An Edhec Risk and Asset Management Research Centre Publication

Investing in Hedge Funds: Adding Value through Active Style Allocation Decisions

SG ASSET MANAGEMENT

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Abstract: In this paper, we introduce a suitable extension of the Black-Litterman Bayesian approach to portfolio construction that allows for the incorporation of active views about hedge fund strategy performance in the presence of non-trivial preferences about higher moments of hedge fund return distributions. We also present a numerical application illustrating how investors can use a multi-factor approach to generate such active views and dynamically adjust their allocation to various hedge fund strategies while staying coherent with a long-term strategic allocation benchmark. Overall the results in this paper strongly suggest that significant value can be added in a hedge fund portfolio through the systematic implementation of active style allocation decisions, both at the strategic and tactical levels.

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Edhec pursues an active research policy in the field of finance. Its **"Risk and Asset Management Research Centre"** carries out numerous research programmes in the areas of asset allocation and risk management in both the traditional and alternative investment universes.

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About the Authors

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Foreword

Since it was set up, in 2001, the Edhec Risk and Asset Management Research Centre has made a point of conducting research that is both independent and pragmatic.

The concern to render our research work relevant and operational led us, in 2003, to publish the first studies on the policies of the European asset management industry. The Edhec European Asset Management Practices survey allowed a comparison to be established between the academic state-of-the-art in the area of portfolio management and risks, and the practices of European managers.

This study was completed in the same year by a review of the state-of-the-art and the practices of European alternative multimanagers, the Edhec European Alternative Multimanagement Practices survey.

In drawing up the latter report, we were able to observe the gap that exists between the conclusions of academic research work and the practices of multimanagers in measuring and reporting on the performance and risks of funds or portfolios of hedge funds. This observation led us to carry out research and a survey on this fundamental dimension of the relationship between investors and managers: the Edhec Funds of Hedge Funds Reporting Survey. This report was presented in February 2005 to pan-European institutional investors and professionals.

In 2005, on the occasion of the Edhec Asset Management Days, which brought together more than 600 professionals in Geneva, we had the honour of presenting our study on the place of structured products in asset management, entitled "Structured Forms of Investment Strategies in Institutional Investors' Portfolios".

The present study, "Investing in Hedge Funds: Adding Value through Active Style Allocation Decisions" is part of the Edhec Risk and Asset Management Research Centre's multi-style/multi-class allocation research programme, which focuses on the benefits, risks and integration methods of the alternative class in asset allocation. From that perspective, Edhec is making a significant contribution to the research conducted in the area of multi-style/multi-class portfolio construction.

I would like to express my sincere thanks to SGAM AI, who have supported our research work in recent years and have enabled us to publish this research.

Noël Amenc
Professor of Finance
Director of the Edhec Risk and Asset Management Research Centre

Executive Summary

Executive Summary

One of the by-products of the bull market of the 90's has been the consolidation of hedge funds as an important segment of financial markets. It was recently announced that the value of the hedge fund industry worldwide had passed the $1 trillion mark for the first time, with approximately 7,000 hedge funds in the world, around 1,000 of which were launched in 2003.

One of the key reasons behind the success of hedge funds in institutional money management is that such alternative investment strategies seem to provide diversification benefits with respect to other existing investment possibilities. In an attempt to fully capitalize on such beta benefits in a top-down approach, investors or (funds of hedge funds) managers must be able to rely on robust techniques for optimization of portfolios including hedge funds. Standard mean-variance portfolio selection techniques are known to suffer from a number of shortcomings, and the problems are exacerbated in the presence of hedge funds. Most importantly it can be argued that the following two aspects require specific care. First, because hedge fund returns are not normally distributed, a mean-variance optimization would be severely ill-adapted. Secondly, the problem of parameter uncertainty needs to be carefully addressed, as the lack of a long history and the non-availability of high frequency data imply that parameter estimation is a real challenge in the case of hedge fund returns.

While both problems (non-trivial preferences about higher moments of asset return distribution and the presence of parameter uncertainty) have been studied independently, what is still missing for active style allocation in the hedge fund universe is a model that would take into account both of these two aspects. Our contribution is precisely to introduce an optimal allocation model that incorporates an answer to both challenges within a unified framework. To this end, we introduce a suitable extension of the Black-Litterman Bayesian approach to portfolio construction that allows for the incorporation of active views about hedge fund strategy performance in the presence of non-trivial preferences about higher moments of hedge fund return distributions.

In a nutshell, we suggest the following approach. We first generate "neutral" views on expected hedge fund returns based on the desire to match a benchmark portfolio composition, where the benchmark is designed based on minimizing the portfolio Value-at-Risk. For this purpose, we use an asset pricing model that incorporates investors' preferences not only on expected return and volatility, but also on higher moments of hedge fund return distributions. Next, we present a simple factor analysis that allows us to obtain a bullish, a bearish or a neutral view concerning the expected return. The next step involves blending such active views with the neutral views, applying a Bayesian statistical approach similar to that introduced by Black-Litterman. Finally, we generate optimal allocations to hedge funds that are consistent with this mixture of neutral and active views.

Executive Summary

We also present a numerical application illustrating how investors can use a multi-factor approach to generate such active views and dynamically adjust their allocation to various hedge fund strategies while staying coherent with a long-term strategic allocation benchmark. We were able to show that the active style selection process, combined with the Black-Litterman portfolio selection method, allows for significant outperformance without a large increase in tracking error. In particular, the implementation of the process led to a 100 basis points excess return over the period January 1997 to December 2004 for a small tracking error (0.86%), leading to a 1.17 information ratio. A more aggressive portfolio version, based on an increase in the parameter defining the relationship between neutral and active views, led to outperformance of almost 200 basis points for a 1.37 tracking error, leading to a 1.41 information ratio. We also show that the optimal design of a hedge fund portfolio based on active allocation decisions to various alternative strategies leads to a significant improvement in the Omega function, a relevant risk-adjusted measure of performance.

The bulk of the message conveyed in this paper is straightforward and has important potential implications for the hedge fund industry: it is only by taking into account the exact nature and composition of an investor's existing portfolio, as opposed to regarding hedge fund investing from a stand-alone approach, that institutional investors can truly customize and maximize the benefits they can expect from investing in these modern forms of alternative investment strategies. Overall the results in this paper strongly suggest that significant value can be added in a hedge fund portfolio through the systematic implementation of active style allocation decisions, both at the strategic and tactical levels. While this fact has long been recognized by market participants, the lack of reliable asset allocation tools has not facilitated the implementation of effective top-down approaches to investment in hedge funds. In this study, we argue that such techniques are actually already available and we show that a suitable extension to the Black-Litterman model can be used to implement active views on hedge fund style performance in a meaningful and consistent approach that avoids the pitfalls of standard optimization procedures.

Résumé

L'une des conséquences de l'envolée des marchés au cours des années 90 a été la reconnaissance de la gestion alternative comme un segment important des marchés financiers. Il a récemment été annoncé que l'industrie de la gestion alternative comptait environ 7000 fonds, 1000 d'entre eux ayant été lancés en 2003 ; le tout représentant plus de 1000 milliards de dollars d'actifs sous gestion.

Le succès des *hedge funds* auprès des investisseurs institutionnels tient en grande partie au fait qu'ils leurs fournissent de nouvelles opportunités de diversification par rapport aux classes d'actifs traditionnelles. Afin de bénéficier pleinement des propriétés de diversification des stratégies alternatives dans le cadre d'une approche *top/down*, les investisseurs et/ou les gérants de fonds de fonds doivent disposer de modèles d'optimisation robustes, permettant la construction de portefeuilles composés pour tout ou partie de *hedge funds*. L'optimisation en moyenne/variance présente un certain nombre de faiblesses, qui sont exacerbées en présence de *hedge funds*. A ce titre, les deux points suivants méritent réflexion. Tout d'abord, parce que les rendements des *hedge funds* ne sont pas distribués selon une loi normale, l'optimisation en moyenne/variance apparaît fort peu appropriée. Ensuite, les problèmes liés à l'incertitude des paramètres doivent faire l'objet d'une attention particulière, étant donné que le manque d'historique et la faible fréquence des données des *hedge funds* impliquent que l'estimation des paramètres est particulièrement ardue dans le cas des *hedge funds*.

Si la non-normalité des rendements et la présence d'incertitude dans l'estimation des paramètres ont d'ores et déjà été étudiées séparément, il manque à ce jour un modèle qui répondrait simultanément à ces deux problèmes et permettrait ainsi de mettre en oeuvre une allocation active à travers les stratégies de *hedge funds*. Notre contribution est précisément d'introduire un modèle d'allocation optimale qui propose une solution à ces deux problèmes, dans un cadre unifié. A cette fin, nous introduisons une extension de l'approche bayesienne proposée par Black-Litterman dans le cadre de la construction de portefeuille qui permet l'intégration de vues actives sur les performances des différentes stratégies en présence de préférences non triviales de la part des investisseurs pour les moments d'ordre supérieurs (i.e. risques extrêmes).

En bref, nous suggérons l'approche suivante. Nous générons dans un premier temps des vues « neutres » sur les rendements espérés des stratégies de *hedge funds* qui sont cohérentes avec la composition du benchmark stratégique, obtenu en minimisant la *Value-at-Risk* du portefeuille. Dans cette optique, nous utilisons un modèle de valorisation des actifs incorporant les préférences des investisseurs non seulement pour les rendements espérés, mais aussi pour la volatilité et les moments d'ordre supérieur de la fonction de distribution des rendements. Ensuite, nous présentons une analyse factorielle permettant d'obtenir des vues sur les rendements des stratégies de *hedge funds* (i.e. hausse, baisse, neutre). L'étape suivante consiste à « mixer » les vues actives et les vues neutres en utilisant une approche bayesienne similaire à celle introduite par Black-Litterman. Cela nous permet finalement de générer des allocations optimales pour les différentes stratégies.

Résumé

Nous présentons également une application numérique illustrant la façon dont les investisseurs peuvent utiliser une approche multi-factorielle pour générer des vues actives et ajuster de façon dynamique leur allocation à travers les différentes stratégies alternatives, tout en restant cohérent avec leur benchmark stratégique. Nous sommes parvenus à montrer que le processus de sélection des stratégies, combiné avec l'approche Black-Litterman pour la construction du portefeuille, permet d'obtenir une sur-performance significative sans pour autant augmenter la *tracking error* de façon sensible. En particulier, la mise en œuvre de cette procédure nous a permis d'obtenir un excès de rendement de l'ordre de 100 points de base sur la période allant de janvier 1997 à décembre 2004, avec une tracking error de seulement 0.86%. Cela nous a ainsi permis de générer un ratio d'information de 1.17. Une version plus agressive de ce portefeuille, basée sur une augmentation du paramètre définissant la relation entre les vues neutres et les vues actives, a conduit à une sur-performance de l'ordre de 200 points de base pour une *tracking error* de seulement 1.37%. Cela nous a permis d'obtenir un ratio d'information de 1.41. Nous montrons également que la construction d'un portefeuille de *hedge fund* optimal basée sur des décisions d'allocation actives à travers les diverses stratégies alternatives entraîne une amélioration significative de la fonction d'Omega, qui est une mesure de performance ajustée du risque pertinente dans l'univers alternatif.

L'essentiel du message transmis dans ce document est clair et il a des implications potentielles importantes pour l'industrie de la gestion alternative : c'est uniquement en prenant en compte la nature et la composition du portefeuille initial de l'investisseur, et non en adoptant une approche de type *« stand alone »* dès lors qu'ils investissent dans les *hedge funds*, que les investisseurs institutionnels pourront profiter pleinement des bénéfices des stratégies alternatives. De façon générale, les résultats présentés dans ce document suggèrent qu'il est possible d'ajouter de la valeur de façon significative dans un portefeuille de *hedge fund* à travers la mise en œuvre systématique de décisions d'allocation, aussi bien au niveau du processus d'allocation stratégique que de celui de l'allocation tactique. Alors que cela fait depuis longtemps l'objet d'un consensus parmi les différents acteurs sur le marché, l'absence d'outils d'allocation fiables n'a pas facilité la mise œuvre d'une approche *top/down* efficiente de l'investissement dans les *hedge funds*. Dans cette étude, nous affirmons que de tels outils existent et nous montrons qu'une extension adéquate du modèle de Black-Litterman peut être utilisée pour mettre en œuvre des vues actives sur la performance des stratégies alternatives selon une approche rationnelle et disciplinée permettant d'éviter les pièges des techniques d'optimisation standard.

Zusammenfassung

Die Etablierung von Hedge Fonds als wichtiges Segment im Finanzsektor war eines der Nebenprodukte des Börsenaufschwungs der 90-er Jahre. Vor kurzem wurde bekannt gegeben, dass der Wert der in Hedge Fonds gehaltenen Positionen zum ersten Mal die Marke von 1 Billionen Euro überschritten hat. Von den rund 7000 Hedge Fonds weltweit wurden alleine 1000 im Jahre 2003 lanciert.

Einer der Hauptgründe für den rasanten Aufstieg der Hedge Fonds im institutionellen Management ist die Erkenntnis, dass diese Investment-Strategien Diversifikationspotential bezüglich existierender Investment-Vehikel zu besitzen scheinen. Um diese Beta-Vorteile in einem Top-Down-Ansatz vollkommen verwerten zu können, benötigen Investoren oder Funds-of-Hedge-Funds-Manager zuverlässige und robuste Optimierungstechniken für Portfolios, die Hedge Fonds beinhalten. Traditionelle Erwartungs-Varianz Portfolioselektion birgt bekanntermaßen eine Reihe von Mängeln, was sich durch die Präsenz von Hedge Funds noch verstärkt. Dies kann vor allem anhand zweier Probleme vergegenwärtigt werden: Erstens sind Hedge-Fonds-Renditen nicht normal verteilt, was den Erwartungswert-Varianz-Ansatz ad absurdum führt. Zweitens muss das Problem der Parameter-Unsicherheit mehr Niederschlag in der Analyse finden. Letzteres wird vor allem dadurch erschwert, dass Hedge-Fonds-Renditen weder lang zurückreichend, noch in hoher Frequenz vorhanden sind.

Während beide Probleme (nicht-triviale Präferenzen für Moment höherer Ordnung und Parameter-Unsicherheit) ausführlich und unabhängig voneinander untersucht wurden, fehlt immer noch ein Modell, welches diese Aspekte zusammen und in Gegenwart von Hedge Fonds betrachtet. Unsere Kontribution ist es, hierfür ein Model zu liefern. Genauer gesagt, führen wir eine Erweiterung des Black-Litterman-Models ein, welche es erlaubt, aktive Vorhersagen von Hedge-Fonds-Strategien mit dem Vorhandensein von nicht-trivialen Präferenzen für höhere Momente von Hedge-Fonds-Rendite-Verteilungen zu vereinen.

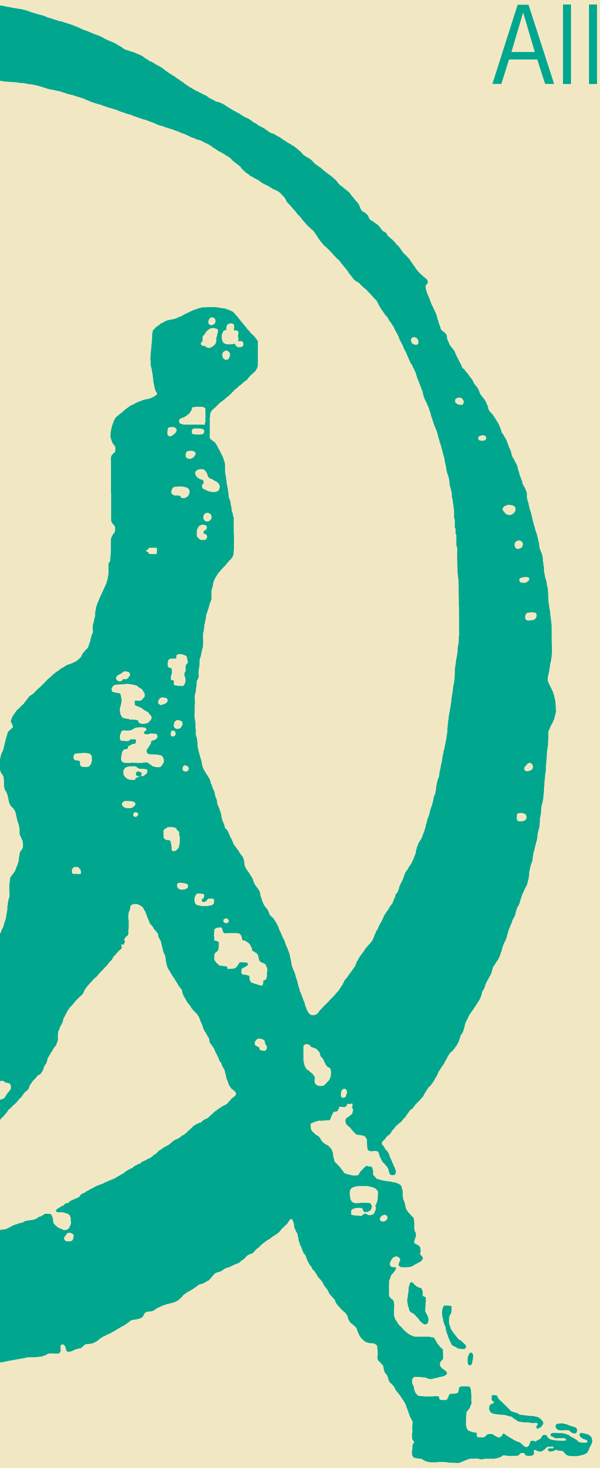
Vereinfacht dargestellt, verwenden wir folgende Idee: Zunächst generieren wir „neutrale" Erwartungen, die darauf basiert sind, eine Benchmark-Allokation zu replizieren, wobei das Benchmark-Portfolio durch das Minimum-Value-at-Risk-Portfolio repräsentiert wird. Zu diesem Zweck haben wir ein Asset-Pricing Modell benutzt, dass nicht nur Präferenzen für Erwartungswert und Varianz berücksichtigt sondern auch jene für höhere Moment der Rendite-Verteilungen. Als nächstes führen wir eine einfache Faktoren-Analyse durch, die es erlaubt positive, negative und neutrale Erwartungen über Renditen auszudrücken. Der nächste Schritt ist, diese Vorhersagen mit den „neutralen" Erwartungen zu mischen, indem ein bayesianischer Ansatz, ähnlich dem von Black-Litterman vorgeschlagenen, angewandt wird. Letztlich erhalten wir optimale Allokationen, die konsistent mit aktiven und neutralen Erwartungen sind.

Zusammenfassung

Des Weiteren, präsentieren wir ein numerisches Verfahren, welches es ermöglicht, die aktiven Erwartungen anhand eines multi-faktoriellen Vorhersage-Modells zu erhalten, wobei die Allokationen dynamisch angepasst werden, ohne dabei den langfristigen strategischen Benchmark außer acht zu lassen. Wir konnten zeigen, dass dieser Vorhersageprozess, angewandt auf ein Black-Litterman-Selektionsmodell, nennenswerte out-of-sample Renditesteigerungen hervorbringt und der Tracking Error mit dem strategischen Benchmark gering ist. Für die Periode von Januar 1997 bis Dezember 2004 beispielsweise, erhielten wir eine Renditesteigerung von 100 Basispunkten bei einem Tracking Error von nur 0,86%, welches zu einem Informationsratio von 1,17 führt. Eine aggressive Strategie (erzielt durch eine Erhöhung des Vertrauensparameters in die aktiven Erwartungen) führte gar zu Renditesteigerungen von fast 200 Basispunkten bei einem Tracking Error von 1,37% (Informationsratio: 1,41). Wir zeigen ebenfalls, dass eine derartige optimale Gestaltung des Hedge-Fonds-Portfolios eine nennenswerte Verbesserung der Omega-Funktion nach sich zieht.

Die Aussage dieser Studie ist einleuchtend und birgt entscheidendes Potential für die Hedge-Fonds-Industrie: nur durch die Inbetrachtnahme der exakten Zusammensetzung der Portfolios von Investoren, können institutionelle Anleger vom Potential moderner Formen alternativer Investment-Vehikel profitieren. Dies steht im Gegensatz zur geläufigen Praxis, welche Hedge Fonds losgelöst von anderen Investments betrachtet. Als Ergebnis dieser Arbeit können wir festhalten, dass nennenswerte Benefize bezüglich eines Hedge Fonds Portfolios erzielt werden können, wenn systematisch aktive Stil-Allokationen implementiert werden, sowohl auf strategischer, als auch auf taktischer Ebene. Während diese Erkenntnis bei Marktteilnehmern längst Konsens findet, wurde bisher kein zuverlässiges Verfahren entwickelt, um effiziente Top-Down-Ansätze für Hedge-Fonds-Investments zu implementieren. Wir zeigen, dass solche Verfahren prinzipiell vorhanden sind und weisen nach, dass eine adäquate Erweiterung des Black-Litterman-Modells eine Implementierung aktiver Vorhersagetechniken im Rahmen von Hedge-Fonds-Investments ermöglicht und gleichzeitig die Probleme üblicher Optimierungsmethoden vermieden werden können.

Investing in Hedge Funds: Adding Value through Active Style Allocation Decisions



Introduction

One of the by-products of the bull market of the 90's has been the consolidation of hedge funds as an important segment of financial markets. It was recently announced that the value of the hedge fund industry worldwide had passed the $1 trillion mark for the first time, with approximately 7,000 hedge funds in the world, around 1,000 of which were launched in 2003.[1]

There seem to be two main reasons behind the success of hedge funds in institutional money management. On the one hand, it is argued that hedge funds may provide abnormal risk-adjusted returns, due to the superior skills of hedge fund managers and flexibility in trading strategies. These benefits have been labeled *return enhancement* or *portable alpha* benefits, and have been the focus of rich academic literature on the presence (or absence) of skills in hedge fund management (see for example Amenc et al. (2003), as well as references therein). A careful quantitative and qualitative due diligence process is needed to ensure that the selection process will increase the likelihood of including high risk-adjusted performance funds in a portfolio. On the other hand, hedge funds seem to provide diversification benefits with respect to other existing investment possibilities. These have been labeled *risk reduction* or *portable beta* benefits. In a nutshell, the diversification, or portable beta, argument states that investors can take advantage of hedge funds' linear and non-linear exposure to a great variety of risk factors, including volatility, credit and liquidity risk, etc., to reduce the risk of their global portfolio.

In an attempt to fully capitalize on such beta benefits in a top-down approach, investors or (funds of hedge funds) managers must be able to rely on robust techniques for optimization of portfolios including hedge funds. Standard mean-variance portfolio selection techniques are known to suffer from a number of shortcomings, and the problems are exacerbated in the presence of hedge funds. Most importantly it can be argued that the following two aspects require specific care. First, because hedge fund returns are not normally distributed, a mean-variance optimization would be severely ill-adapted, except in the case of an investor endowed with quadratic preferences. Secondly, the problem of parameter uncertainty needs to be carefully addressed, as the lack of a long history and the non-availability of high frequency data imply that parameter estimation is a real challenge in the case of hedge fund returns. There are many reasons to believe that the main challenge for optimal allocation models is in fact in estimating expected returns, as opposed to higher moments, of asset class returns. First, there is a general consensus that expected returns are difficult to obtain with a reasonable estimation error. Hedge fund performance measurement biases have largely been documented in the literature (see Fung and Hsieh (2000, 2002) among other examples). What makes the problem worse is that optimization techniques are very sensitive to differences in expected returns, so that portfolio optimizers typically allocate the largest fraction of capital to the asset class for which estimation error in the expected returns is the largest (e.g., Britten-Jones (1999) or Michaud (1998)).

1. These numbers have been extracted from the *2004 Alternative Fund Service Review Survey*, as reported in the weekly publication *International Fund Investment*, issue 116, May 17, 2004.

Introduction

There is already a considerable amount of academic literature that has addressed the question of hedge fund portfolio optimization. Many studies have actually shown that in a mean-variance framework, mixing hedge funds with traditional assets leads to an enhancement of the return of the traditional portfolio with a constant risk level, or conversely, to a reduction in the risk level of the traditional portfolio with a constant return level (see Georgiev et al. (2002), Terhaar et al. (2002), Amenc and Martellini (2002) or Alexander and Dimitriu (2004) among other examples). This might easily be explained by the fact that hedge funds typically present low volatility, together with low correlation with traditional asset classes and relatively high returns. However, low volatility is not a free lunch. It is actually possible to show, through a statistical model integrating fatter tails than those of the normal distribution, that minimising the second order moment (the volatility) can be accompanied by a significant increase in extreme risks (Sornette et al. (2000)). This is confirmed in Amin and Kat (2003) where the authors find empirical evidence that low volatility is generally obtained at the cost of lower skewness and higher kurtosis. As stressed in Cremers et al. (2005), in the presence of asymmetric and/or fat-tailed return distribution functions, the use of mean-variance analysis can potentially lead to a significant loss of utility for investors.

Many attempts have therefore been made to better account for the specific risk features of hedge funds. Building on Markowitz's initial proposition, some of them used downside risk measures such as semi-deviation or lower partial moments (see McFall Lamm (2003)) or robust estimations of extreme risks to define the risk dimension. In the latter case, some authors opted for the modified Value-at-Risk (see Favre and Galeano (2002) and McFall Lamm (2003)) based on the expansion introduced in Cornish and Fisher (1937). Others have preferred Conditional-Value-at-Risk based on empirical distribution of hedge fund returns (see De Souza and Gokcan (2004), or Agarwal and Naik (2004)) or using Extreme Value Theory (see Bacmann and Gawron (2004)).

Another avenue of investigation involves maximizing objective functions accounting for investors' aversion to extreme risks. In Morton et al. (2003), the authors focus on a family of expected utilities that are a weighted sum of the probability of achieving a benchmark and expected shortfall relative to a benchmark. In the Multiple Objective Approach introduced in Davies et al. (2004), the authors explicitly integrate not only volatility but also skewness and kurtosis in their polynomial optimization model. The objective is to find an optimal balance between conflicting goals such as the maximization of expected returns, and/or skewness, the minimization of volatility and/or kurtosis.

The last stream of research, finally, consists of maximizing alternative ratios such as the Omega ratio. The principal interest of such ratios is to take into account the whole return distribution function. While in Favre-Bulle and Pache (2003) the authors use the empirical returns of hedge funds, Passow (2004)) suggests modeling higher order moments applying the Johnson cumulated densities to calculate a so-called Johnson-Omega. The advantage of the Johnson-Omega is to extract persistent information from track record and exploit the persistence of higher order moments (i.e., up to 4). It therefore overcomes the instabilities within non-parametric approaches to Omega, and allows for more reliable portfolios.

Introduction

As transpires from the literature review above, significant progress has been made in the direction of extending the mean-variance approach, which is ill-suited for hedge funds, to more general setups that take into account higher moments of return distributions. In the same vein, and even though the focus in that literature was not on an application to hedge funds, significant work has been carried out on the impact on optimal portfolio decisions of parameter uncertainty induced by the estimation of expected returns.

One of the most fruitful directions is the literature on optimal allocation with incomplete information about security parameters. In a continuous-time setting, this literature starts with Detemple (1985), Dothan and Feldman (1985), and Gennotte (1985). For technical reasons, those papers consider a myopic (logarithmic) investor who has perfect information about the standard deviation of securities dynamics, but does not know the expected return. The investor has priors on expected returns. The investor observes prices and upgrades priors accordingly, in a Bayesian way. Financial econometricians seem to agree on the fact that it is feasible to obtain good estimates of variance parameters, but much harder to estimate expected returns. Based on that, and other reasons of a technical nature, most of the continuous-time literature with incomplete information focuses on the problem of incomplete information on the expected returns of securities. Recently, Brennan (1998), Brennan and Xia (2001), Cvitanic et al. (2005), and Xia (2001) have been able to extend the results of the aforementioned papers to the case in which investors have CRRA preferences, possibly non-myopic.

A parallel line of research starting with Kandel and Stambaugh (1996), Pastor and Stambaugh (1999, 2000), Barberis (2000), and Pastor (2000) considers the problem of incomplete information, both about expected return and variance of the securities, in discrete-time settings, but the objective of the investor (mean-variance optimization) does not generate hedging components in the optimal demands (as in the Brennan-Xia papers). They use numerical techniques to upgrade priors about both mean and variance in a Bayesian way. Also related is an influential paper by Black and Litterman (1990, 1992) who introduce, in a static mean-variance setting, a methodology that allows investors to account for uncertainty in their priors on expected returns, expressed in terms of deviation from neutral equilibrium CAPM-based estimates. The Black-Litterman approach, because of its technical tractability and conceptual simplicity, has become a widely used tool in the context of active asset allocation decisions.

While both problems (non-trivial preferences about higher moments of the asset return distribution and the presence of parameter uncertainty) have been studied independently, what is still missing for active style allocation in the hedge fund universe is a model that would take both of these two aspects into account. One pragmatic solution that has been used in the hedge fund literature to overcome the problem of large estimation risk in the estimated expected returns is to focus on selecting the one portfolio on the efficient frontier for which no information on expected returns is required, i.e. the portfolio with the minimum amount of risk. This approach has been followed for example by Amenc and Martellini (2002), or Alexander and Dimitriu (2004). This approach, however, does not allow for the inclusion of active views on expected returns on hedge fund strategies.

Introduction

Our contribution to this literature on optimal allocation decisions when hedge funds are part of an investor's opportunity set is precisely to introduce an optimal allocation model that incorporates an answer to both challenges within a unified framework. To this end, we first introduce a suitable extension of the Black-Litterman Bayesian approach to portfolio construction that allows for the incorporation of active views about hedge fund strategy performance in the presence of non-trivial preferences about higher moments of hedge fund return distributions. We also present a numerical application illustrating how investors can use a multi-factor approach to generate such active views and dynamically adjust their allocation to various hedge fund strategies while staying coherent with a long-term strategic allocation benchmark.

The bulk of the message conveyed in this paper is straightforward. It is only by taking into account the exact nature and composition of an investor's existing portfolio, as opposed to regarding hedge fund investing from a stand-alone approach, that institutional investors can truly customize and maximize the benefits they can expect from investing in these modern forms of alternative investment strategies. Overall the results in this paper strongly suggest that significant value can be added in a hedge fund portfolio through the systematic implementation of active style allocation decisions, both at the strategic and tactical levels.

The rest of this paper is organized as follows. In the next section, we present a formal model of an active hedge fund style allocation decision based on a suitable extension of the Black-Litterman approach. We then design a strategic style allocation benchmark that will be used to generate implied neutral estimates of expected returns that are needed in the context of the Bayesian portfolio construction process. In the following section, we present a multifactor approach that can be used to generate active views on hedge fund strategy performance, and we present results on active style allocation decisions based on these views and the portfolio construction process introduced in the previous sections. A conclusion can be found in the final section.

A Bayesian Model of Active
Hedge Fund Style Allocation Decisions

In this section, we present a formal model of active style allocation that can be used in the hedge fund universe. We first review the Black-Litterman model, and then extend it to a setup where higher moments of return distribution are taken into account.

Review of the Black-Litterman approach

Since the seminal work of Markowitz (1952) there is a strong consensus in portfolio management on the trade-off between expected return and risk. In the Markowitz world, the risk is represented by the standard deviation. Given the investor's specific risk aversion, optimal portfolios and the so-called efficient frontier can be derived. Based on this mean-variance approach, Sharpe (1964) and Lintner (1965) designed an equilibrium model, the Capital Asset Pricing Model (CAPM), aimed at describing asset returns. Assuming homogenous beliefs, every investor holds the market portfolio derived from this equilibrium model.

Subsequently, Black and Litterman (1990, 1992) proposed a formal model based on the desire to combine neutral views consistent with market equilibrium and individual active views. They introduce confidence levels on the prior distribution and on individual beliefs and obtained the joint distribution. Based on a Bayesian approach, the expected return incorporates market views and individual expectations. The Black-Litterman approach in its original form can be summarized as the following multi-step process (see Idzorek (2004)).

Based on the risk aversion coefficient (λ), the historical variance covariance matrix (Σ) and the vector of market capitalization weights (ω_M), the vector of implied equilibrium returns in excess of the risk-free rate can be obtained as:[2]

$$\Pi = \lambda \Sigma \omega_M$$

This, of course, is equivalent to using a standard Capital Asset Pricing Model. The parameter λ can thus be rewritten as:[3]

$$\lambda = \left(\frac{E(R_M) - R_F}{\sigma_M^2} \right)$$

where the indices F and M denote the risk-free rate and the market portfolio respectively.

Individual active views, for example based on forecasting procedures, are then introduced. These k views can either be relative or absolute and are represented in the $kx1$-vector Q. The kxn-projection matrix P will be used to define these views:

$$Q = P \cdot R_A$$

A confidence level will be associated with each of the views expressed in Q. Thus, the individual beliefs can be described by a view distribution.[4]

$$P \cdot R_A \sim N(Q, \Omega)$$

In the same way, the confidence in the equilibrium model and the derived implied returns can be defined. Consequently, we obtain the prior equilibrium distribution:

$$R_N \sim N(\Pi, \tau\Sigma)$$

Ω and τ have to be calibrated. We will come back to this problem in the penultimate section.

2. More generally, this reverse-engineering mechanism can be applied to infer the expected return estimate that can support *any* given asset allocation, and not necessarily an asset allocation corresponding to market cap weightings.
3. Note that λ is the market risk aversion coefficient.
4. In all those Bayesian approaches normal distributions are assumed.

A Bayesian Model of Active Hedge Fund Style Allocation Decisions

Following the Bayesian rule the two distributions are combined (cf. Satchell and Scowcroft (2000)) to yield the following distribution:

$$R_{BL} \sim N\left(E(R_{BL}),\left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\right)$$

where

$$E(R_{BL}) = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right] \quad (1)$$

This distribution incorporates both the neutral (equilibrium) and the active views. Taking this expected return as an input, the optimal Black-Litterman portfolio weights (ω_{BL}) are then given by:

$$\omega_{BL} = (\lambda\Sigma)^{-1} E(R_{BL})$$

An Extension to Higher Moments

While the Black-Litterman model is well-suited for portfolio construction in the context of active asset allocation decisions, it suffers from an important limitation, namely that it is based on the Markowitz model, where volatility is used as the definition of risk.

In order to apply a Black-Litterman approach to hedge funds, we first propose to extend the Black-Litterman model by taking higher moments into account so as to turn it into a four-moment portfolio selection model.

Portfolio allocation with higher moments

The first step is to derive the neutral expected returns that enter the Black-Litterman formula. In order to be coherent with the non-normality assumption and the consideration of higher moments, $\Pi = \lambda\Sigma\omega_M$ cannot be the appropriate formula to do so.

Going back to the basics of utility-maximization, we note that the mean-variance approach is, provided non-normal asset returns, only a second order approximation of a general utility function.

In order to take higher moments into account, we consider any arbitrary utility function. The investor is assumed to be maximizing the utility emanating from wealth invested in a portfolio with return denoted by R. The fourth-order Taylor expansion gives us:

$$U(R) = \sum_{k=0}^{4}\left[\frac{U^{(k)}(E(R))}{k\,!}(R - E(R))^k\right] + o\left[(R - E(R))^4\right]$$

where $U^{(k)}$ denotes the k^{th} derivative of the function U. Taking the expectation on both sides yields:

$$E[U(R)] \approx U(E(R)) + \frac{U^{(2)}(E(R))}{2}\mu^{(2)}(R) + \frac{U^{(3)}(E(R))}{6}\mu^{(3)}(R) + \frac{U^{(4)}(E(R))}{24}\mu^{(4)}(R)$$

with the centralized moments:

$$\mu^{(2)}(R) = E\left[(R - E(R))^2\right]$$
$$\mu^{(3)}(R) = E\left[(R - E(R))^3\right]$$
$$\mu^{(4)}(R) = E\left[(R - E(R))^4\right]$$

Thus, we can approximate any utility function of a portfolio return as a function of expected portfolio return and standard deviation, but also as a function of third and fourth moments of the portfolio return distribution. The mean-variance corresponds to a second order approximation that is, of course, less exact. The new maximization problem yields:

$$\max_{\omega} \Phi\left(\mu(R_P),\mu^{(2)}(R_P),\mu^{(3)}(R_P),\mu^{(4)}(R_P)\right)$$

Such that $\sum_{i=1}^{n} \omega_i = 1 - \omega_0$.

A Bayesian Model of Active Hedge Fund Style Allocation Decisions

with:

$$\mu(R_p) = E(\omega' R) + \omega_0 R_0 = \omega' E(R) + \omega_0 R_0$$
$$\mu^{(2)}(R_p) = \omega' E\left[(R - E(R))(R - E(R))'\right]\omega = \omega' \Sigma \omega \tag{2}$$
$$\mu^{(3)}(R_p) = \omega' E\left[(R - E(R))(R - E(R))' \otimes (R - E(R))'\right](\omega \otimes \omega) = \omega' \Omega_\omega$$
$$\mu^{(4)}(R_p) = \omega' E\left[(R - E(R))(R - E(R))' \otimes (R - E(R))' \otimes (R - E(R))'\right](\omega \otimes \omega \otimes \omega) = \omega' \Psi_\omega$$

where $\otimes$ denotes the Kronecker-product, $R = (R_1,\ldots,R_n)'$ the vector of asset returns, $\omega = (\omega_1,\ldots,\omega_n)'$ the vector of portfolio proportions invested in these assets, ω_0 the position held in the risk-free asset (return R_0) and $\mu = (\mu_1,\ldots,\mu_n)'$ the mean return vector. Ω_ω is the vector of co-skewnesses for the weighting vector ω and Ψ_ω the vector of co-kurtosises respectively (see next section for more details). We take this matrix representation from Jondeau and Rockinger (2004) who used it in a different approach.

Solving the problem as for example in Hwang and Satchell (1999) we obtain:

$$\mu - R_0 = \alpha_1 \beta^{(2)} + \alpha_2 \beta^{(3)} + \alpha_3 \beta^{(4)} \tag{3}$$

with $\beta^{(i)}$ the vectors of portfolio betas defined as (see next section and Martellini and Ziemann (2005) for an interpretation of these beta coefficients):

$$\beta^{(2)} = \frac{\Sigma\omega}{\mu^{(2)}(R_p)} \quad \beta^{(3)} = \frac{\Omega_\omega}{\mu^{(3)}(R_p)} \quad \beta^{(4)} = \frac{\Psi_\omega}{\mu^{(4)}(R_p)} \tag{4}$$

The variables α_i can be understood as the risk premia associated with covariance, coskewness and cokurtosis respectively. Hence, we have obtained a four-moment-Capital Asset Pricing Model similar to that of Hwang and Satchell (1999).

One possibility to obtain the $\hat{\alpha}_i$ estimates is a simple GLS-regression. However, since our model will be applied to data including hedge funds, for which data is not available at a frequency greater than monthly, we are constrained to small samples. In this case, one is naturally led to mitigate sample risk through the introduction of a specific model, which of course comes at the cost of specification risk. As in Jondeau and Rockinger, we assume a specific representative utility function. Due to its appealing parameter interpretation, we chose the constant absolute risk aversion function $U(W) = -e^{(-\lambda W)}$.

The risk premia are then given by (cf. Hwang and Satchell (1999) or Jondeau and Rockinger (2004)):

$$\alpha_1 = \frac{\lambda \mu^{(2)}(R_p)}{A} \quad \alpha_2 = \frac{\lambda^2 \mu^{(3)}(R_p)}{2A} \quad \alpha_3 = \frac{\lambda^3 \mu^{(4)}(R_p)}{6A} \tag{5}$$

with

$$A = 1 + \frac{\lambda^2}{2}\mu^{(2)}(R_p) - \frac{\lambda^3}{6}\mu^{(3)}(R_p) - \frac{\lambda^4}{24}\mu^{(4)}(R_p) \tag{6}$$

The risk aversion parameter λ can be calibrated with respect to historical data. Here we use long-term estimates based on stock return estimates over the 1900-2000 period for 16 countries by Dimson, Marsh and Staunton (2002) to obtain $\lambda = 2.14$, based on a 6.20% risk premium and 17% volatility (see 34-1 page 311 in Dimson, Marsh and Staunton (2002)). The various co-moments are estimated over the whole sample period January 1997 to December 2004.

It is important to note that the vectors of portfolio betas as well as the risk premia (α_i) are functions in the weighting vector ω (see equations (2)-(6)). Thus, equation (3) gives us a deterministic relationship between the expected returns (left-hand side) and the weighting vector (right-hand side). Built on the hypothesis of homogenous expectations,

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this equation represents a pricing relationship for a distinct market segment and can be used in two different ways:

i) to obtain expected returns[5] based on exogenous portfolio weights.
ii) to obtain the weighting vector based on exogenous expected returns.

The idea is that equation (3) yields a non-linear equation system with *n* equations and *n* unknowns that can be solved using standard numerical methods.

In what follows, we will create "neutral portfolios weights" based on minimizing the portfolios' Value-at-Risk (next section). Using equation (3), those views will then be taken in order to obtain "neutral" expected returns (approach i)). The penultimate section presents a prediction model that will be used to create active views (expressed in terms of returns). The latter will be incorporated with the neutral views applying the known Bayesian approach (equation (1)). Finally, based on those Black-Litterman returns, equation (3) allows us to obtain the resulting Black-Litterman portfolio weights (approach ii)).

5. These are obtained endogenously and sometimes called implied equilibrium returns.

Designing a Strategic
Style Allocation Benchmark

The key to the implementation of a structured top-down approach to investing in hedge funds is to properly define a target asset allocation across various hedge fund styles. A variety of investment constraints and objectives need to be taken into account in the design of the target allocation. In particular, the target allocation should be designed so as to allow for an optimal mix with the client's existing stock and bond portfolio.

In the real world, investors are not only interested in maximizing expected return and minimizing volatility, but also in limiting the loss with a given probability (1-α). This is the reason why our objective shall focus on a measure like Value-at-Risk at (1-α)% which is defined as the negative value of the α-quantile of the underlying return distribution. Assuming a normal distribution this measure is simply given by:

$$VaR(1-\alpha) = -(\mu + z_\alpha \sigma)$$

with z_α the α-quantile of the standard normal distribution.

A classic way to analyze and formalize the benefits of investing in hedge funds is to note the improvement in the risk-return trade-off they allow when included in a traditional long-only stock and bond portfolio. Applying the foregoing Value-at-Risk formula comes back to making the restrictive assumption that asset returns are normally distributed. Recent research has shown that the returns of alternative funds are clearly not Gaussian (see for example Brooks and Kat (2002)). In the case of portfolios that include derivative instruments, the assumption of Gaussian returns is not in fact tenable. Even if the return of the traditional asset classes were Gaussian, the return of funds using derivative instruments or dynamic strategies relating to those traditional classes would not be. Derivative instruments generally generate cash flows that are non-linear functions of the underlying asset value, and it is well known that a non-linear function of a Gaussian variable is not distributed in a Gaussian manner.

Methodology

In this section, we present a method for optimal allocation decisions to hedge funds that can be used in the absence of specific views on hedge fund strategy returns when higher moments of hedge fund return distribution need to be taken into account.

The reasons why it is necessary to go beyond volatility as a measure of risk is that i) hedge fund returns are notoriously non-Gaussian (see for example Fung and Hsieh (1997)), and ii) the minimization of the 2nd order moment (i.e., the volatility) is often accompanied by a significant increase in higher moments (see Sornette et al. (2000)).

The definitions of higher order central portfolio moments can be used to express in turn the variance *Var(R),* skewness *S(R)* and (excess) kurtosis *K(R)* in the following way:

$$Var(R) = \mu^{(2)}(R)$$

$$S(R) = \frac{\mu^{(3)}(R)}{\left[\mu^{(2)}(R)\right]^{3/2}}$$

$$K(R) = \frac{\mu^{(4)}(R)}{\left[\mu^{(2)}(R)\right]^{2}} - 3$$

In the rest of the paper, we look at the most widely-used set of hedge fund strategies, namely Long/Short Equity, Event Driven, Relative Value and CTA. According to CSFB Tremont, these four broad categories made up 91 percent of assets under management in

Designing a Strategic Style Allocation Benchmark

the hedge fund industry at the end of 2003. Likewise, in the CISDM hedge fund database, funds in these strategies constitute 85 percent of total assets under management by single hedge funds. As a proxy for the return on these hedge fund strategies, we use the *Edhec Alternative Indexes*.[6] Each of these indexes can be thought of as the best one-dimensional summary of the information contained in competing hedge fund indexes for the corresponding strategy.[7]

Strategies such as Relative Value or Event Driven present low levels of volatility but significantly negative skewness and significantly positive excess kurtosis (see Illustration 1).

Illustration 1: Analysis of the return distribution of hedge fund strategies' monthly returns (from January 1997 through December 2004)[8]

	Return	Volatility	Skewness	Exc. Kurtosis
Long/Short Equity	0.96%	2.14%	0.07	0.99
CTA Global	0.74%	2.70%	0.10	-0.16
Relative Value	0.81%	0.99%	-1.25	3.55
Event Driven	0.93%	1.71%	-2.08	10.49
Proxy Stock Markets	0.63%	4.86%	-0.50	0.04
Proxy Bond Markets	0.27%	1.03%	-0.79	1.41

This is the reason why we should incorporate higher moments in the Value-at-Risk measure. This can be done through a method called a Cornish-Fisher expansion (see Jaschke (2002) for a detailed description) which approximates distribution percentiles in the presence of non-Gaussian higher moments.

The Cornish-Fisher expansion is derived from the general Gram-Charlier expansion, using the standard normal distribution as the reference function. For a four-moment approximation of α-percentiles the following formula is given:

$$\tilde{z}_\alpha = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)K - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^2$$

where S denotes the sample skewness, K the sample's excess-kurtosis and the α-percentile of the standard normal distribution. $\tilde{z}_\alpha$ denotes the modified α-percentile. This approximation is built on the hypothesis that the underlying distribution is close to a normal distribution. We obtain the modified Value-at-Risk measure with confidence $(1 - \alpha)$:

$$VaR_{mod}(1-\alpha) = -(\mu + \tilde{z}_\alpha\,\sigma)$$

where σ denotes the standard deviation and μ the mean of the sample. In what follows, we will use a confidence level of 95%.

We also impose portfolio constraints, ensuring for example that:

- No fund or (FoHF) is allocated less than x% and more than y% of the overall portfolio, where x and y are suitably defined numbers based on the portfolio structure.
- Not less than 5% and not more than 40% is allocated to a single hedge fund strategy, where each FoHF exposure to the individual hedge fund strategy is measured through style analysis.

It has actually been argued (see Jagannathan and Ma (2003)) that the presence of portfolio constraints, in addition to avoiding corner solutions in optimization techniques, allows one to achieve a better trade-off between specification error and sampling error similar to what can be achieved by statistical shrinkage techniques (see Jorion (1986) and Ledoit (1999)). This also allows us to mitigate biases in the FoHF selection and allocation process.

6. In the case of Global Macro and CTA, we construct a portfolio that is equal weighted in the indexes for these two strategies.
7. For more information, see www.edhec-risk.com, where monthly data on Edhec Alternative Indexes can be downloaded, as well as Amenc and Martellini (2003).
8. We used the S&P 500 to proxy equity market returns, and a portfolio made up of 20% US Government bonds, 45% mortgage bonds, and 35% corporate bonds to proxy bond market returns. The Edhec Alternative Indexes were used to proxy the different hedge fund strategies.

Designing a Strategic Style Allocation Benchmark

Consequently, the objective function in order to obtain the optimal neutral portfolio weight vector ω is given by:

$$\omega = \operatorname{argmin} VaR_{mod}(95\%) \quad \text{s.t.} \quad \omega_i \in [0.05;0.40]\ \forall i \qquad (7)$$

Note that all the elements of the Value-at-Risk of a portfolio are either linear (expected return) or non-linear functions (variance, skewness, kurtosis) in the weighting vector so that the foregoing program is justified (see equation (2)).

Contrasted Diversification Benefits of Various Hedge Fund Strategies

As evidenced in Amenc et al. (2003), different alternative investment strategies present dramatically different exposures to various risk factors. It is thus important to identify the strategies that are likely to offer the best diversification benefits with respect to traditional asset classes. To do so, we have to assess the extent to which they could lead, in a portfolio context, to a marginal reduction in symmetric risk (measured by volatility) as well as in asymmetric and in fat-tail risks (measured in terms of skewness and kurtosis).

As in Martellini and Ziemann (2005), we define the following co-moments (co-variance, co-skewness and co-kurtosis):[9]

$$CoV(R_i, R_j) = E[(R_i - E(R_i))(R_j - E(R_j))]$$
$$CoS(R_i, R_j) = E[(R_i - E(R_i))(R_j - E(R_j))^2]$$
$$CoK(R_i, R_j) = E[(R_i - E(R_i))(R_j - E(R_j))^3]$$

If we denote the initial portfolio as P, and the new portfolio as $P' = (1 - \varepsilon)P + \varepsilon A$, the marginal impact of the introduction of some small amount ε invested in a new asset A (e.g., a hedge fund or hedge fund portfolio) on the second moment of portfolio distribution is, as a first order approximation:

$$Var(R_{P'}) - Var(R_P) = -2\varepsilon Var(R_P) + 2\varepsilon CoV(R_A, R_P)$$

From this, we obtain the following condition, which states that the introduction of the new asset A has led to a decrease in portfolio variance if and only if the beta of this asset with respect to the initial portfolio is less than one.

$$Var(R_{P'}) \leq Var(R_P) \Longleftrightarrow \beta^{(2)}_{A/P} = \frac{CoV(R_A, R_P)}{Var(R_P)} \leq 1$$

Similarly, we obtain that the marginal impact of the introduction of some small amount ε invested in a new asset A on the third moment of portfolio distribution is, as a first order approximation:

$$\mu^{(3)}(R_{P'}) - \mu^{(3)}(R_P) = \mu^{(3)}((1-\varepsilon)R_P + \varepsilon R_A) - \mu^{(3)}(R_P) \underset{\varepsilon \to 0}{\approx} -3\varepsilon\mu^{(3)}(R_P) + 3\varepsilon CoS(R_A, R_P)$$

which leads to the following conditions:

$$\text{If } \mu^{(3)}(R_P) > 0, \mu^{(3)}(R_{P'}) \geq \mu^{(3)}(R_P) \Longleftrightarrow \beta^{(3)}_{A/P} \equiv \frac{CoS(R_A, R_P)}{\mu^{(3)}(R_P)} \geq 1$$

$$\text{If } \mu^{(3)}(R_P) < 0, \mu^{(3)}(R_{P'}) \geq \mu^{(3)}(R_P) \Longleftrightarrow \beta^{(3)}_{A/P} \equiv \frac{CoS(R_A, R_P)}{\mu^{(3)}(R_P)} \leq 1$$

Finally, we obtain that the marginal impact of the introduction of some small amount ε invested in a new asset A on the fourth moment of portfolio distribution is, as a first order approximation:

$$\mu^{(4)}(R_{P'}) - \mu^{(4)}(R_P) = \mu^{(4)}((1-\varepsilon)R_P + \varepsilon R_A) - \mu^{(4)}(R_P) \underset{\varepsilon \to 0}{\approx} -4\varepsilon\mu^{(4)}(R_P) + 4\varepsilon CoK(R_A, R_P)$$

9. With these definitions, it should be noted that CoV $(X, X) = \mu^{(2)}(X) = Var(X)$, but CoS $(X, X) = \mu^{(3)}(X) \neq S(X)$ and CoK $(X, X) = \mu^{(4)}(X) \neq K(X)$.

Designing a Strategic Style Allocation Benchmark

which leads to the following condition:

$$\mu^{(4)}(R_{P'}) \leq \mu^{(4)}(R_P) \Longleftrightarrow \beta^{(4)}_{A/P} \equiv \frac{\mathrm{CoK}(R_A, R_P)}{\mu^{(4)}(R_P)} \leq 1$$

Hence, a low or negative second, third, and fourth moment beta indicates good diversification potential, in the sense of a potential for a decrease in the average risk of the overall portfolio (i.e., volatility), in the bias towards lower-than-average returns (i.e., skewness), and in fat tails (i.e., kurtosis), respectively.[10]

For the sake of illustration, we focus on the standpoint of a specific investor with a given initial asset allocation, e.g., 20% in stocks and 80% in bonds.[11] This is consistent with the situation of a typical institutional investor in the presence of liability constraints. While no existing commercial index is perfectly suited to representing a specific investor's liability structure, bond indices are typically the asset class most correlated with institutional investors' liabilities, and as a result should be predominant in institutional investors' allocation. The following table illustrates the hedge fund higher moment betas with the initial portfolio (20% stocks and 80% bonds):

Illustration 2: Higher Moment Betas – from 01/1997 through 12/2004[12]

	Long/Short Equity	CTA Global	Relative Value	Event Driven
Beta Covariance $\beta^{(2)}_{A/P}$	0.99	0.24	0.41	0.69
Beta Coskewness $\beta^{(3)}_{A/P}$	1.29	-0.50	0.48	1.57
Beta Cokurtosis $\beta^{(4)}_{A/P}$	0.88	0.55	0.37	0.62

As can be seen from illustration 2, we assume that the CTA Global and Relative Value indices dominate the optimal minimum Value-at-Risk portfolio since they yield the highest potential in terms of diversifying the corresponding portfolio return moments. As our approach is going to account for time-varying portfolio weights (a rolling window will be used), it is useful to analyze time-conditional properties of the higher moment betas. The general idea is to obtain regression coefficients that are linearly linked to the above-mentioned higher order betas. These coefficients can then be modeled by a Kalman Smoother technique. The following regressions are natural candidates for this approach:

$$\begin{aligned} R_i &= c_1 + \alpha_1 R_B \\ R_i &= c_2 + \alpha_2 R_B^2 \\ R_i &= c_3 + \alpha_3 R_B^3 \end{aligned} \tag{8}$$

Where R_i denotes the return of index i and R_B the benchmark return here represented by the initial stocks-bonds portfolio. The regression coefficients are directly linked to the higher moment betas:

$$\begin{aligned} \beta^{(2)} &= \alpha_1 \\ \beta^{(3)} &= \alpha_2 \cdot \frac{\mathrm{Var}(R_B^2)}{\mu_B^{(3)}} \\ \beta^{(4)} &= \alpha_3 \cdot \frac{\mathrm{Var}(R_B^3)}{\mu_B^{(4)}} \end{aligned} \tag{9}$$

For simplicity, we assume that the variances associated with the normal, squared and cubic benchmark returns are stationary. The Kalman Smoother is a technique that is often used in the framework of *ex-post* modeling of regression coefficients, whereas GARCH approaches focus on the smoothing of the regression errors. Additionally, the Kalman Smoother is known to be much more reactive than the GARCH models since it responds directly to shocks. Applying the Kalman

10. This is under the assumption of a portfolio with negative skewness, which is the case in our sample for the stock and bond proxies (see Illustration 1).
11. We used the S&P 500 to proxy equity market returns, and a portfolio made up of 20% US Government bonds, 45% mortgage bonds, and 35% corporate bonds to proxy bond market returns.
12. These results were obtained using the series of indices of indices published by Edhec. It is worth stressing that single hedge funds following these strategies might present different diversification profiles. However, as highlighted in Learned and Lhabitant (2002), portfolios made up of 5 to 10 hedge funds appear to be fairly representative of their investment universe. In an attempt to diversify idiosyncratic risk away, most multi-managers generally construct FoHF composed of more than 15 single hedge funds. Results presented in illustration 1 are therefore relevant for most investors.

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Smoother to the three foregoing regressions (equation (7)) delivers time-conditioned coefficients (alphas) and, consequently, using equation (6), higher order betas for each data point. In order to analyze the stability of the latter we present the observed standard deviations.

Illustration 3:
Standard deviations of Higher Moment Betas modeled by Kalman Smoother techniques – from 01/1997 through 12/2004

	Long/Short Equity	CTA Global	Relative Value	Event Driven
Beta Covariance $\beta^{(2)}_{A/P}$	0.17	0.43	0.11	0.16
Beta Coskewness $\beta^{(3)}_{A/P}$	1.62	2.43	0.87	1.70
Beta Cokurtosis $\beta^{(4)}_{A/P}$	0.19	0.53	0.19	0.16

Interestingly, the even co-moment betas (beta covariance and beta cokurtosis) appear to be rather stable. On the other hand, the odd co-moment beta seems to be less stable. This is consistent with the fact that even moments (variance and kurtosis) are natural measures of dispersion (i.e., risk), while odd moments (expected return, skewness) are measures of location, which are notoriously less stable. This suggests that hedge funds' ability to diversify traditional asset portfolios both in terms of reduction in variance and kurtosis is rather robust with respect to sample changes.

Based on these standard deviations of the higher moment betas and the higher moment betas themselves, confidence intervals could be built up. However, in the framework of our study, in the absence of pre-selection of distinct strategies, the higher moment betas only illustrate diversification potential and are in some way predictors for the optimal portfolio decomposition. Following Illustrations 2 and 3 we assume that the CTA Global and Relative Value indices dominate the minimum Value-at-Risk. Especially in the case of the covariance– and the cokurtosis-beta, we have relatively stable results so that, on an out-of-sample basis, improvements in terms of diversification are expected. The following empirical analysis aims to find out whether this proves to be the case.

Results

The minimum-Value-at-Risk portfolio

We rebalanced the optimally designed portfolio (see objective function (equation (7)) every 3 months, using a 36-month rolling window analysis in the optimization procedure. We further assume that the investor is not willing to allocate more than 15% to hedge funds. Since the history of the Edhec Alternative Indexes starts in January 1997, the first calibration period was January 1997 – December 1999. We therefore disposed of out-of-sample returns for the optimally designed portfolios from January 2000 onward. The following illustration presents the evolution of the composition of these portfolios on the out-of-sample period (i.e., from January 2000 through December 2004).

Illustration 4:
Evolution of the Composition of the Optimally Designed Portfolio (from 01/ 2000 through 12/2004)

	Long/Short Equity	CTA Global	Relative Value	Event Driven
2000				
Q1	15%	40%	40%	5%
Q2	15%	40%	40%	5%
Q3	15%	40%	40%	5%
Q4	15%	40%	40%	5%
2001				
Q1	15%	40%	40%	5%
Q2	5%	40%	40%	15%
Q3	5%	40%	40%	15%
Q4	5%	40%	40%	15%
2002				
Q1	5%	40%	40%	15%
Q2	5%	40%	40%	15%
Q3	5%	40%	40%	15%
Q4	5%	40%	40%	15%
2003				
Q1	5%	40%	40%	15%
Q2	5%	40%	40%	15%
Q3	5%	40%	40%	15%
Q4	5%	40%	22%	33%
2004				
Q1	5%	40%	15%	40%
Q2	5%	40%	15%	40%
Q3	5%	32%	23%	40%
Q4	5%	15%	40%	40%

Designing a Strategic Style Allocation Benchmark

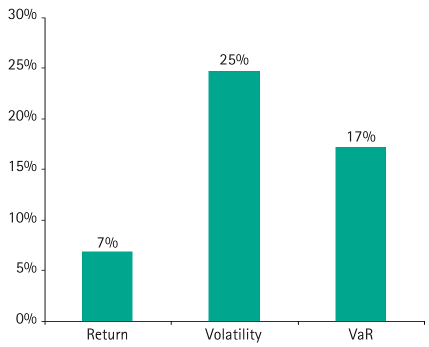
To assess the benefits provided by the hedge fund portfolio in terms of both average and extreme risk diversification, we computed the decrease in volatility and in modified Value-at-Risk it entailed over the out-of-sample sample period.

Illustration 5:
Diversification Benefits – from 01/2000 through 12/2004

	Optimally Designed Portfolio
Return	30%
Volatility	-8%
VaR	-13%

As can be seen from Illustration 5, normal and extreme risks are reduced by 8% and 13% when traditional asset classes are mixed with our optimally designed portfolio. At the same time, the return increases by 30%. These results confirm that hedge fund strategies provide institutional investors with appealing diversification properties. However, most institutional investors are still reluctant to invest a significant proportion of their assets in hedge funds. They must therefore strive to make the best out of their limited exposure to hedge fund strategies. The construction methodology we have presented is precisely designed to maximize the benefits obtained by investors in terms of diversification.

Illustration 6:
Improvement in Diversification Benefits relative to an off-the-shelf product – from January 2000 through December 2004



Comparison with investment in off-the-shelf funds of hedge funds

As a means of comparison, we conducted the same experiment with the Edhec Funds of Hedge Fund index as a proxy for a typical investment in hedge funds. We expect the diversification benefits allowed by the introduction of hedge funds in the portfolio to be much weaker with this alternative compared to the optimal approach used above because of the lack of specific focus on an optimal strategy allocation.

As expected, investors would have improved the reduction in normal and extreme risks that they would have obtained with off-the-shelf products by 25% and 17% *on an out-of-sample basis.*

Robustness Analysis

Because of all sorts of frictions and associated costs, most investors would be reluctant to adjust their strategic asset allocation decisions to hedge funds at a quarterly frequency. In an attempt to find out whether the improvements we obtained in terms of both normal and extreme risks are due to our rebalancing scheme (i.e., our portfolio is rebalanced quarterly), we repeated the previous experiment with semi-annual and annual rebalancing frequencies. We then contrasted the results we obtained with semi-annual and annual rebalancing frequency with the ones presented above. Interestingly, we observe that the results are particularly robust since the optimally designed portfolio constructed with semi-annual rebalancing frequency allowed for a 7.58% and 12.66% reduction in normal and extreme risks

Designing a Strategic Style Allocation Benchmark

respectively. The optimally designed portfolio constructed with annual rebalancing frequency, on the other hand, allowed for a 7.57% and 12.58% reduction in normal and extreme risks respectively. It should be noted that these results do not come as a surprise given the fact that in all cases the average composition of the optimally designed portfolios is almost perfectly identical.

Illustration 7:
Testing the impact of the rebalancing scheme

Rebalancing Frequency	Quarterly	Semi-Annual	Annual
Return Evolution	29.52%	30.26%	30.26%
Volatility Evolution	-7.62%	-7.58%	-7.57%
VaR Evolution	-12.70%	-12.66%	-12.58%

This robustness check allows us to conclude that the improvement we obtained in terms of diversification benefits with our optimally designed portfolios are not due to the rebalancing schemes but to the optimal weighting of hedge fund strategies.

We subsequently tested the robustness of our results when the initial allocation of the investor varies. As can be seen in Illustration 8, the benefits obtained with our approach in terms of extreme risk reduction, and to a certain extent in terms of normal risk reduction, remain stable. In other words, alternative diversification suits all types of traditional investors.

Illustration 8:
Testing the impact of the investor's initial allocation

% Equity	10%	15%	20%	25%	30%	35%	40%
Return Evolution	20%	23%	30%	37%	43%	53%	68%
Volatility Evolution	-7%	-6%	-8%	-9%	-10%	-11%	-12%
VaR Evolution	-15%	-12%	-13%	-14%	-14%	-15%	-15%

Illustration 9:
Testing the impact of the allocation to hedge fund strategies

% Alternative Investment	30%	25%	20%	15%	10%	5%
Impact on Return	75%	63%	50%	30%	25%	12%
Impact on Volatility	-8%	-8%	-7%	-8%	-4%	-2%
Impact on VaR	-21%	-18%	-15%	-13%	-8%	-4%

Finally, we tested the impact of an increase in the allocation made to hedge fund strategies. Not surprisingly, the higher the allocation made to hedge fund strategies, the higher the benefits in terms of normal and extreme risk reduction (see Illustration 9).

In this section, we have argued that a customized portfolio based on hedge fund strategy indices outperforms a one-size-fits-all solution in terms of diversification benefits. An added advantage of style indices is that they allow investors to perform style timing, i.e., dynamically change their allocation to various hedge fund strategies in response to changes in their expectations about expected returns on these strategies. In other words, the benefits of active style rotation can be added to the benefits of diversification to ensure the design of an even better investment solution.

Generating Active Views
on Hedge Fund Strategies and Application to Active Style Allocation Decisions

There is now a consensus in empirical finance that expected asset returns, and also variances and covariances, are, to some extent, predictable. Pioneering work on the predictability of asset class returns in the U.S. market was carried out by Keim and Stambaugh (1986), Campbell (1987), Campbell and Shiller (1988), Fama and French (1989), and Ferson and Harvey (1991). Subsequently, some authors started to investigate this phenomenon on an international basis by studying the predictability of asset class returns in various national markets (see, for example, Bekaert and Hodrick (1992), Ferson and Harvey (1993, 1995), Harvey (1995), and Harasty and Roulet (2000)).

While there has been a significant amount of research on the predictability of traditional asset classes, and the implications in terms of tactical asset allocation strategies, until recently very little was known about the predictability of returns emanating from alternative vehicles such as hedge funds. In a recent paper, Amenc et al. (2003) examine (lagged) multi-factor models for the return on nine hedge fund indexes. The factors are chosen to measure the many dimensions of financial risks: market risks (proxied by stock prices, interest rates and commodity prices), volatility risks (proxied by implicit volatilities from option prices), default risks (proxied by default spreads) and liquidity risks (proxied by trading volume). They show that a parsimonious set of models captures a very significant amount of predictability for most hedge fund styles. They also find that the benefits in terms of tactical style allocation portfolios are potentially very large. Even more spectacular results are obtained both for an equity-oriented portfolio mixing traditional and alternative investment vehicles, and for a fixed-income-oriented portfolio mixing traditional and alternative investment vehicles.

The conclusion from this research is that there is at least as much evidence of predictability in hedge fund returns as there is in stock and bond returns. In what follows, we present the results of a simple experiment showing how the presence of predictability in hedge fund returns can be exploited in the context of an active asset allocation process. In an attempt to alleviate the risk of model specification, we perform a simple in-sample analysis using the historical relationship between a set of factors and hedge fund returns. The spirit of the experiment we now conduct is actually not to put an emphasis on a specific econometric process that could be used to generate signals about hedge fund style returns. It is rather to put an emphasis on the fact that sophisticated and reliable active asset allocation models are available (e.g., the Black-Litterman model), which can be further extended to account for the presence of extreme risk measures.

Forming Active Views on Hedge Fund Returns

In this section we will propose an approach in order to detect individual active views on various hedge fund styles. For each hedge fund strategy, we obtain a bullish, a bearish or a neutral view concerning the expected return based on a lagged factor analysis.

The predictive factors we use are:

- Implied Volatility (VIX) - CBOE SPX Volatility VIX
- First differences of the implied volatility
- Commodity Index - Goldman Sachs
- Term Spread - Lehman US Treasury 5-7 years minus Lehman US Treasury 1-3 years
- Credit Spread - Lehman US Universal: High Yield Corp. Red Yield minus Lehman US Treasury 1-3 years

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- Value vs. Growth - S&P 500 Barra/Value minus S&P 500 Barra/Growth
- Small Cap vs. Large Cap - S&P 600 Small Cap minus S&P 500 Composite
- S&P 500 Composite return
- T-Bill - Merrill Lynch T-Bill 3 month
- US Dollar - US MAJOR CURRENCY MAR 73 = 100 (FED) EXCHANGE INDEX
- Bond return volatility - calculated over one month Lehman US Aggregate returns.

The approach we follow is based on a simple univariate conditional factor analysis. The idea is to consider each factor separately and to estimate the correlation between the hedge fund returns and the three-month lagged factor values. Over the sample period from October 1996 to September 2004, the values for each factor are classified into three different states of nature based on the corresponding distribution: low (0%-33%), medium (33%-66%) and high (66%-100%).

Next, we consider the hedge fund returns for each strategy, conditional on the state of nature of the three-month lagged factor values. One of the letters H (for high), M (for medium) or L (for low) is associated with each combination, as a function of the difference between the conditional and the unconditional mean:[13]

- H indicates that the conditional mean return on the strategy is significantly greater than the unconditional one, suggesting a bullish view according to the factor.[14]
- L indicates that the conditional mean is significantly smaller than the unconditional one, suggesting a bearish view according to the factor.
- M indicates that there is no significant difference between conditional and unconditional mean.

We report the result of this analysis in Illustration 10.

This suggests for example that when value and size spreads are tight, when markets are posting average returns, or when term and credit spreads are narrow at time t, long/short equity funds tend to achieve high returns 3 months later. This is again consistent with the fact that these funds typically hold – for liquidity issues – long positions in small companies (respectively growth stocks) and short positions in large companies (respectively value stocks). Similarly, when stock markets are moving strongly in one direction (i.e., bull or bear), the correlation between individual stocks tends to increase, producing profits since returns generated on the long positions are offset by those generated on short positions. Finally, since small companies and growth stocks tend to show greater sensitivity to credit and term spreads, long/short equity funds are negatively exposed to the widening of these spreads.

Illustration 10: Conditional Factor Analysis of Hedge Fund Returns

	Implied Volatility			Diff. Implied Volatility			Commodity Index			Term Spread			Credit Spread			Value minus Growth			Small minus Large			S&P 500			T-Bill			US Dollar			Lehman Bond Return		
	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high
Long/Short Equity	M	M	M	L	M	M	M	M	M	M	H	L	M	H	L	H	M	L	H	M	M	L	H	M	M	M	M	M	M	M	M	M	M
CTA Global	H	M	L	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	H	L	M	M	M	M	M	M	M	M	H	M	M	M	L
Relative Value	L	M	M	L	M	M	M	M	M	M	H	L	M	H	L	H	M	M	M	M	M	L	M	M	L	M	H	M	M	M	M	M	M
Event Driven	L	M	M	M	M	H	M	M	M	M	H	M	M	H	M	H	M	M	M	M	M	L	H	M	M	M	M	M	M	M	M	M	M

13. Each combination is characterized by a strategy, a factor and a state of nature. With four strategies, eleven factors and three states of nature, we obtain 132 combinations.
14. We use a 20% significance level so as to generate a reasonably large number of H and L values.

Allocation to Hedge Funds in the Context of Surplus Optimisation: a More Robust Approach

Conditionally on the state of nature of the different factors, active views on hedge fund returns are determined by the combination of individual factor influences. More specifically, we assign the value +1 to H (high), the value –1 to L (low) and the value 0 to M and calculate the sum S of these numbers over all factors. The view on the strategy is bullish if this number is positive (indicating more H occurrences than L occurrences for the strategy at a given date) and bearish if it is negative (indicating more L occurrences than H occurrences for the strategy at a given date). The corresponding value Q_i in the view-vector will be defined as:

$$Q_i = (1 + x \cdot sign(S))\Pi_i$$

where Π_i is the implied expected return on strategy i (cf., previous section). This scheme implies that a bullish active bet is obtained (with respect to a neutral estimate) when S is positive, and a bearish view is obtained (with respect to a neutral estimate) when S is negative. The confidence in our view will be determined by the corresponding i^{th} diagonal value Ω_{ii} in the Variance-Covariance matrix of the view-distribution. In what follows, we define it as:

$$\Omega_{ii} = \left(1 - \frac{|S|}{K}\right)\Sigma_{ii}$$

where K is the total number of factors (11 in this case). Based on this formulation, it appears that if all factors agree on a bullish or bearish view for a given strategy, then $|S| = K$ and the uncertainty on the view goes to zero, as is normal since confidence is maximal in that case. Note also that we use the variance of the returns on the hedge fund strategy Σ_{ii} as a multiplicative scaling factor. The intuition is that an investor perceives more uncertainty on a view that relates to a riskier asset, as it is more likely to experience large moves in a given interval of time. Finally, the matrix P is defined so that each line corresponds to an active view and consists of zeros in all but the i's place (view on the i's asset).

Note that the parameter x is completely arbitrary. The parameters x and τ determine the relative weight between neutral portfolio weights and the view-adjusted ones, and these degrees of freedom can be adjusted to generate an active portfolio that will deviate more or less with respect to the benchmark portfolio. More specifically, since the standard deviation of the individual view is perfectly determined by the above relationship, we use the parameter τ in order to calibrate the model in terms of the trade-off between neutral and active views. The higher τ, the lower the relative confidence in the neutral view and the more weight is put on individual beliefs. Since the parameter x has a function similar to the parameter τ, we chose to fix the former arbitrarily at 40% whereas τ will be discussed in the application below.

Implications for Active Style Allocation Decisions

Our opportunity set is composed of the same four Edhec Hedge Fund indices as above. The sample period is from January 1997 to December 2004. We use the optimal minimum VaR portfolio obtained in section 3 as the reference benchmark portfolio from which neutral implied expected return estimates will be obtained.

In order to take higher moments in hedge fund return distributions, as well as preferences about these higher moments, into account (through a CARA utility function), we apply the four-moment portfolio selection model presented in the second section. We also create active views by applying the prediction procedure presented above.

Allocation to Hedge Funds in the Context of Surplus Optimisation: a More Robust Approach

In order to obtain the Black-Litterman portfolios we pursue as follows:

1) We use equation (3) to obtain the implied returns (Π) derived from the "neutral" weights of the minimum Value-at-Risk portfolio (ω_N) presented above:

$$\Pi - R_0 = \alpha_1\beta^{(2)} + \alpha_2\beta^{(3)} + \alpha_3\beta^{(4)}$$

where alphas and betas are functions in ω_N (see the second section for details).

2) Having defined P, Q and Ω we apply equation (1) in order to obtain the Black-Litterman return vector $E(R_{BL})$.

3) Finally, equation (3) will be used in order to obtain the resulting Black-Litterman portfolio:

$$E(R_{BL}) - R_0 = \alpha_1\beta^{(2)} + \alpha_2\beta^{(3)} + \alpha_3\beta^{(4)} + \varepsilon$$

where alphas and betas are functions of the Black-Litterman weighting vector obtained by minimizing the sum of squared residuals ($SSR = \varepsilon'\varepsilon$).

All elements except the parameter τ have already been determined. Our approach concerning this parameter consists of controlling the tracking error of the portfolio with respect to the benchmark portfolio above. We have considered three different Black-Litterman active portfolios associated with different levels of tracking error. The corresponding parameter values τ are 1, 5, and 20.

Illustration 11 presents summary statistics for the corresponding portfolios. The active style selection process, combined with the Black-Litterman portfolio selection method, allows for significant outperformance without a large increase in tracking error, as can be seen from the information ratio values. The excess performance, as well as the tracking error, increase in τ, as expected.

In view of the non-trivial preference of investors for the third and fourth order moments of return distribution functions (cf. Scott and Horvath (1980)), hedge fund performance cannot be analyzed within a mean/variance framework. In an attempt to assess the benefits of our approach we therefore decided to consider a measure that takes account of the whole return distribution, namely the Omega ratio. This measure was recently introduced in Keating and Shadwick (2002) and is defined as follows:

$$\Omega(MAR) = \frac{\int_{MAR}^{b}\left[1 - F(x)\right]dx}{\int_{a}^{MAR}\left[F(x)\right]dx}$$

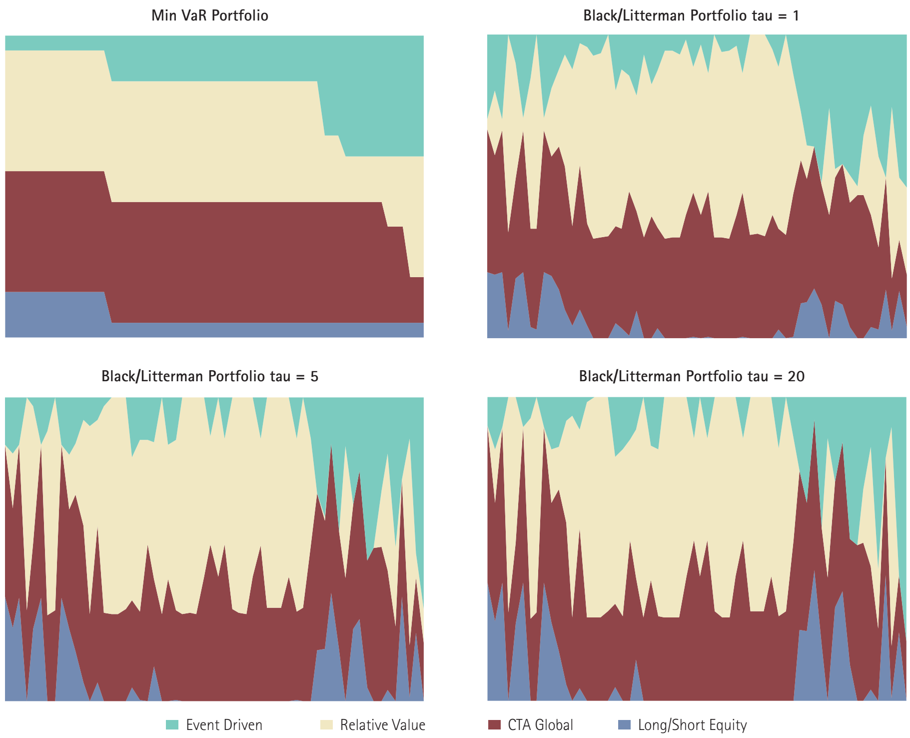
where x is a random variable and F is the cumulative return distribution function of the asset that is being evaluated. The constants a and b respectively represent the lower and upper boundaries of the distribution function. *MAR* corresponds to the minimal acceptable return.

**Illustration 11:
Performance of Active Allocation Strategies**

	PF minVaR	PF Black Litterman $\tau = 1$	PF Black Litterman $\tau = 5$	PF Black Litterman $\tau = 20$
Mean annual return	8.79%	9.79%	10.48%	10.71%
Volatility	4.29%	4.32%	4.39%	4.46%
VaR (95%)	1.33%	1.21%	1.17%	1.19%
Sharpe ratio (r = 3%)	1.58	1.80	1.93	1.95
Tracking error		0.86%	1.24%	1.37%
Information ratio		1.17	1.37	1.41

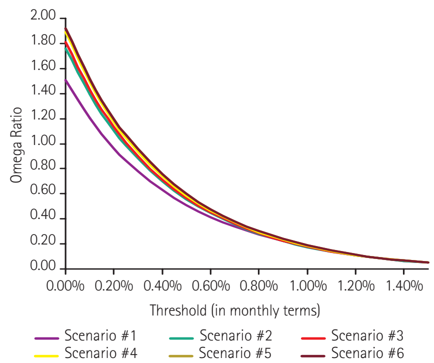
Allocation to Hedge Funds in the Context of Surplus Optimisation: a More Robust Approach

Illustration 12:
Portfolio Allocations



In order to avoid any arbitrary choice of loss threshold (since this level depends on investors' preferences), we decided to plot the Omega functions. We contrasted the results obtained in six different scenarios. In the first scenario, we consider a traditional investor holding 80% bonds and 20% equities. In the second scenario, we consider an investor who invests 85% of his wealth in a traditional portfolio made up of 80% bonds and 20% equities (i.e., resulting in an allocation of 68% for bonds and 17% for equities), and 15% in an equally weighted portfolio made up of hedge fund strategies. In the third scenario, we consider an investor who invests 85% of his wealth in a traditional portfolio made up of 80% bonds and 20% equities (i.e., resulting in an allocation of 68% for bonds and 17% for equities), and 15% in a portfolio made up of hedge fund strategies and optimized according to the min-modified VaR procedure described above. In the fourth, fifth and sixth scenarios, we consider an investor who invests 85% of his wealth in a traditional portfolio made up of 80% bonds and 20% equities (i.e., resulting in an allocation of 68% for bonds and 17% for equities), and 15% in a portfolio made up of hedge fund strategies optimized with the procedure including active views presented in this section. τ values were set to 1, 5, 20, respectively.

Illustration 13:
Risk-Adjusted Performance Analysis[15]



15. It should be noted that a polynomial smoothing procedure was used for presentation purposes.

Allocation to Hedge Funds in the Context of Surplus Optimisation: a More Robust Approach

As expected, the introduction of an equally-weighted portfolio of hedge fund style indices leads to a significant improvement in the Omega function. This strongly suggests that diversification benefits can be largely amplified through the use of an optimally designed portfolio of hedge fund strategies. This is particularly true when the loss threshold lies between 0% and 5%. The introduction of active views further allows for an improvement in the Omega function, with benefits in terms of risk-adjusted performance increasing with τ.

It should be emphasized that the results presented here abstract away from a number of issues. First, because we have performed an in-sample factor analysis of hedge fund returns, we do not claim that the process used in this paper to generate active style allocation views is efficient and should be applied in practice. We rather believe that the implementation of dynamic asset allocation strategies requires the use of a more sophisticated econometric process, which should put great care on avoiding the pitfalls of data mining.

Besides, our analysis does not take into account the presence of transaction costs. Because of significant shifts in asset allocation (as evidenced in illustration 12), we expect such frictions to be a major issue when implementing a style rotation strategy in the hedge fund universe. It is clearly the case that investable supports used in the process should display a high level of liquidity.

Overall, the purpose of the exercise was not to promote a specific style rotation strategy, but rather to illustrate the fact that, given a set of active views on hedge fund strategy returns, which may come either from a qualitative or a quantitative analysis, optimal active asset allocation decisions can be implemented on the basis of a sound and sophisticated investment process.

Conclusion

In this paper, we present evidence that active strategy allocation decisions can generate significant benefits in the context of hedge fund investing. Given the low allocation typically made to alternative investment strategies (i.e., generally 5% to 15% of the global allocation), investors must try to maximize the benefits of their hedge fund portfolios.

As shown in this paper, this can be done by customizing an optimal hedge fund portfolio and by taking into account the investor's original allocation to stocks and bonds.

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Edhec Risk and Asset Management Research Centre

Edhec is one of the top five business schools in France and was ranked 12th in the *Financial Times* Masters in Management Rankings 2005 owing to the high quality of its academic staff (over 100 permanent lecturers from France and abroad) and its privileged relationship with professionals that the school has been developing since it was established in 1906. Edhec Business School has decided to draw on its extensive knowledge of the professional environment and has therefore concentrated its research on themes that satisfy the needs of professionals. Edhec is one of the few business schools in Europe to have received the triple international accreditation: AACSB (US-Global), Equis (Europe-Global) and AMBA (UK-Global). Edhec pursues an active research policy in the field of finance. Its "Risk and Asset Management Research Centre" carries out numerous research programmes in the areas of asset allocation and risk management in both the traditional and alternative investment universes.

The choice of asset allocation

The **Edhec Risk and Asset Management Research Centre** structures all of its research work around asset allocation. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of active management based solely on stock picking as a source of performance. On the other, the appearance of new asset classes (hedge funds, private equity), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity in both conceptual and operational terms. This strategic choice is applied to all of the centre's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; measuring the performance of funds while taking the tactical allocation dimension of the alphas into account; taking extreme risks into account in the allocation; or studying the usefulness of derivatives in constructing the portfolio.

An applied research approach

In a desire to ensure that the research it carries out is truly applicable in practice, Edhec has implemented a dual validation system for the work of the **Edhec Risk and Asset Management Research Centre**. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the centre's advisory board. This board is made up of both internationally recognised researchers and the centre's business partners. The management of the research programmes respects a rigorous validation process, which guarantees both the scientific quality and the operational usefulness of the programmes.

To date, the centre has implemented six research programmes:

Multi-style/multi-class allocation

This research programme has received the support of Misys Asset Management Systems, SG Asset Management and FIMAT. The research carried out focuses on the benefits, risks and integration methods of the alternative class in asset allocation. From that perspective, Edhec is making a significant contribution to the research conducted in the area of multi-style/multi-class portfolio construction.

Performance and style analysis

The scientific goal of the research is to adapt the portfolio performance and style analysis models and methods to tactical allocation. The results of the research carried out by Edhec thereby allow portfolio alphas to be measured not only for stock picking but also for style timing. This programme is part of a business partnership with the firm EuroPerformance (part of the Fininfo group).

Indices and benchmarking

Edhec carries out analyses of the quality of indices and the criteria for choosing indices for institutional investors. Edhec also proposes an original proprietary style index construction methodology for both the traditional and alternative universes. These indices are intended to be a response to the critiques relating to the lack of representativity of the style indices that are available on the market. Edhec was the first to launch composite hedge fund strategy indices as early as 2003. The indices and benchmarking research programme is supported by AF2I, Euronext, BGI, BNP Paribas Asset Management and UBS Global Asset Management.

Edhec Risk and Asset Management Research Centre

Asset allocation and extreme risks

This research programme relates to a significant concern for institutional investors and their managers – that of minimising extreme risks. It notably involves adapting the current tools for measuring extreme risks (VaR) and constructing portfolios (stochastic check) to the issue of the long-term allocation of pension funds. This programme has been designed in co-operation with Inria's Omega laboratory. This research programme also intends to cover other potential sources of extreme risks such as liquidity and operations. The objective is to allow for better measurement and modelling of such risks in order to take them into consideration as part of the portfolio allocation process.

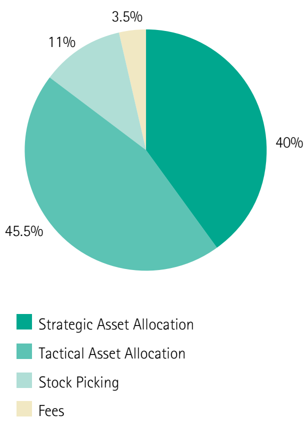
Asset allocation and derivative instruments

This research programme focuses on the usefulness of employing derivative instruments in the area of portfolio construction, whether it involves implementing active portfolio allocation or replicating indices. "Passive" replication of "active" hedge fund indices through portfolios of derivative instruments is a key area in the research carried out by Edhec. This programme is supported by Eurex and Lyxor.

ALM and asset management

This programme concentrates on the application of recent research in the area of asset-liability management for pension plans and insurance companies. The research centre is working on the idea that improving asset management techniques and particularly strategic allocation techniques has a positive impact on the performance of Asset-Liability Management programmes. The programme includes research on the benefits of alternative investments, such as hedge funds, in long-term portfolio management. Particular attention is given to the institutional context of ALM and notably the integration of the impact of the IFRS standards and the Solvency II directive project. This programme is sponsored by AXA IM.

Percentage of variation between funds



Source: Edhec (2002) and Ibbotson, Kaplan (2000).

Research for business

In order to facilitate the dialogue between the academic and business worlds, the centre has recently undertaken four major initiatives:

- Opening of a web site that is entirely devoted to the activity of international research into asset management. www.edhec-risk.com is aimed at a public of professionals who wish to benefit from Edhec's expertise and analyses in the field of applied portfolio management research, such as detailed summaries, from a business perspective, of the latest academic research on risk and asset allocation as well as the latest industry news assessed in the light of the results of the Edhec research programme. www.edhec-risk.com is also the official site for the Edhec Indices.
- Launch of Edhec-Risk Advisory, the consulting arm of the research centre focusing on risk management issues within the buy-side industry, and offering a wide range of services aimed at supporting fund managers and their service providers in the fields of operational risk, best execution, structured products, alternative investment due diligence and risk management system implementation.
- Launch of Edhec Investment Research, in order to support institutional investors and asset managers in implementing the results of the Edhec Risk and Asset Management Research Centre's research. Edhec Investment Research proposes asset allocation services in the context of a "core-satellite" approach encompassing alternative investments.
- Launch of Edhec Alternative Investment Education, which is the exclusive official CAIA association course provider for Europe.

SG Asset Management

(www.sgam.com)

SG Asset Management is one of the world's leading asset managers with 313 billion in assets under management as at 30th September 2005.

A subsidiary of the Société Générale Group, SG AM is a global player with a balanced and robust business model based on:

• its multi-center structure: 2,600 employees, including 600 managers and analysts, are located at the heart of the markets in Continental Europe and the United Kingdom, and in major centers (each with 600 employees) in the United States and Asia;
• a business that covers all asset classes: equities, fixed income, balanced, and alternative investment;
• access to all types of investor: institutions, distributors, corporates and individuals, all of whom benefit from SG AM's leading-edge expertise and a local service.

Thanks to cross-selling (more than a third of net inflows at 30th June 2005) and a focus on quality and constant innovation, SG AM has developed value-added management solutions tailored to clients' specific needs and which optimize performance and control risk.

Since 2000, SG AM has been rated AM2+ by Fitch Ratings, the top rating awarded to an asset management company for the whole of its structure. For investors, this rating is a guarantee of the professionalism of SG AM's teams and the quality of its international organization.

SG Asset Management Alternative Investments

(www.sgam-ai.com)

SG AM Alternative Investments, 100% subsidiary of SG Asset Management, manages €30 bn of assets in the largest scope of alternative investments: hedge funds, structured products, private equity and real estate.

The sustained development of SG AM Alternative Investments results of a successful combination of the processes of an active asset manager and of the capital markets culture allying innovation and risk monitoring. With more than 200 collaborators in the world and benefiting from the strategic and financial support of Société Générale Group, SG AM Alternative Investments has become a leader among the global players in alternative management.

SG AM Alternative Investments has built a global hedge fund management platform with investment professionals located in Paris, New-York and Tokyo. Since 2000, year of establishment of this activity, the company has developed a comprehensive product offer with 3 main ranges of products: Fund of Hedge Funds (diversified and specialized on Long-Short or Relative Value strategies), Single Strategy hedge funds (Long-short, market neutral, volatility arbitrage, directional trading, portable alpha) and Multi-Strategy funds (the funds invest in a selection of 9 internal single strategies and have risk/returns profile from Libor+100 to Libor+400 with daily liquidity). More recently, the company entered the incubation activity by launching a hybrid product with characteristics of private equity and fund of fund.

As of September 30th 2005, SG AM Alternative Investments manages EUR 4.9 bn of hedge funds assets.

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