Integrating Multiple Commodities in a Model of Stochastic Price Dynamics

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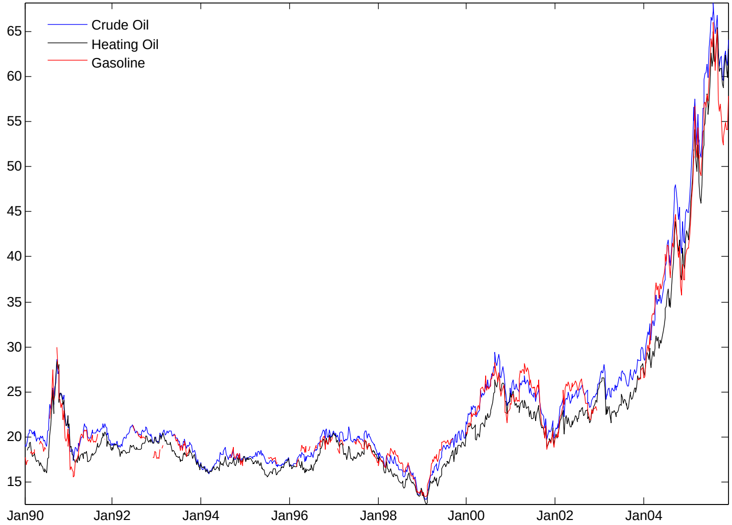
Motivation



12 Months Futures Prices

Motivation (cont'd)

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- Our Contribution: Integrated arbitrage-free model for related (co-integrated) commodity prices
- Who should care: Everybody with a joint exposure in more than one commodity market

Features of the Model

- Spot price model in continuous time
- Essentially-affine latent factor structure

$$\ln S_{k,t} = \delta_k \, x_t + \delta_k^0 + s_k(t)$$

$$dx_t = (a^P - K^P x_t)dt + dZ_t^P$$

- One non-stationary factor representing the long-term equilibrium
- $N - 1$ mean-reverting factors representing deviations from the equilibrium

Features of the Model (cont'd)

- Closed form (affine) solutions:

$$
\begin{aligned}
F_k(x_t, t; T) &= e^{E_t^Q[\ln S_{k,T}] + \frac{1}{2} V_t^Q[\ln S_{k,T}]} \\
&= e^{\mathcal{A}_k^Q(t,T) x_t + \mathcal{B}_k^Q(t,T)}
\end{aligned}
$$

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- Incorporating stylized facts of commodity prices
  - Changing shape of term structure of futures prices
  - Mean-reversion
  - Inverse term structure of volatilities
  - Seasonality
  - Co-integration

Model Implementation

- Crude oil, heating oil, and gasoline futures traded at NYMEX
- Sample period 01/1990 - 12/2005, weekly observations
- Maturities one to 18 (12) months

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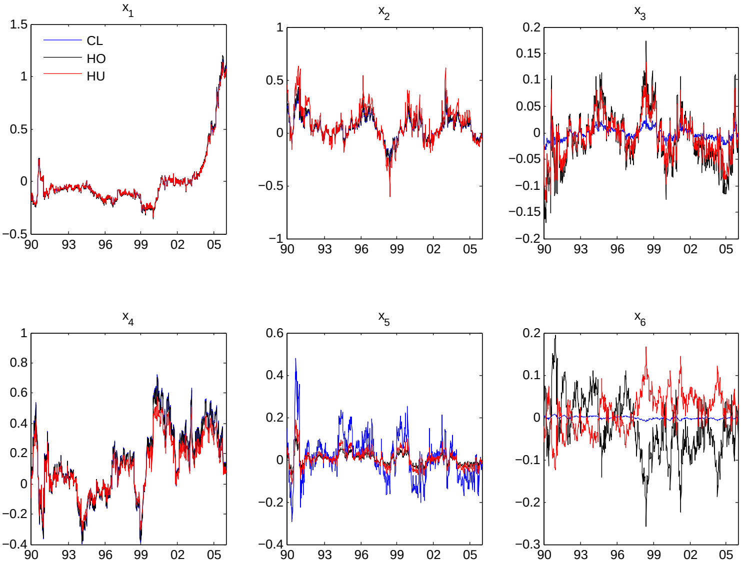
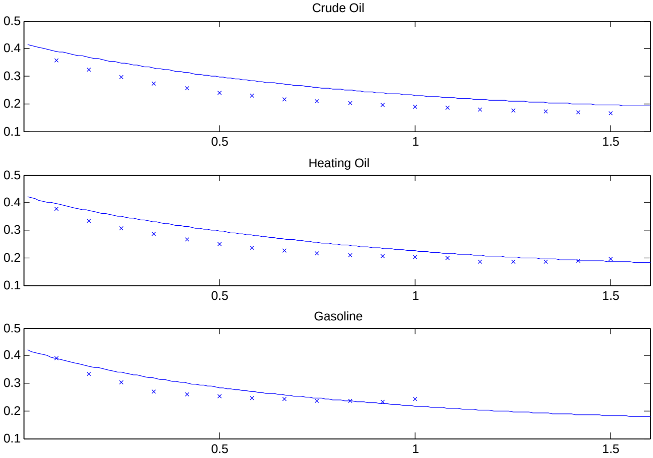
Estimation details:

- Six factor model (1+5)
- Trigonometrical functional form for seasonality adjustment
- Kalman filter ML estimation of parameters

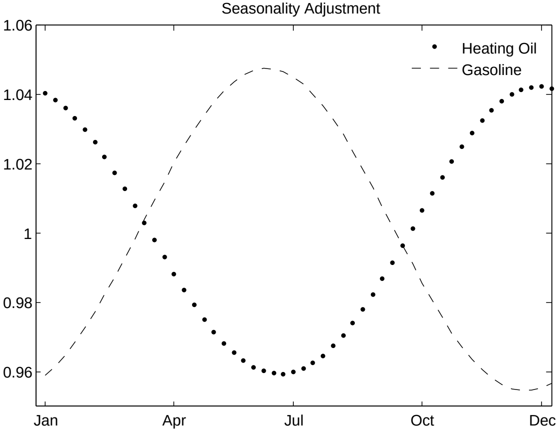
Results: Filtered State Variables



Results: Term Structure of Volatilities



Results: Seasonality Adjustment



Application I: Risk Measurement

- Assume a refinery processing crude oil into heating oil and gasoline
- Financial representation of this refinery:

$$PV(T) = \sum_{t=1}^{T} D(0,t)[-S_{1,t} + \alpha S_{2,t} + (1-\alpha)S_{3,t}]$$

- Risk horizon 30 years
- Crack ratio $\alpha = 0.33$ (3:2:1)
- Average sized refinery: 850,000 barrels per week
- Profit margin 10%
- Risk free rate 2%

Application I: Risk Measurement (cont'd)

Benchmark: Schwartz/Smith (2000) with correlation across commodities

$$\begin{aligned} \ln S_{CL,t} &= x_{1,t} + x_{4,t} \\ \ln S_{HO,t} &= x_{2,t} + x_{5,t} \\ \ln S_{HU,t} &= x_{3,t} + x_{6,t} \end{aligned}$$

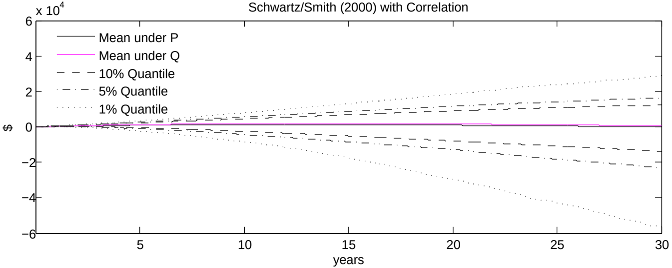
with

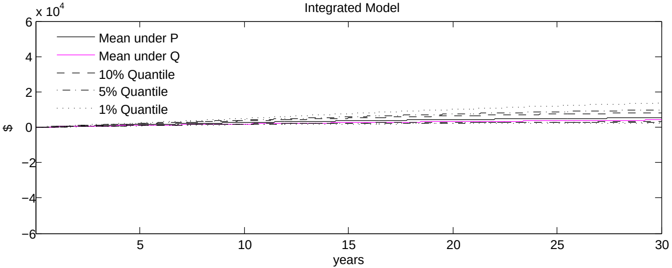
$$\begin{aligned} dx_{i,t} &= \mu_i dt + \sigma_i dZ_i \quad \textit{for } i = 1, 2, 3 \\ dx_{i,t} &= -\kappa_i x_i dt + \sigma_i dZ_i \quad \textit{for } i = 4, 5, 6 \end{aligned}$$

and

$$< dZ_i, dZ_j >= \rho_{i,j} dt$$

Application I: Risk Measurement (cont'd)





Application II: Hedging

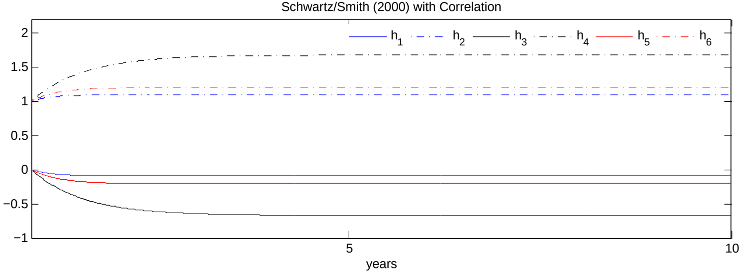
- Risk management of refinery
- Hedging of long term exposures with short term futures
- Monthly uncertain cash-flow

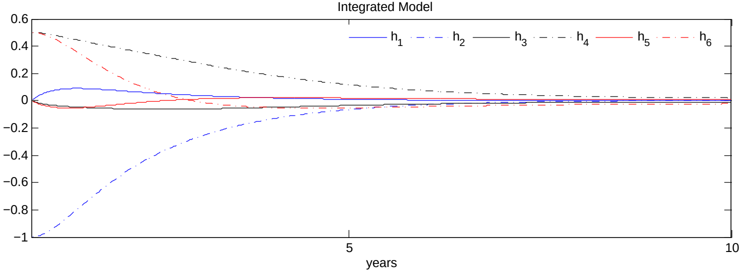
$$CF_T = -S_{1,T} + \alpha S_{2,T} + (1-\alpha) S_{3,T}$$

- Use six futures with distinctive underlyings or maturities
- Hedge ratios h_{jt} are given by

$$\sum_{j=1}^{6} h_{jt} \frac{\partial F_j(t, T_j)}{\partial x_i} = \frac{\partial E_t^Q[CF_T]}{\partial x_i} \qquad \forall i = 1, ..., 6$$

Application II: Hedging (cont'd)





Summary

- Modelling multiple commodity prices in a consistent stochastic framework
- Latent factor model capturing stylized empirical facts
- Estimation of a six factor model for crude oil, heating oil, and gasoline
- Identification of a long-term component driving all markets
- Several short term factors affect markets identically/differently
- Applications show economic significance