excess mean return. Method 1 is the CAPM alpha. Method 2 is an extension of CAPM to account for the presence of stale prices. Method 3 is Leland (1999) modification on CAPM alpha. Methods 4 and 5 are, respectively, a single- and multi-index factor models.

Strategy	Meth 0	Meth 1	Meth 2	Meth 3	Meth 4	Meth 5	Mean Alpha	St. Dev. Alpha
Ev. Dist.	10.42	2.83	-0.68	2.20	1.53	-0.14	2.69	4.02
Ev. Risk	12.62	6.26	4.67	5.84	7.82	6.67	7.31	2.80
Ev. Driven	11.76	5.05	2.77	4.55	5.74	4.07	5.66	3.15
FoF Div.	11.80	4.10	0.93	3.73	-0.82	-2.01	2.96	4.96
FoF Niche	11.35	4.83	2.32	4.42	5.70	3.26	5.31	3.19
FoF	10.70	3.26	0.13	2.86	-0.20	-3.06	2.28	4.72
Mkt Neutr. Arb	16.10	10.82	9.66	10.47	7.16	12.04	11.04	2.96
Mkt Neutr. L/S	11.50	6.46	6.45	6.45	9.50	9.41	8.30	2.15
Mkt Neutr.	10.51	5.64	4.60	5.53	9.12	8.61	7.33	2.39
Short Sale	5.85	13.34	13.90	14.19	1.59	31.57	13.41	10.27
Average	11.26	6.26	4.48	6.02	4.72	7.04	6.63	2.47

To get a better insight about how these average alpha values for the indices compare to those obtained on the sample of funds, we compute the distribution of average alpha across all hedge funds in the database (see figure 9).

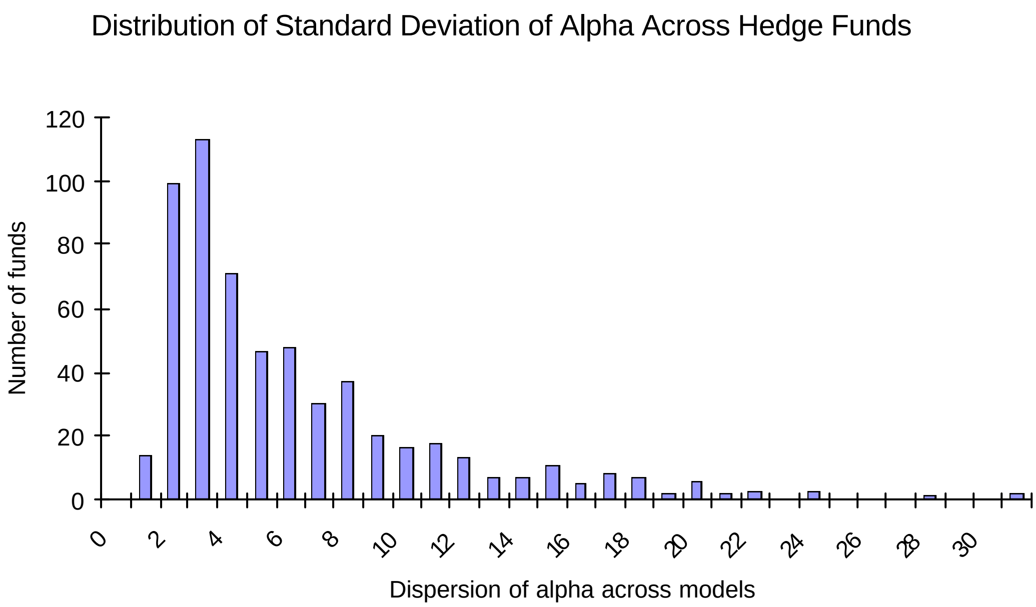
Figure 9: Cross-Section of Average Alphas. The mean of that distribution is 2.77%, the standard deviation is 11.13%.



The mean of that distribution is 2.77%, the standard deviation is 11.13%. (Note that we do not include here method 0; this is why the numbers are lower than in the following tables.) This seems to indicate that the average hedge fund is likely to generate positive risk-adjusted return, when the risk-adjustment is performed with an average of asset pricing models. The conclusion that hedge funds yield on average positive alpha needs, however, to be balanced by the presence of survivorship, selection and ins tant history biases, which account for a total approaching at least 4.5% annual, as recalled earlier. Therefore, the average alpha net of these biases is a negative -1.73% = 2.77% - 4.5% < 0. On the other hand, 261 (out of 581 hedge funds) have an average alpha across methods larger than 5%, which seems to indicate the presence of positive abnormal return for at least some funds in the sample, even after accounting for the presence of the biases.

In the same vein, we compute the distribution of standard de viation of alpha across the sample of hedge funds (figure 10). The mean of that distribution is 5.77%, the standard deviation is 5.33%. It should be noted that one fund has a dispersion of alpha across methods larger than 30%.

Figure 10: Cross-Section of Standard Deviation of Alphas. The mean of that distribution is 5.77%, the standard deviation is 5.33%.



In what follows, we perform an ex-post experiment, where we focus on model risk and assume away estimation risk. In other words, we address the following question: given ex-post alpha estimates for 1996-2000, what would have been the optimal allocation to hedge funds in the period, accounting for the fact that different models disagree on alpha estimates? We refer the reader to Cvitanic et al. (2003) for the impact of estimation risk.

While it is commonly assumed that a "standard" level of risk-aversion is captured by a value *1-a=3* or *4* and a conservative upper bound for the value of the risk-aversion parameter should be 10, it is well-known that values as high as 20 are needed to fit the equity premium (this is the *equity premium puzzle* from Mehra and Prescott (1985)). In this paper, we use $a=-15$ as a base case value; this is consistent with a (68.2%,31.8%) Merton (1973) allocation to the risk-free versus risky asset. We also perform some comparative static analysis on the risk aversion *a* parameter. We use the mean T-Bill rate on the period (5.06%) as an estimate of the risk-free rate *r*. The average return on the S&P500 on the period is 18.23%, and will be regarded as the value for the expected return on the traditional portfolio m_p, which we assume identical to the true population value for the moment ($d_p = 0$). The annual volatility on the S&P500 volatility is estimated at $s_p = 16.08\%$ from monthly data over the period 1996-2000. Finally, we arbitrarily fix the time-horizon at a $T=5$ value. Given the presence of lockup periods imposed by most hedge fund managers, it is hardly feasible in practice to invest in hedge funds over a very short horizon.

Based on the above estimates and assumptions on parameters, we compute in table 8 the base case optimal allocation to T-Bills, S&P500 and hedge funds such as predicted by the model. We also detail the origin of the funds invested in hedge funds.

Table 8: Optimal Portfolio Strategies - The Base Case. This table features the percentage holding in the passive portfolio (column 2), active portfolio (column 3) and the risk-free asset (column 5), as well as the relative holdings in the active versus passive portfolio (column 4). It also features the changes in holdings due to the introduction of the hedge fund (called delta in columns 6 and 7). We use the following set of base case parameter values: *r=5.06%;* m_p *=18.23%;* $d_p = 0$ (we assume away sample risk);*T=5; a= -15.*

Strategy	Holding in Passive	Holding in Active	Relative Holding A versus P	Holding in Risk-Free	Delta Passive	Delta Risk-Free Risk-Free
Ev. Dist.	28.14%	16.03%	36.29%	55.83%	3.70%	12.33%
Ev. Risk	18.62%	96.02%	83.76%	-14.64%	13.22%	82.80%
Ev. Driven	22.49%	57.13%	71.76%	20.38%	9.35%	47.78%
FoF Div.	28.70%	13.20%	31.51%	58.10%	3.14%	10.06%
FoF Niche	23.66%	54.54%	69.75%	21.80%	8.19%	46.35%
FoF	29.30%	11.56%	28.30%	59.13%	2.54%	9.02%
Mkt Neutr. Arb	29.31%	45.33%	60.73%	25.36%	2.53%	42.79%
Mkt Neutr. L/S	24.23%	202.21%	89.30%	-126.44%	7.61%	194.60%
Mkt Neutr.	27.91%	160.45%	85.18%	-88.35%	3.94%	156.51%
Short Sale	42.57%	11.80%	21.71%	45.62%	-10.73%	22.54%
Av. Fund	30.84%	28.64%	48.15%	40.51%	1.00%	27.64%

From the results in table 8, we find that the average hedge fund receives a 28.64% allocation. The strategies that exhibited spectacular performances, e.g., the market neutral long/short index posting an average return higher than 12% for a volatility around 2%, with a 8.30% average alpha and a low 2.15% dispersion around that value, receives a share of the portfolio that goes beyond the 100%. A rational investor would essentially want to borrow at the risk-free rate to invest in such a fund.

More generally, our analysis provides formal backing to the widely spread notion that investors should use hedge funds as substitutes for some portion of their risk-free asset holdings. The origin of the 28.64% now allocated to the average active portfolio is 27.64% from the risk-free asset versus 1.00% from the traditional portfolio (remember that the beta of the average fund is 0.03).

We then penalize the estimated expected return on all hedge funds by 4.5%, a reasonable estimate of effect of survivorship, selection and instant history biases, and obtain the following results (under the same base case parameter values).[15]

From the results in table 9, we find that the willingness of investors to hold hedge funds has severely declined as a result of expected returns being deflated by 4.5%. The holdings in the average hedge fund have decreased from 28.64% down to 9.20%. This is actually consistent with what most investors allocate to hedge funds. From informal conversations with various people in industry, it actually appears that most asset allocators would heuristically argue for a 10 to 20% allocation to hedge funds as a reasonable number Moreover, the optimal holdings in any hedge fund exhibiting an average alpha lower than 4.5% are now negative.

[15] In principle, the presence of biases not only affects expected return on hedge funds, but also beta and volatility estimate (see section 2.1.1.3.). For simplicity, however, we focus on the impact on the alpha estimate in this section.

Table 9: Optimal Portfolio Strategies - Impact of Biases. This table features the percentage holding in the passive portfolio (column 2), active portfolio (column 3) and the risk-free asset (column 5), as well as the relative holdings in the active versus passive portfolio (column 4). It also features the changes in holdings due to the introduction of the hedge fund (called delta in columns 6 and 7). We use the following set of base case parameter values: *r=5.06%;* m_p *=18.23%*; $d_p = 0$ (we assume away sample risk); *T=5; a= -15*. We penalize the estimated expected return on a hedge fund by 4.5%, a reasonable estimate of effect of survivorship, selection and instant history biases.

Strategy	Holding in Passive	Holding in Active	Relative Holding A versus P	Holding in Risk-Free	Delta Passive	Delta Risk-Free Risk-Free
Ev. Dist.	34.32%	-10.75%	-45.60%	76.43%	-2.48%	-8.27%
Ev. Risk	26.76%	36.94%	57.99%	36.31%	5.09%	31.85%
Ev. Driven	29.93%	11.67%	28.05%	58.40%	1.91%	9.76%
FoF Div.	33.49%	-6.90%	-25.96%	73.42%	-1.64%	-5.26%
FoF Niche	30.59%	8.34%	21.43%	61.07%	1.25%	7.09%
FoF	34.31%	-11.22%	-48.61%	76.91%	-2.46%	-8.76%
Mkt Neutr. Arb	30.34%	26.86%	46.96%	42.80%	1.50%	25.36%
Mkt Neutr. L/S	28.36%	92.52%	76.54%	-20.88%	3.48%	89.04%
Mkt Neutr.	30.32%	62.01%	67.16%	7.67%	1.52%	60.49%
Short Sale	38.97%	7.84%	16.75%	53.19%	-7.13%	14.97%
Av. Fund	31.52%	9.20%	22.59%	59.28%	0.32%	8.88%

In closing, we therefore argue that this model provides investors with reasonable estimates of optimal allocation to hedge funds in the presence of uncertainty over the fund's abnormal return. Other authors have focused on

2.2 Tactical Asset Allocation with Hedge Funds

There is now a consensus in empirical finance that expected asset returns, and also variances and covariances, are, to some extent, predictable. Pioneering work on the predictability of asset class returns in the U.S. market was carried out by Keim and Stambaugh (1986), Campbell (1987), Campbell and Shiller (1988), Fama and French (1989), and Ferson and Harvey (1991). More recently, some authors started to investigate this phenomenon on an international basis by studying the predictability of asset class returns in various national markets (see, for example, Bekaert and Hodrick (1992), Ferson and Harvey (1993, 1995), Harvey (1995), and Harasty and Roulet (2000)).

While there has been a significant amount of research on the predictability of traditional asset classes, and the implications in terms of tactical asset allocation strategies, very little is known about the predictability of returns emanating from alternative vehicles such as hedge funds. Also, and not surprisingly given the absence of academic evidence on the predictability of hedge fund returns, very little is known about the performance of tactical asset allocation strategies involving hedge funds. In particular, while the out-of-sample performance of optimal strategic asset allocation decisions based on alternative investment vehicles has recently been documented by Amenc and Martellini (2002), there is no such available evidence of the ability to generate superior risk-adjusted returns based on timing among various hedge fund styles.

In this section, we attempt to fill this gap by documenting evidence of predictability in hedge fund index returns. More specifically, we examine (lagged) multi-factor models for the return on nine hedge fund indexes. The factors are chosen to measure the many dimensions of financial risks: market risks (proxied by stock prices, interest rates, commodity prices), volatility risks (proxied by implicit volatilities from option prices), default risks (proxied by default spreads) and liquidity risks (proxied by trading volume). We show that a parsimonious set of models captures a very significant amount of predictability for most hedge fund styles. We also find that the benefits in terms of tactical style allocation portfolios are potentially very large. Even more spectacular results are obtained both for an equity-oriented portfolio mixing traditional and alternative investment vehicles, and for a fixed-income oriented portfolio mixing traditional and alternative investment vehicles. These results do not seem to be significantly affected by the presence of reasonably high transaction costs.

2.2.1. Evidence of Predictability in Hedge Fund Returns

To represent the alternative investment universe, we chose to use data from Credit Swiss First Boston/Tremont (CSFB/Tremont).

Given that we are searching for evidence of predictability in hedge fund returns with the goal of implementing a style allocation strategy, we attempt to find the best possible trade-off between quality of fit and robustness. With a focus on attempting to avoid the pitfalls of data snooping, we use the following methodology.

- Step 1: rather than trying to screen hundreds of variables through stepwise regression techniques, which usually leads to high in-sample R-squared but low out-of-sample R-squared (robustness problem), we instead choose to select a short list of 10 meaningful variables. These variables are selected on the basis on previous evidence of their ability to predict asset returns, or their natural influence on asset returns.

    - Yield on T-Bill 3-month rate. Fama (1981) and Fama and Schwert (1977) show that this variable is negatively correlated with future stock market returns. It serves as a proxy for expectations of future economic activity.
    - Dividend yield (proxied by the dividend yield on S&P stocks). It has been shown to be associated with slow mean reversion in stock returns across several economic cycles (Keim and Stambaugh (1986), Campbell and Shiller (1998), Fama and French (1998)). It serves as a proxy for time variation in the unobservable risk premium since a high dividend yield indicates that dividends have been discounted at a higher rate.
    - Default spread (proxied by changes in the monthly observations of the difference between the yield on long term Baa bonds and the yield on long term AAA bonds). This captures the effect of default premium. Default premiums track long-term business cycle conditions: higher during recessions, lower during expansions (Fama and French (1998)).
    - Term spread (proxied by monthly observations of the difference between the yield on 3-month Treasuries and 10-year Treasuries).
    - Implicit volatility (proxied by changes in the average of intra-month values of the VIX).[16]
    - Market volume (proxied by changes in the monthly market volume on then NYSE). The last three variables have been identified by Amenc, Curtis and Martellini (2001) and Schneeweis and Spurgin (1999) as important factors explaining hedge fund returns.
    - Oil price (closely related to short-term business cycles).[17]

[16] VIX, introduced by CBOE in 1993, measures the volatility of the U.S. equity market. It provides investors with up-to-the-minute market estimates of expected volatility by using real-time OEX index option bid/ask quotes. This index is calculated by taking a weighted average of the implied volatilities of eight OEX calls and puts. The chosen options have an average time to maturity of 30 days.

- US equity factor (proxied by the return on the S&P 500 index).
- World equity factor (proxied by the return on the MSCI World Index ex US).
- Currency factor (proxied by changes in the level of a volume-weighted exchange index of currencies versus US dollar).

- Step 2: we test for the sub-set of variables that allows for a good trade-off between quality of fit and robustness. Our methodology is as follows. We select the sub-set of variables that allows for at least 5% in-sample explanation power (quality of fit). We have tested not only for the explanation power of the raw variables Z_i, for $i=1,...,10$, but also for changes in the variables $Z_{i,t-1} - Z_{i,t-2}$, one-month lag $Z_{i,t-1}$, two-month lag $Z_{i,t-2}$, three-month lag $Z_{i,t-3}$, moving average $\frac{1}{3}Z_{i,t-1} + \frac{1}{3}Z_{i,t-2} + \frac{1}{3}Z_{i,t-3}$, as well as combinations of the above. The explanation power at this stage is simply measured in terms of in-sample R-squared of regressions of the nine CSFB/Tremont hedge fund indexes on a sub-set of permutations (1-month lag, 2-month lag, 3-month lag, changes in the variable, moving average) on one of the 10 aforementioned variables.

In addition to these variables, we also add the lagged return on each index as a potential regressor. If there is some evidence of positive or negative serial-correlation in the time-series of the index returns, then the lagged value of the index may indeed help in predicting current values. There are various ways of testing for the presence of serial correlation in a given time-series. In particular, serial correlation can be measured in terms of the *Hurst exponent*. The Hurst exponent is a convenient summary of statistics of persistence in time-series data that has been applied in broad areas of economics, finance and natural sciences. Here, we use the R/S method for estimating the Hurst Index of the time-series of returns on hedge fund indexes (see Peters (1991)).

As a result of this two-step process, we select, for each index, a very limited number of factors (2 or 3) that predict the return on that index most closely. It actually turns out that the same sub-set of a small number of variables (6 in this instance; see table 10) is used for all indexes.

[17] Data on crude oil price can be found at www.eia.doe.gov/emeu/cabs/chron.html.

Table 10: Predictive Model for the Tremont Hedge Fund Indexes. This table provides the information on which exact model is used for each hedge fund index, with beta parameter values and associated probabilities (in parenthesis). When a variable is not used in the model, we display the mention "NA".

	$Ri,_{t-1}$	$MA(S\&P)_{t-1}$	Oil_{t-1}	$\Delta 3m_{t-1}$	ΔVIX_{t-1}	Vol_{t-1}	$MA(MSCI)_{t-1}$
HF1	0.1768 (0.24)	0.2917 (0.006)	0.0015 (0.008)	-0.0174 (0.04)	NA	NA	NA
HF2	NA	NA	NA	NA	NA	NA	NA
HF3	0.1893 (0.14)	NA	0.0064 (0.002)	NA	NA	NA	-0.0001 (0.003)
HF4	NA	0.1980 (0.38)	0.0003 (2.10^{-5})	-0.0037 (0.012)	NA	NA	NA
HF5	0.3193 (0.015)	NA	0.0004 (0.017)	NA	NA	NA	NA
HF6	NA	0.2566 (6.10^{-4})	0.0008 (3.10^{-5})	NA	-0.0008 (0.004)	-3.10^{-8} (3.10^{-5})	NA
HF7	-02651 (0.06)	0.9590 (0.002)	0.0014 (0.044)	NA	NA	-6.10^{-8} (0.05)	NA
HF8	NA	NA	NA	NA	NA	NA	NA
HF9	NA	NA	NA	NA	NA	NA	NA

There are three out of nine indexes for which it has not appeared feasible to have a robust predictor model using the shortlist of variables defined above. In other words, we have not been able to identify a single set of variables that yields sufficient explanation power (R-squared greater than 5%), while allowing for significant robustness. These indexes are dedicated short bias, long-short equity and managed futures. On the other hand, for the other six remaining indexes, a significant explanation power can be obtained. This can be seen from the fact that most coefficients are significant at the 95% level.

We also perform out-of-sample testing of the model. The methodology is as follows. We calibrate the models displayed above using a rolling window of the previous 60 months, that is, we dynamically re-estimate the coefficients each month using the past 60 months of observation. The calibration period is January 1994 to December 1998, and the back-testing period is January 1999 to December 2000. Table 11 provides information on the performance of the predictive models for the Tremont hedge fund indexes. The first column contains the in-sample R-squared of the regression. The second column contains the hit ratios of the model, that is the percentage of time the predicted direction is valid, i.e., the index goes up (resp. down) when the model predicts it will go up (resp. down).

Table 11: In-Sample and Out-of-Sample Performance of the Predictive Models for the Tremont Hedge Fund Indexes. The first column contains the in-sample R-squared of the regression. The second column contains hit ratios for the models, that is the percentage of time predicted direction is valid, i.e., the index goes up (resp. down) when the model predicts it will go up (resp. down).

	R^2	*HR(1)*
Convertible Arbitrage	51.8%	87.5%
Dedicated Short Bias	NA	NA
Emerging Markets	25.1%	50.0%
Equity Market Neutral	14.8%	95.8%
Event Driven	15.7%	79.2%
Fixed-Income Arbitrage	53.4%	62.5%
Global Macro	22.0%	54.2%
Long-Short Equity	NA	NA
Managed Futures	NA	NA

As we can see from table 11, hit ratios are very high, all above 50% and one at 95.8%. We have found clear statistical evidence of predictability in hedge fund returns. We now attempt to test whether there is also economic significance in the predictability of hedge fund return by investigating the implications in terms of a tactical asset allocation model.

2.2.2. Implications for Tactical Style Allocation Decisions

Tactical asset allocation is a form of conditional asset allocation, which consists of re-balancing portfolios around long run asset weights depending on conditional information.

Given that optimal dynamic portfolio decision rules in the presence of predictable returns can be solved explicitly only under somewhat restrictive assumptions (see in particular Brennan, Schwartz and Lagnado (1997), Campbell and Viceira (1998) or Barberis (2000)), we choose instead to test the economic significance of return predictability in terms of over-performance of style allocation models in a static mean-variance framework.[18] We base our tactical style allocation strategies on the conditional expected returns obtained from the predictive models presented in the previous section. In the case of the 3 indexes for which no forecasting model could be calibrated, we simply use the unconditional expected return as a forecast of the expected return. This amounts to regressing the return on these indexes on a constant variable (which leads to a 0 R-squared). In principle, it would also be desirable to allow for the possibility that the state variables predict changes in risk. By limiting ourselves in ruling out the use of lagged variables to forecast covariances, portfolio choice can only be affected by improved conditional forecasts of changes in expected returns. The loss of generality involved, might, however, be limited. Campbell (1987), Harvey (1989) and Glosten, Jagannathan and Runkle (1993) have tested the ability of the state variables to predict risk, and have found only limited effects that are

[18] A promising avenue for future research would be to use the non-parametric approach recently introduced by Brandt (1999) and Ait-Sahalia and Brandt (2001)).

dominated by the impact of forecasts on expected returns. Besides, we expect covariance to be more stable in the case of hedge funds compared to unmanaged assets because managed funds typically have a target level of volatility (enforced via VaR or other limits).

In the same vein, correlations also tend to be more stable because dynamic hedge fund strategies adapt to changes in the market environment. For example, fixed income arbitrage managers will reduce leverage when equity index volatility goes up, since equity index volatility is usually associated with spread widening in credit markets.

We consider the following two optimization programs

$$\text{P1}: \ \underset{w_1,\dots,w_9}{\text{Max}} \ IR = \frac{E\left(R_p - R_{Bench} \mid F_{t-1}\right)}{\text{Var}\left(R_p - R_{Bench}\right)}$$

$$\text{P2}: \ \underset{w_1,\dots,w_9}{\text{Max}} \ E\left(R_p - R_{Bench} \mid F_{t-1}\right) \text{s.t} \sqrt{\text{Var}\left(R_p - R_{Bench}\right)} \leq \overline{TE}$$

where $R_p = \sum_{i=1}^{9} w_i R_i$, *i=1,...,9,* is a portfolio invested in each of the nine hedge fund indexes, R_{Bench} is the return on a benchmark portfolio defined as an equally-weighted portfolio invested in the 9 Tremont indexes[19] and F_{t-1} is the conditioning information, i.e., an information set containing the past values of the predictive factors. Given the multi-dimensional regression $R_{i,t} = Z'_{i,t-1} b_i + e_{i,t}$ fully described in section 2.3, the conditional expected return prediction is given by $Z'_{i,t-1} b_i$, and is used for implementing dynamic style allocation.

The first problem P1 consists of a simple maximization of the information ratio *IR*, defined as the ratio of the conditional expected excess return of the portfolio over the benchmark divided by the tracking error (i.e., the volatility of the difference between the portfolio and the benchmark) (see Grinold and Kahn (2000) for more on the information ratio). It is well known that maximizing the information ratio is often likely to generate strategies with very low tracking error (simply because the tracking error appears in the denominator of the information ratio, so that minimizing that quantity allows the information ratio to be increased significantly). We have therefore also considered another problem (P2), which consists of the maximization of the conditional excess return subject to a tracking error constraint. This allows us to test different tracking error targets. Note also that we impose the usual portfolio ($\sum_{i=1}^{9} w_i = 1$ and positivity *(* $w_i \geq 0$ *, for i=1,...,9)* constraints.

The testing of a tactical style allocation model is of predictive nature in essence: we perform out-of-sample testing of the performance of the model by using estimates for the hedge fund index returns obtained from the predictive models, the coefficients of which are dynamically re-estimated on a monthly basis using a 57-month rolling window. The calibration period is again

[19] We have also tested using the value-weighted Tremont Global Index as a benchmark and obtained similar results.

January 1994 to December 1998, while the back-testing period is January 1999 to December 2000.

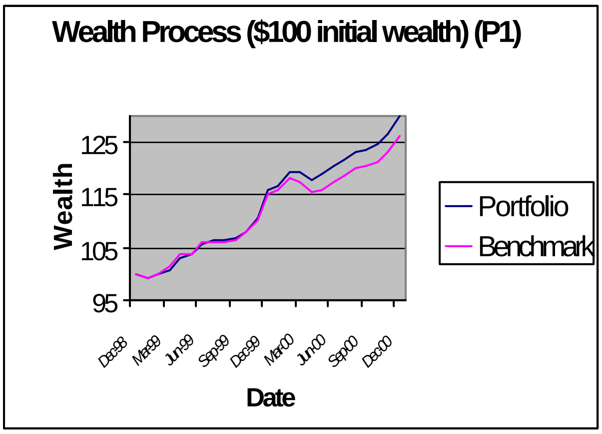
Table 12 provides a summary of the performance of both tactical style allocation models (P1 and P2). A hit ratio that is different from the one we computed before appears in line 5. This one is the percentage of time that the return on the tactical style allocation portfolio is greater than the return on the benchmark (we denote it as HR(2) to distinguish it from HR(1) which is the percentage of time that the predicted direction is valid). Mean and standard deviation are expressed in percentage per year, and obtained from monthly data through a multiplicative factor of 12 and square-root of twelve, respectively.

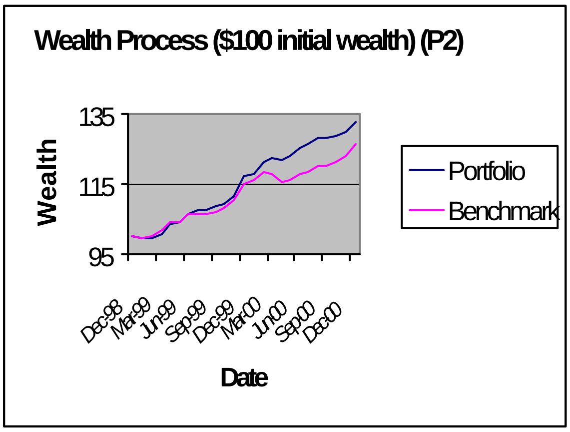
Table 12: Performance of the TSA Model for the Tremont Hedge Fund Indexes. This table provides a summary of the performance of both tactical style allocation models (P1 and P2). The Hit Ratio HR(2) is the percentage of time that the return on the tactical style allocation portfolio is greater than the return on the benchmark. Mean and standard deviation are expressed in percentage per year, and obtained from monthly data through a multiplicative factor of 12 and squared-root of twelve, respectively.

	$P1$	$P2\left(\overline{TE}=2\%\right)$
Annualized Return on Benchmark	11.78%	11.78%
Annualized Volatility on Benchmark	4.32%	4.32%
Annualized Return on TSA Portfolio	13.2%	14.48%
Annualized Volatility on TSA Portfolio	4.24%	4.12%
Hit Ratio *HR (2)*	83.3%	66.7%
Annualized Excess Return	1.4%	2.7%
Annualized Tracking Error	0.58%	1.8%
Information Ratio	2.44	1.5

In both cases, the ex-post information ratio obtained is very high (see Grinold and Kahn (2000), chapter 5, for a heuristic empirical distribution of information ratios among active portfolio managers). The two graphs below (see figures 11 and 12) display the evolution of $100 invested in the benchmark and the TSA portfolio (P1 and P2, respectively), from January 1998 to December 2000.

Figures 11 and 12: Performance of TSA Portfolios. The picture on the left-hand side displays the evolution of $100 invested in the benchmark and the TSA portfolio (problem P1) from January 1998 to December 2000. The picture on the right-hand side displays the evolution of $100 invested in the benchmark and the TSA portfolio (problem P2) from January 1998 to December 2000.





The benefits of alternative investments can be better understood when hedge funds are combined with traditional assets in a diversified portfolio. To test whether superior tactical style allocation can be achieved by considering a global investment universe mixing alternative and traditional style indexes, we perform the following experiments: we consider a style timing portfolio invested in S&P 500 growth, S&P 500 value and S&P 400 mid-cap for the traditional part, and in Tremont dedicated short bias, Tremont market neutral and Tremont long/short. We refer the reader to Amenc, El Bied and Martellini (2002) for an example of style timing in a fixed-income oriented universe.

In an equity-oriented universe, i.e., we try to implement a style timing portfolio invested in S&P 500 growth, S&P 500 value, and S&P 400 mid-cap for the traditional part, and in Tremont market neutral and Tremont long/short. We consider two benchmarks, one being the value-weighted portfolio (S&P 500), the other being an arbitrary strategic asset allocation (SAA) benchmark: 25% S&P growth, 25% S&P value, 20% S&P 400 mid-cap, 15% equity market neutral and 15% event driven. We use the same methodology as above to come up with predictive models for traditional investment styles.

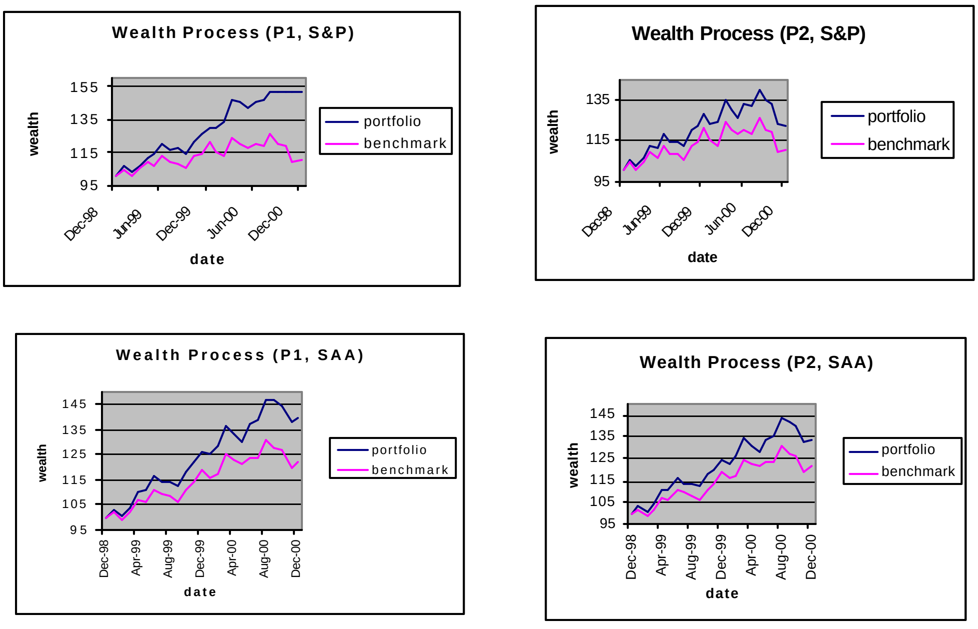
We then compute optimal portfolios using both maximization of the information ratio (program P1) and maximization of the excess expected return under tracking error constraint (program P2), and we test both the S&P benchmark and the strategic asset allocation benchmark. Table 13 summarizes the results.

Table 13: Performance of the TSA Model for the Equity Oriented Universe. This table provides a summary of the performance of both tactical style allocation models. The Hit Ratio HR(2) is the percentage of time that the return on the tactical style allocation portfolio is greater than the return on the benchmark. Mean and standard deviation are expressed in percentage per year, and obtained from monthly data through a multiplicative factor of 12 and squared-root of twelve, respectively. IR in column 5 stands for information ratio; it is the ratio of the annual excess return (column 30 over the annual tracking error (column 4). One benchmark is the S&P 500, the other is an arbitrary strategic asset allocation (SAA) benchmark: 25% S&P growth, 25% S&P value, 20% S&P 400 mid-cap, 15% equity market neutral and 15% event driven.

	$HR\,(2)$	Annual Excess Return	Annual TE	IR
S & P 500; Max IR	66.7%	15.6%	9.3%	1.68
S & P 500; Max $E\left(R_p - R_{Bench} \mid F_{t-1}\right)$s.t $TE \le 2\%$	62.5%	4.9%	3.8%	1.3
SAA; Max IR	62.5	7.0%	4.2%	1.66
SAA; Max $E\left(R_p - R_{Bench} \mid F_{t-1}\right)$s.t $TE \le 2\%$	66.7%	4.7%	3.4%	1.41

The four graphs below (see figures 13 and 16) display the evolution of \$100 invested in the benchmark and the TSA portfolio (P1 and P2, respectively), from January 1998 to December 2000.

Figure 13 to 16: Performance of TSA Portfolios in an Equity-Oriented Universe. The picture on the top left-hand side displays the evolution of \$100 invested in the S&P 500 and the TSA portfolio (program P1) from January 1998 to December 2000. The picture on the top right-hand side displays the evolution of \$100 invested in the S&P 500 and the TSA portfolio (program P2) from January 1998 to December 2000. The picture on the bottom left-hand side displays the evolution of \$100 invested in the SAA benchmark and the TSA portfolio (program P1) from January 1998 to December 2000. The picture on the bottom right-hand side displays the evolution of \$100 invested in the SAA benchmark and the TSA portfolio (program P2) from January 1998 to December 2000.



One might have wondered in the past whether documenting the predictability of hedge fund returns could be regarded as anything but a purely academic exercise. Obviously, some specific features of hedge fund investing do not facilitate the implementation of tactical allocation strategies. In particular, the absence of liquidity and the presence of lockup periods typical to investments in hedge funds are likely to prevent investors from implementing any kind of dynamic allocation among funds. In an attempt to estimate the actual percentage of funds applying strict investment constraints, we have decided to analyze the current conditions of access to funds by investors. The results of this analysis are striking: we have listed a relatively high total of 714 out of 2000 hedge funds covered by Altvest[20] that claim to offer monthly liquidity and no lockup period as of July 2002.[21] It should be noted, however, that drastic conditions of minimum assets under management apply to individual funds or funds of funds, which still impose binding constraints to investors willing to re-allocate a large fraction of their wealth among various alternative investment vehicles. In that context, the results presented in this paper can perhaps be regarded as most relevant for multi-strategy managers, who can freely dynamically allocate funds among various accounts of funds.

We actually have reasons to believe that the future of hedge fund style timing is even brighter than its past or present. The hedge fund industry is still relatively new, and market conditions are evolving at a frightening pace. While in its infancy the world of alternative investment strategies consisted of a disparate set of managers following very specific strategies, significant attempts at structuring the markets have occurred over the last decade. Following an early lead by CSFB/Tremont, we have also witnessed the creation of investment products designed to track the performance of hedge fund indexes (see Amenc and Martellini (2001) for an overview of existing hedge fund indexes and evidence of the contrasted view they offer). In particular, Zurich Capital Markets has launched in 2001 a series of hedge fund indexes that consist of equally weighted portfolios of funds that satisfy a number of qualitative criteria for institutional investment as well as a statistical classification procedure for style classification, under the supervision of an independent advisory board.[22] Investable portfolios, i.e., replicating portfolios with an approximate 2.5% tracking error, are available for each of these 5 indexes with monthly liquidity provided by Zurich Capital Markets, and no lockup period imposed to investors. In a similar spirit, S&P, MSCI and HSBC Republic Investments expect to launch index funds of funds with a high level of liquidity in the very near future.

3. Hedge Funds as Portable Alpha Vehicles

Given that most active managers still have dominant passive exposure to their benchmark, instead of paying high fees on the passively managed part of their portfolio, the core-satellite

[20] Altvest is one of the main hedge fund databases for managers with at least 1 year track record and $2 million of assets under management).

[21] There is evidence that the trend is towards more, and not less, hedge funds imposing lockup periods, as more and more investors seek to invest in a small number of funds. In this context, it is generally new funds that offer accommodating conditions to investors, while the most successful ones can afford to impose strict redemption rules and lockup periods.

[22] The five hedge fund strategies currently covered are Convertible Arbitrage, Merger Arbitrage, Distressed Securities, Event Driven and Hedged Equity. These indexes differ from existing hedge fund indexes by focusing only on those funds/managers that are 1) strategy pure in their style 2) have a two-year minimum performance track record and 3) sufficient assets under management to demonstrate organizational and managerial infrastructure, scalable strategies and the ability to raise funds from sophisticated investors.

approach suggests to passively invest in a low-fee index fund (or an enhanced index product) as a core portfolio and in a variety of satellite active managers with higher tracking error.

3.1. The Core/Satellite Approach

In this section, we review the basics and mathematics of an efficient core/portfolio approach. Core and satellite portfolio construction is recognized as an effective strategy for institutions that want to diversify their portfolios without giving up the potential for higher returns generated by selected active management strategies. It also provides the framework for targeting and controlling those areas where investors are willing to take more risk in a cost-efficient manner.

Most active managers still have dominant passive exposure to their benchmark. Instead of paying high fees on the passively managed part of their portfolio, the core-satellite approach suggests to passively invest in a low-fee index fund (or an enhanced index product) as a core portfolio and in a variety of satellite active managers with higher tracking error. In the limit, investors may want to invest in market-neutral managers who provide only portable alpha benefits without passive exposure to the index, so that they only compensate active managers for their abnormal returns, not for their passive exposure to rewarded sources of risk.

Let us first consider a core-satellite approach with a single satellite portfolio. The mathematics of a core-satellite approach is then straightforward. The overall portfolio $P = wS + (1-w)C$, where *w* is the fraction invested in the satellite (S), and *1-w* is the fraction invested in the core (C). We now calculate the tracking error with respect to a benchmark B. We have $P - B = wS + (1-w)C - B = w(S-B) + (1-w)(C-B)$. If we now assume that the core portfolio is perfectly replicating the benchmark, we have *C=B*, then we have $P - B = w(S-B)$. As a result, $TE(P) = \sqrt{\mathrm{var}(P-B)} = w\sqrt{\mathrm{var}(S-B)} = wTE(S)$.

Let us consider the following example. We assume an investor has a target level of risk relative to a given benchmark, such as a 2.5% tracking error budget. Two options are possible. Either the investor hires one manager with a tracking error equal to 2.5% for the entire portfolio, or the investor forms a passive core portfolio and leave 20% in an aggressively managed satellite with a $12.5\% = \frac{TE(P)}{w} = \frac{2.5\%}{20\%}$ tracking error. The latter solution is more cost-efficient, as 80% of the portfolio will be passively managed in the framwork of a low-cost indexing strategy. The next step consists in deriving the optimal proportion *w** to invest in satellite versus core portfolio. We solve the problem in the context of a simple mean-variance analysis. The optimization program reads $U = E(P-B) - l\,s^2(P-B) = IR(P) \times TE(P) - l\,TE^2(P)$, where *IR(P)* is the ijnformation ratio of the portfolio P with respect to the benchmark, i.e., $IR(P) = \frac{E(P-B)}{s\,(P-B)} = \frac{E(P-B)}{TE(P)}$.

One can actually show the following proposition. If the core portfolio perfectly replicates the benchmark, then the information ratio of the overall portfolio *IR(P)* is actually independent of the proportion in core versus satellite and equal to the information ratio of the satellite portfolio *IR(S)* (as long as that proportion *1-w>0*). To see this, just note that

$$IR(P) = \frac{E(wS + (1-w)C - B)}{s\,(wS + (1-w)C - B)} = \frac{wE(S-B)}{wTE(S)} = IR(S)$$

We may rewrite the optimization program as $U(w) = IR \times w \times TE(S) - l w^2 TE^2(S)$, and the first-order condition reads $\frac{\partial U}{\partial w}\left(w^*\right) = 0 \Rightarrow w^* = \frac{IR}{2\,l\,TE(S)}$.

For example, let us assume that the tracking error of the active fund is 5%, that the Information Ratio (*IR*) is 0.5, and that the coefficient of risk-aversion with respect to relative risk is $l = 0.2$. Then, the he optimal proportion invested in the active portfolio is

$$w^* = \frac{IR}{2\,l\,TE(S)} = \frac{.5}{2 \times .2 \times 5\%} = 25\%$$

The resulting tracking error is $TE(P) = 25\% \times 5\% = 1.25\%$.

The next step is to extend the analysis to the case of a satellite $S = \sum_{i=1}^{n} w_i S_i$ invested in a number n of active portfolio managers S_i according to the proportions w_i. The excess return on the satelitte portfolio is then $S - B = \sum_{i=1}^{n} w_i (S_i - B)$, and the tracking error of the satellite portfolio reads $TE(S) = \left(\sum_{i,j=1}^{N} w_i w_j s_{ij} - 2\sum_{i=1}^{N} w_i s_{iB} + s_B^2 \right)^{1/2}$, where s_{ij} is the covariance between portfolio managers S_i and S_j, and s_B is the volatility of the benchmark.

One can then find the optimal fraction invested in each active manager within the satellite portfolio so as to achieve the higest possible Information Ratio. One can show (see for example Scherer (2002)) that the optimal condition is that the ratio of return to risk contribution is the same for all managers, which reads

$$\frac{w_k a_k}{\left(w_k^2 s_{a_k}^2 + \sum_j w_k w_j s_{kj} \right) \Big/ TE(S)} = \frac{w_l a_l}{\left(w_l^2 s_{a_l}^2 + \sum_j w_l w_j s_{kj} \right) \Big/ TE(S)}$$

As a conclusion, a satellite/core approach seems to be perfectly suited for investors who attempt to use hedge funds to add portable alpha benefits to their long-only portfolio without modifying their passive exposure to a reference index, as it allows for a separate control on the tracking error of the satellite and core portfolios, so as to ensure that the overall portfolio is consistent with a target level of deviation with respect to the chosen benchmark.

3.2. Example

One natural version of the satellite portfolio approach specifies that investors should focus on market-neutral managers who provide only portable alpha benefits without passive exposure to the index.

In this section, we introduce a new form of active portfolio strategy, based on style timing, as opposed to stock picking. It is based upon a sophisticated and robust dynamic econometric approach to forecast S&P style returns and turn bets into portfolio decisions while controlling for the level of tracking error. We also show how such a strategy perfectly fits in the context of a multi-style core/satellite portfolio construction approach.

3.2.1. Tactical Style Allocation – A New Form of Market Neutral Strategy

Superior performance can be generated by timing or security selection decisions (see exhibit 1 for a classification of active investment strategies).

Market Timing and Stock Picking

- Stock returns can be decomposed into a systematic and a specific component (Sharpe's (1963) market model)

$$R_{i,t} - r_{f,t} = \underbrace{b_i\left[R_{M,t} - r_{f,t}\right]}_{\text{systematic}} + \underbrace{e_{i,t}}_{\text{specific}}$$

- Two forms of active strategies
  - *Market timing:* aims at exploiting predictability in systematic return
  - *Stock picking:* aims at exploiting predictability in specific return
- Academic evidence
  - There is ample evidence of predictability in systematic component (Keim and Stambaugh (1986), Campbell (1987), Campbell and Shiller (1988), Fama and French (1989), Ferson and Harvey (1991), etc.)
  - There is little evidence of predictability in specific component (more noïsy) in the absence of private information

Exhibit 1

Practitioners have recognized the potential of market timing, and started to engage in tactical asset allocation strategies as early as the 1970s. Tactical Asset Allocation strategies were traditionally concerned with allocating wealth between two asset classes, typically shifting between stocks and bonds. This severely limits the potential benefits of active allocation in a context of bear equity markets. Recently, more complex *style* timing strategies have been successfully tested and implemented.

Today, the notion of equity style management is widely accepted in the investment community, and the concept of equity styles permeates the way most professional investors think about the stock markets. In the academic community, it has also been recognized (Fama and French (1992)) that Sharpe's CAPM (1964) needs to be extended to account for the presence of other pervasive risk factors, i.e., size and book-to-market factors (see exhibit 2).

TAA versus TSA

- Extension of the market model to account for size and book-to-market factors (Fama and French (1992))

$$R_{i,t} - r_{f,t} = \underbrace{b_{i,M}\left[R_{M,t} - r_{f,t}\right]}_{\text{systematic - market}}$$
$$+ \underbrace{b_{i,B/M}\left[R_{B/M,t} - r_{f,t}\right] + b_{i,size}\left[R_{size,t} - r_{f,t}\right]}_{\text{systematic - style}} + \underbrace{e_{i,t}}_{\text{specific}}$$

- Three forms of active strategies
  - *Tactical Asset Allocation:* exploits evidence of predictability in market factor
  - *Tactical Style Allocation:* exploits evidence of predictability in style factors
  - *Stock picking:* exploits evidence of predictability in specific risk

Exhibit 2

A new classification of active portfolio strategies can be developed to account for the presence of style factors in equity returns (see table 14).

Table 14: Classification of Active Portfolio Strategies

	Systematic - market	Systematic - style	Specific
Form of active strategy	***Tactical Asset Allocation***	***Tactical Style Allocation***	***Stock picking***
Mutual fund – stock picking	X (discretionary)	X (discretionary)	X
Hedge fund – stock picking long short	X	X (discretionary)	X
Hedge fund – stock picking equity market neutral	0	X (discretionary)	X
Mutual fund – market timing	X (discretionary or systematic)	0	0
TSA – market neutral	0	X (systematic)	0

The presence of a X (resp. a 0) indicates that the fund manager uses (resp. does not use) the specified form of strategy

This detailed analysis of active portfolio strategies naturally leads to the introduction of a new form of equity market neutral strategies, based on style timing, as opposed to stock picking. It should be noted that this is not an entirely new approach.

On the one hand, most mutual fund managers actually make discretionary, and sometimes unintended, bets on styles as much as they make bets on stocks. In other words, they perform TAA, TSA and stock picking at the same time in a somewhat confusing “mélange des genres”.

On the other hand, recent proprietary research has emphasized the benefits of active strategies that only focus on systematic tactical style timing (see for example Amenc, El Bied and Martellini (2002) and Amenc, Malaise, Martellini and Sfeir (2002)).

In what follows, we summarize the findings in Amenc, Malaise, Martellini and Sfeir (2002). They calibrate forecasting models on style differentials:

- S&P Growth – S&P500
- S&P Value – S&P500
- S&P 400 (Mid Cap)
- S&P 600 (Small Cap) – S&P500

Their approach is based upon standard proven econometrics and implemented using dedicated statistical packages (SAS and Eviews) according to the following multi-step process.

Step 1: Setting Up the Database

To forecast style (differential) returns, they screen a universe of meaningful variables. These variables are chosen on the basis of the previous evidence of their ability to predict asset returns, as well as their natural influence on asset returns.

Step 2: Selecting the Variables

They select for each style a limited set of variables with significant predictive power. Predictive power is measured in-sample in terms of Schwartz Information Criterion, and out-of-sample in terms of hit ratio, i.e. percentage of times when predictive sign of differential is accurate.

Step 3: Building the Model

For each style, they select a preferred model while maintaining a set of competing models that they use as benchmarks and potential alternative statistical criteria to penalize for degrees of freedom and test various combinations of the short-list of predictors chosen in step 2 on an in-sample and out-of-sample basis.

Step 4: Improving the Model

Their models are based on multi-variable regression analysis, and complemented with state-of-the-art econometrical techniques:

- Tests of heteroskedasticity: They use White’s test to detect the presence of heteroskedasticity. The correction for heteroskedasticity involves weighted (or generalized) least squares. More specifically, they use White‘s correction, except in the presence of residual auto-correlation where they rather use Newey and West‘s corection.

- Tests of auto-correlation in the regression residuals (Ljung-Box Q-statistics and Breusch-Godfrey Lagrange Multiplier test). In the presence of serial correlation in the residuals, they perform a regression analysis with ARMA disturbance.

- Tests of seasonality. There is some evidence of seasonality in style differentials at the monthly and quarterly levels. Small cap stocks tend to outperform in January. For large-cap stocks, the outperformance of value stocks occurs mostly in January and the underperformance of value stocks occurs throughout the entire fourth quarter, with November being most significant. They systematically test for the presence of seasonality in style differential returns and account for it in our models when necessary.

- Tests of co-integration (Dickey-Fuller and Philipps-Perron for the presence of unit roots, Engle-Granger for the presence of co-integration). In the presence of co-integration between style differential and a predictive variable, an error correction model is implemented.

Step 5: Checking the Robustness Improving the Model

- Checking the robustness of the model through time. They use a Chow test to test for stability of regression coefficients between two periods. When they find significant evidence of parameter unstability, they use a Kalman filter analysis, which is a general form of a linear model with dynamic parameters, where priors on model parameters are recursively updated in reaction to new information.

- Checking the robustness of the linear specification. They also estimate the probability of a positive/negative sign differential through a logit regression. This allows us to increase the level of confidence in the model.

- Checking the robustness of the distributional assumption. They perform a Jarque-Bera test for evidence of non-normality in the residuals. When appropriate, they use bootstrapping as a non-parametric way of estimating confidence intervals.

Step 6: Updating the Model

A model is regarded as satisfactory as long as the coefficients remain significant and hit ratios are good. Decisions of updating the model are triggered by two (one) consecutive months with (strongly) decreasing t-stats and/or t-stat below a reasonable confidence level, and/or three consecutive errors on predicted sign of style differential.

When one of these events happens, they take this an indication of a paradigm shift. They then re-do the whole analysis, and select the new best performing models in these new market conditions.

3.2.2. Performance

In this section, we show how these econometric bets can be turned into market-neutral portfolio decisions, and also how the resulting portfolio can be optimally added to a traditional portfolio in a core/satellite approach.

For that, we use an otpimizer and maximize expected returns subject to a tracking error constraint with respect to the S&P500. Given that the TSA portfolio is meant to be included within a larger portfolio, with the remaining fraction passively invested in the S&P500, we feel comfortable with a relatively high level of tracking error fixed at 15% ex-ante. In our experiment, the realized (i.e., ex-post) tracking error on the optimal TSA portfolio actually turned out to be 12.44%.

3.2.2.1. Performance of the TSA Strategy as a Stand-Alone Portfolio

We first provide in table 15 a summary of risk-return measures on the TSA portfolio over the period.

Table 15: Summary of risk-return measures on the TSA portfolio from January 1999 to June 2002

Annual Average Return	14.15%
Annual Std dev	14.66%
Annual Variance	2.15%
Annual Semi variance	0.54%
Sharpe Ratio*	0.69
Information Ratio	1.59
Hit Ratio**	67%

* We have used a 4% Risk Free Rate for the computation of the Sharpe Ratio
** Hit Ratio accounts for the percentage of times that the TSA Portfolio beats the S&P 500

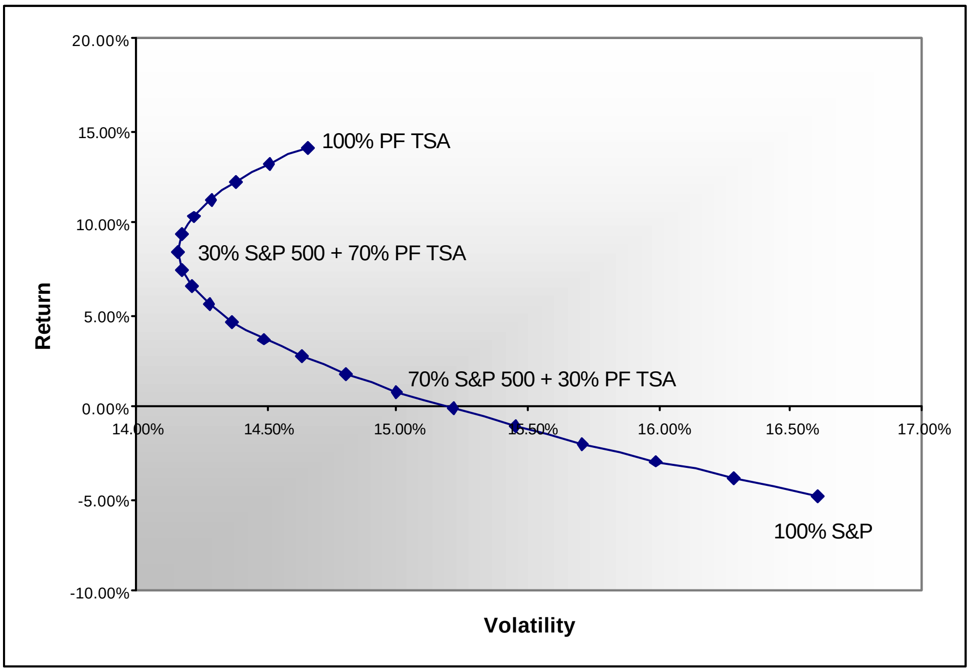
The performance of the TSA portfolio is 14.15% average return with a 14.66% volatility, a comfortable risk-return tradeoff that translates (under the assumption of a 4% risk-free rate) into a 0.69 Sharpe ratio. It should be noted that downside risk, as measured by semi-variance is relatively low. As a comparison, the performance of the S&P500 on the period was a –4.83% average return for a 16.60% annualised volatility, resulting in a –0.53 Sharpe ratio (we use again a 4% risk-free rate).

3.2.2.2. Performance of the TSA Strategy in a Core/Satellite Approach Combined with a Passive S&P500 Portfolio

We now test for the impact of inclusion of the TSA strategy as a satellite with respect to a core portfolio passively invested in the S&P500. The TSA portfolio can actually reduce risk without heavily sacrificing expected returns (up months in equity-friendly market:: 88.89%, down months in down markets: 58.33%, up market outperformance: 55.56%, down market outperformance:

70.83%). The following figure (figure 17) shows all possible portfolios that are obtained as we let the percentage of the TSA strategy vary from 0% to 100%.

Figure 17: Performance of the overall portfolio from January 1999 to June 2002 as a function of the percentage invested in the core portfolio versus the TSA strategy



We focus on a portfolio invested at 70% in S&P500 and at 30% in the TSA strategy. The performance of that portfolio is 0.87% average return with a 15% volatility.

Such a portfolio significantly outperforms the S&P500 over the period, as can be seen from an information ratio equal to 1.5264 (see also the distribution of returns in the Appendix 2). The alpha of the portfolio computed as the intercept of a regression of the portfolio returns on the S&P returns reaches an annual 4.66% abnormal performance. This strongly signals that a tactical style timing strategy is well-suited for a portable alpha strategy that can dramatically enhance the performance of an equity portfolio when implemented via a core/satellite approach.

We focus on a risk control methodology adapted to a satellite/core approach. The Tracking Error in the core portfolio is by definition 0, since the core portfolio is assumed to be passively invested in the S&P500. As noted above, we have a separate control on the tracking error of the satellite portfolio, which is implemented to ensure that the overall portfolio (core + satellite) does not deviate too much with respect to the benchmark (S&P500).

We actually get a relatively low tracking error (lower than 5%) when the satellite strategy represents up to 40% of the overall portfolio, as can be seen from table 16 below that shows the tracking error of the overall portfolio as a function of the percentage invested in the core portfolio versus the TSA strategy.

Table 16: Tracking error of the overall portfolio as a function of the percentage invested in the core portfolio versus the TSA strategy

% in Core	100%	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%
TE	0.00%	1.24%	2.49%	3.73%	4.97%	6.22%	7.46%	8.70%	9.95%	11.19%	12.44%

The tracking error is of course 0 when 100% are invested in the core strategy, while it is equal to 12.44%, the ex-post tracking error of the TSA strategy, when 100% is invested in the satellite. When the satellite TSA strategy represents 30% of the overall portfolio, the tracking error is 3.73%.

3.2.2.3. Introducing S&P Futures

Even higher portable alpha performance can be achieved through the introduction of S&P500 futures. The use of such futures contracts, consistent with the legislation on market risk hedging, can allow for more active tactical allocation decisions in the satellite portfolio, as they would allow an investor to profit more aggressively of forecasts of negative returns on all S&P style indices.

To test for the impact of the introduction of S&P500 futures contract on the TSA strategy, we perform the following experiments. For all months such that the econometric model was forecasting that no style index would perform the LIBOR (i.e., from February 2001 to September 2001 included, and in June 2002), we short S&P500 futures for 100% of the value of the portfolio. While such a strategy would not be viable in a long-only perspective at the level of the entire portfolio, it can be made consistent at the global portfolio level in a core/satellite approach.

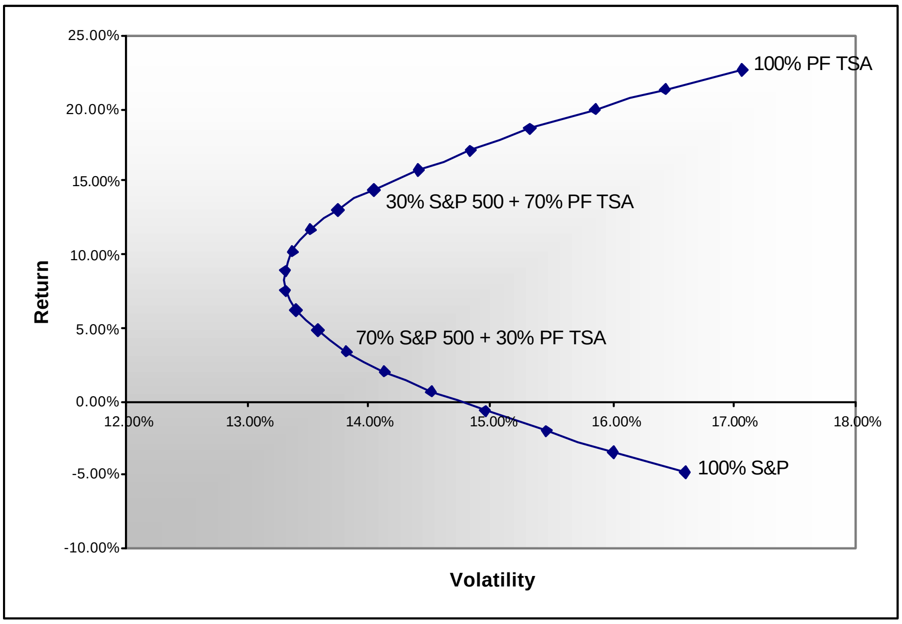
We focus again on a portfolio invested at 70% in S&P500 and at 30% in the TSA strategy. The performance of that portfolio is 3.43% average return with a 13.82% volatility. (That performance does not take into account the interest paid on the cash position.) Such a portfolio significantly outperforms the S&P500 over the period, as can be seen from an annual 6.29% alpha. The tracking error is also increased, and now reaches 6.18%. The following figure (figure 18) shows all possible portfolios that are obtained as we let the percentage of the TSA strategy vary from 0% to 100% when the use of S&P500 futures is allowed.

In table 17, we present a synthetic overview of the performance of all afore mentioned portfolios.

Table 17: Summary of risk-return measures on all portfolios from January 1999 to June 2002
(*: we have used a 4% risk-free rate in the computation of the Sharpe ratio)

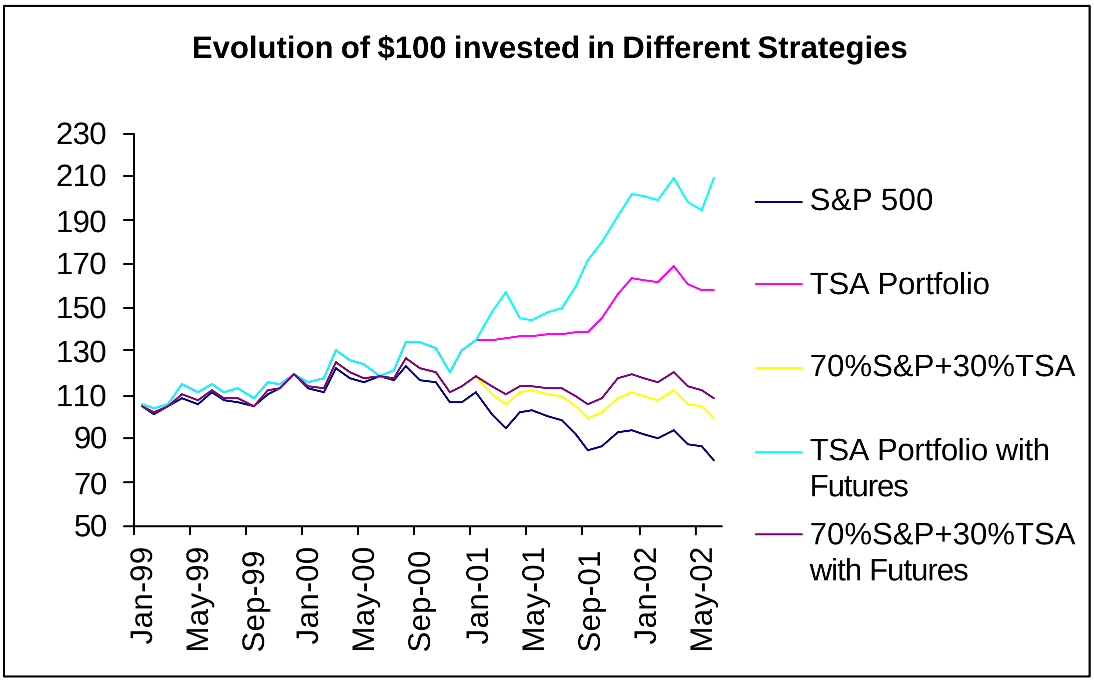
	S&P500	TSA	70% S&P500 + 30% TSA	TSA with S&P500 Futures	70% S&P500 + 30% TSA with Futures
Annual Average Return	-4.83%	14.15%	0.87%	22.69%	3.43%
Annual Std dev	16.60%	14.66%	15.00%	17.06%	13.82%
Tracking Error	0.00%	12.44%	3.73%	20.61%	6.18%
Sharpe Ratio*	-0.53	0.69	-0.21	1.10	-0.04
Information Ratio	n.a.	1.53	1.53	1.33	1.33
CAPM Alpha	0.00%	15.53%	4.66%	20.96%	6.29%

Figure 18: Performance of the overall portfolio from January 1999 to June 2002 as a function of the percentage invested in the core portfolio versus the TSA strategy when use of S&P500 futures is allowed



We also present in figure 19 the evolution of $100 invested in January 1999 in each strategy.

Figure 19: Evolution of $100 invested in various strategies from January 1999 to June 2002. Note: from January 1999 to February 2000, the performance of the TSA portfolio is identical to the performance of the TSA portfolio with futures.



Conclusion

Because the returns of alternative investment strategies exhibit in general low correlation with that of standard asset classes, it is expected that hedge funds will take on a significant share in active allocation strategies. While in its infancy the world of alternative investment strategies consisted of a disparate set of managers following very specific strategies, significant attempts at structuring the industry have occurred over the last decade which now allow active asset allocation models to apply to hedge funds as well as to traditional investment vehicles. In particular, investable portfolios replicating broad-based hedge funds indexes are today available with a sufficient level of liquidity.

Among the reasons that explain the growing institutional interest in hedge funds, there is first an immediate and perhaps superficial one: hedge funds always gain in popularity when equity market bull runs end, as long-only investors seek protection on the downside. This certainly explains in part the rising demand for hedge funds in late 2000 and early 2001. A more profound reason behind the growing acceptance of hedge funds is the recognition that they can offer a more sophisticated approach to investing through the use of derivatives and shortselling. Such added flexibility potentially offers two advantages to investors, and it has been recognized that these correspond to two fundamental reasons why people should invest in alternative investment vehicles (see for example Schneeweis and Spurgin (1999) and Amenc, Curtis and Martellini (2001)). First, the added in flexibility in terms of asset classes and strategies may lead to low correlations of hedge fund returns with traditional asset classes. This is true in particular for so-called "non-directional" alternative strategies such as equity market neutral or fixed-income arbitrage. Hence, investors seek to invest in hedge funds for their betas, i.e., because the low exposure of hedge fund returns with respect to standard asset classes helps them diversify away their traditional long-only portfolio. This is a risk diversification motive. Furthermore, the added flexibility in terms of assets and strategies provides hedge fund managers with more freedom to exert their skills and expertise in an attempt to generate positive alpha. This is a return enhancement motive: investors seek to invest in hedge funds because the expect some positive abnormal risk-adjusted performance generated by hedge funds through superior managers' skills to add positive return on the performance of their traditional long-only portfolio.

What percentage of their portfolio should investors allocate to alternative investment vehicles given their low betas and uncertain potential for positive alphas, and how to optimally select these hedge funds? In this paper, we discuss the state-of-the art techniques for optimal asset allocation to traditional and alternative investment vehicles, and we specifically account for the difficulties in estimating risk/return parameters from hedge fund return data. We first present various techniques allowing an investor to better assess the contrasted diversification properties of hedge funds. In particular, we introduce a multi-factor framework for the assessment of which funds should be included for which portfolio. We also present various competing models allowing investors to get a quantitative estimate of the optimal fraction of a given portfolio that should be allocated to hedge funds, in a context where only imperfect estimates of hedge fund expected returns are available. We not only discuss optimal strategic asset allocation decisions; we also explain how tactical asset decisions can also be made in a portfolio mixing traditional and alternative investment vehicles. Finally, we show how hedge fund can be used as portable alpha vehicles in a core/satellite portfolio approach.

References

- Agarwal, V., and N. Naik, 2000, Multi-period performance persistence analysis of hedge funds, Journal of Financial and Quantitative Analysis, 35, 327-342.

- Agarwal, V., and N. Naik, 2001, Performance evaluation of hedge funds with option-based and buy-and-hold strategies, working paper, LBS.

- Ait-Sahalia, Y., M. Brandt, 2001, Variable selection for portfolio choice, *Journal of Finance*, 56, 1297-1351.

- Amenc, N., S. Curtis, and L. Martellini, 2001, The alpha and omega of hedge fund performance measurement, working paper, EDHEC-MISYS Risk and Asset Management Research Center.

- Amenc, N., S. El Bied, and L. Martellini, 2002, Evidence of predictability in hedge funds returns", with N. Amenc and S. El Bied, *Financial Analysts Journal*, forthcoming.

- Amenc, N., P. Malaise, L. Martellini, and D. Sfeir, 2002, Tactical style allocation: a new form of market neutral strategy, working paper, USC.

- Amenc, N., and L. Martellini, 2001, The brave new world of hedge fund indexes, working paper, EDHEC-MISYS Risk and Asset Management Research Center.

- Amenc, N., and L. Martellini, 2002, Portfolio optimization and hedge fund style allocation decisions, *Journal of Alternative Investments*, 5, 2, 7-20, 2002.

- Amenc, N., L., Martellini, and M., Vaissie, 2002, The risks and benefits associated with investing in hedge funds, forthcoming, *The Journal of Asset Management*.

- Amin, G., and H. Kat, 2001, Hedge fund performance 1990-2000. Do the 'money machines' really add value?", working Paper, University of Reading.

- Amin G., Kat, H, 2002, Portfolios of hedge funds: what investors really invest in, ISMA discussion paper no 07.

- Anson, M. J. P., 2000, Hedge fund risk management for institutions, in *Managing hedge fund risk*, Risk Books.

- Asness, C., R. Krail, and J. Liew, 2001, Do hedge funds hedge?, working paper, AQR Capital Management.

- Barber, J.R., M.L. Copper, 1995, Immunization using principal component analysis, *Journal of Portfolio Management*, 99-105.

- Barberis, N., 2000, Investing for the long run when returns are predictable, Journal of Finance, 55, 225-264.

- Bekaert, G., and R. Hodrick, 1992, Characterizing predictable components in excess returns on equity and foreign exchange markets, *Journal of Finance*, 47, 467-509.

- Billingsley R.S., Chance D.M., 1996, Benefits and limitations of diversification among commodity trading advisors, *The Journal of Portfolio Management*, Fall, 65-80.

- Black, F., and R. Litterman, 1991, Global asset allocation with equities, bonds and currencies, Fixed-Income Research, Goldman Sachs.

- Black, F., and R. Litterman, 1992, Global portfolio optimization, *Financial Analysts Journal*, September-October 1992.

- Brennan, M., E. Schwartz and R. Lagnado, 1997, Strategic asset allocation, *Journal of Economic Dynamics and Control*, 21, 1377-1403.

- Britten-Jones, M., 1999, The sampling error in estimates of mean-variance efficient portfolio weights, *Journal of Finance*, 54, 2.

- Brandt, M., 1999, Estimating portfolio and consumption choice: a conditional Euler equations approach, *Journal of Finance*, 54, 1609-1645.

- Brooks, C., and Kat H., 2001, The statistical properties of hedge fund index returns and their implications for investors, *working paper,* The University of Reading, ISMA Centre.

- Campbell, J., 1987, Stock returns and the term structure, *Journal of Financial Economics*, 18, 373-399.

- Campbell, J., and R. Shiller, 1988, Stock prices, earnings, and expected dividends, *Journal of Finance*, 43, 661-676.

- Campbell, J., and L. Viceira, 1998, Who should buy long-term bonds, NBER working paper 6801.

- Chan, L., J. Karceski, and J. Lakonishok, 1999, On portfolio optimization: forecasting covariances and choosing the risk model, *Review of Financial Studies*, 12, 937-74.

- Chung, S. Y., 2000, What is the optimal portfolio risk measurement? A review of value-at-risk, in *Managing hedge fund risk,* Risk Books.

- Connor, G., and R. Korajczyk, 1993, A test for the number of factors in approximate factor models, *Journal of Finance*, 48, 1263-91.

- Cornish, and Fisher, 1937, Moments and cumulants in the specification of distributions, Review of the International Statistical Institute, p. 307-320.

- Cvitanic, J., A. Lazrak, L. Martellini, and F. Zapatero, 2002, Optimal allocation to hedge funds : an empirical analysis, *Quantitative Finance*, forthcoming.

- Cvitanic, J., A. Lazrak, L. Martellini, and F. Zapatero, 2003, Revisiting Treynor and Black (1973): an intertemporal model of active management, working paper, USC.

- Dimson, E., 1979, Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics*, 7.

- Dybvig, P., 1988a, Inefficient dynamic portfolio strategies, or how to throw away a million dollars in the stock market, *Review of Financial Studies*, 1., 67-88.

- Dybvig, P., 1988b, Distributional analysis of portfolio choice, *Journal of Business*, 61, 369-393.

- Elton, E., and M. Gruber, 1973, Estimating the dependence structure of share prices - implications for portfolio selection, *Journal of Finance*, 28, 1203-1232.

- Fama, E., 1981, Stock returns, real activity, inflation, and money, *American Economic Review*, 545-565.

- Fama, E., and K. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics*, 25, 23-49.

- Fama, E., and K. French, 1998, Value versus growth: the international evidence, *Journal of Finance*, 53, 6, 1975-2000.

- Fama, E., and K. French, 1992, The cross-section of expected stock returns, *Journal of Finance*, 442-465.

- Fama, E., and W. Schwert, 1977, Asset returns and inflation, *Journal of Financial Economics*, 115-46.

- Favre, L., and Galinao, J.A., Portfolio allocation with hedge funds - Case study of a Swiss institutional investor, internal research document, UBS, 2000.

- Fedrigo, I., T. Marsh, and P. Pfleiderer, 1996, Estimating factor models of secutity returns : how much difference does it make?, working paper.

- Ferson, W., and C. Harvey, 1991, Sources of predictability in portfolio returns, *Financial Analysts Journal*, May/June, 49-56.

- Ferson, W., and C. Harvey, 1993, The risk and predictability of international equity returns, *Review of Financial Studies*, 6, 527- 566.

- Ferson, W., and C. Harvey, 1995, Predictability and time-varying risk in world equity markets, Research in Finance, 13, 25-88.

- Fung, W., and Hsieh D., 1997a, Empirical characteristics of dynamic trading strategies, *Review of Financial Studies*, 10, 275-302.

- Fung, W., and Hsieh D., 1997b, Is mean-variance analysis applicable to hedge funds?, Working Paper, Fuqua School of Business, Duke University.

- Fung, W., and D. Hsieh, 2001, Benchmarks of hedge fund performance: information content and measurement biases, *Financial Analyst Journal*, forthcoming.

- Glosten, L., R. Jagannathan, and D. Runkle, 1993, On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance*, 48, 1779-1801.

- Grinold, R., and R. Kahn, 2000, Active portfolio management: a quantitative approach to providing superior returns and controlling risk, Second Edition, McGraw Hill.

- Harasty, H., and J. Roulet, 2000, Modeling stock market returns -- an error correction model, *Journal of Portfolio Management*, Winter, 33-46.

- Harvey, C., 1989, Time-varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics*, 24, 289-317.

- Harvey, C., 1995, Predictable risk and returns in emerging markets, *Review of Financial Studies*, 773-816.

- Henker, T., 1998, Naive diversification for hedge funds, *The Journal of Alternative Investments*, Winter, 33-38.

- Henker T., Martin, 1998, Naive and optimal diversification for managed futures, *The Journal of Alternative Investments*, Fall.

- Indjic, D., 2002, Strategic asset allocation with portfolios of hedge funds, *AIMA Journal*, December, 29-33.

- Jagannathan R., and T. Ma, 2000, Covariance matrix estimation: myth and reality, working paper, Northwestern University.

- Johnstone, I, 2001, On the distribution of the largest eigenvalue in principal components analysis, *Annals of Statistics*, forthcoming.

- Jorion, P., 1985, International portfolio diversification with estimation risk, *Journal of Business*, 58, 259-278.

- Jorion, P., 1986, Bayes-Stein estimation for portfolio analysis, *Journal of Financial and Quantitative Analysis*, 21, 3, 279-292.

- Karavas, V., 2000, Alternative investments in the institutional portfolio, *Journal of Alternative Investments,* 3, 11-26.

- Keim, D., and R. Stambaugh, 1986, Predicting returns in the stock and bond markets, *Journal of Financial Economics*, 17, 357-390.

- Laloux, L., P. Cizeau, J. P. Bouchard, and M. Potters, Random matrix theory, *Risk Magazine*, 12, 69.

- Learned, M., and F.-S. Lhabitant, 2002, Diversification: How much is enough?, Research Paper number 52, FAME - International Center for Financial Asset Management and Engineering.

- Ledoit, O., 1999, Improved estimation of the covariance matrix of stock returns with an application to portfolio selection, working paper, UCLA.

- Leland, H., 1999, Beyond mean-variance: risk and performance measures for portfolios with nonsymmetric distributions, *working paper*, Haas School of Business, U.C. Berkeley.

- Lhabitant, F., 2001, Hedge funds investing: a quantitative look inside the black box, working paper, Union Bancaire Privee.

- Liang, B., 2001, Hedge fund performance: 1990-1999, *Financial Analysts Journal,* January/February, 11-18.

- Litterman, R. and J. Scheinkman, 1991, Common factors affecting bond returns, *Journal of Fixed Income*, 1, 54-61.

- Litterman, R., and K. Winkelmann, 1998, Estimating covariance matrices, Goldman Sachs Risk Management Series.

- Lo, A., 2001, Risk management for hedge funds, introduction and overview, *Financial Analysts Journal,* Nov.-Dec., 16-23.

- Longin F., Solnik B., 1995, Is the correlation in international equity returns constant?, *Journal of International Money and Finance*, 14.

- Marchenko, V., and L. Pastur, 1967, Eigenvalue distribution in some ensembles of random matrices , Math. USSR Sbornik 72, 536-567.

- Markowitz, H., 1952, Portfolio selection, Journal of Finance, 7, 77-91.
- Mehra, R., and E. Prescott, 1985, Equity premium: a puzzle, *Journal of Monetary Economics*, 15, 145-161.
- Merton, R. C., 1973, An intertemporal capital asset pricing model, *Econometrica*, 41, 867-888.
- Michaud, R., 1998, Efficient asset management: a practical guide to stock portfolio optimization and asset allocation, Harvard Business School Press, 1998.
- Merton, R., 1980, On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics,* 8, pp. 323-362.
- Park J., S., Brown, and W. Goetzmann, 1999, Performance benchmarks and survivorship bias for hedge funds and commodity trading advisors, *Hedge Fund News*, August 1999.
- Peters, E., 1991, Chaos and order in capital markets, John-Wiley & Sons, New York.
- Roll R., and S. Ross, 1980, An empirical investigation of the arbitrage pricing theory, *Journal of Finance*, XXXV, 5.
- Ross, S., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory*, December, 341-360.
- Ruddick, S., 2002, Diversification overkill, working paper, Albourne Partners, January.
- Scherer, B., 2002, Portfolio Construction and Risk Budgeting, RiskBooks.
- Schneeweis, T., and R. Spurgin, 1999, Multi-factor models in managed futures, hedge fund and mutual fund estimation, working paper, CISDM, University of Armhest.
- Schneeweis, T., and R. Spurgin, 2000, The benefits of index option-based strategies for institutional portfolios, working paper, CISDM, University of Armhest.
- Scholes, M., and J. Williams, 1977, Estimating betas from nonsynchronous data, *Journal of Financial Economics*, 5.
- Singer, B., R. Staub, and K. Terhaar, 2002, The appropriate policy allocation for alternative investments, UBS Global Asset Management working paper.
- Sharpe, W., 1963, A Simplified model for portfolio analysis, *Management Science*, 277-293.
- Sharpe, W., 1964, Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance*, 19, 425-442.

- Sharpe, W., 1988, Determining a fund's effective asset mix, *Investment Management Review*, 2, 6, 56-69.

- Sharpe, W., 1992, Asset allocation: management style and performance measurement, *Journal of Portfolio Management*, 18, 7-19.