Change in Regime and Optimal Domestic-Global Portfolio Diversification

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Motivations

- Volatility-correlation relationship
  - Roll, 1988; King and Wadhwani, 1990; Erb *et al.* 1994; Lin *et al.* 1994; Longin and Solnik, 1995; Karolyi and Stulz, 1996; Boyer *et al*. 1999; Longin and Solnik, 2001; Jacquier and Marcus, 2001; Ang and Bekaert, 2002; Forbes and Rigobon, 2002; Bae *et al.* 2003; Das and Uppal, 2004
  - High volatility + high correlation

Motivations

- Applications of MRS (Markov-switching) models
  - Ramchand and Susmel (1998), Ang and Bekaert (2002), Das and Uppal (2004) and Baele (2005)
  - Ramchand and Susmel (1998) observe that the correlation between the U.S. domestic stock market and other national markets was strongest when the former was experiencing high volatility
  - Dual-state setting: HV/LV

In Contrast with Prior Studies

- Two components of the risk at play in domestic-global portfolio
  - (1) the global market risk factor (systematic risk)
  - (2) the domestic market risk factor (nonsystematic risk)
- Dual-state specification (HV/LV states) versus Four-state specification

Certain Questions Addressed in this Study

- Is the magnitude of domestic-global correlations consistent across various combinations of volatility regimes?

- If no such consistency exists, what are the relationships among various market volatility regimes and correlations?

Certain Questions Addressed in this Study

- If domestic-global correlations are state dependent, can a state-varying framework help investors design a more effective strategy for investing in international stock markets?

Model Specifications

- Bivariate GARCH Model
  - Optimal Asset Allocation via a Time-Varying System
- Bivariate SWARCH Model
  - Optimal Asset Allocation via a State-Varying System

Bivariate GARCH Model

$$r_t^w = \theta_0^w + \sum_{i=1}^{i=p} \theta_i^w r_{t-i}^w + e_t^w$$

$$r_t^d = \theta_0^d + \sum_{i=1}^{i=p} \theta_i^d r_{t-i}^d + e_t^d$$

$$e_t \mid \psi_{t-1} = \begin{bmatrix} e_t^w \\ e_t^d \end{bmatrix} \mid \psi_{t-1} \sim BN(0, \quad H_t)$$

$$H_t = \begin{bmatrix} h_t^w & h_t^{w,d} \\ h_t^{w,d} & h_t^d \end{bmatrix}$$

Bivariate GARCH Model

Variance at t.

$$h_t^w = \alpha_0^w + \sum_{j=1}^{q} \alpha_{t-j}^w (e_{t-j}^w)^2 + \sum_{l=1}^{m} \beta_{t-l}^w h_{t-l}^w$$

Variance at t-1

$$h_t^d = \alpha_0^d + \sum_{j=1}^{q} \alpha_{t-j}^d (e_{t-j}^d)^2 + \sum_{l=1}^{m} \beta_{t-l}^d h_{t-l}^d$$

$$h_t^{w,d} = \rho \times (h_t^w \cdot h_t^d)^{1/2}$$

Bivariate GARCH Model

- Key limitations of GARCH models
  - Structural changes
  - Diebold (1986), Lamoureux and Lastrapes, (1990), Nelson (1991) and Engle and Mustafa (1992), Bollerslev and Engle (1986), Schwert and Seguin (1990), Bollerslev et al. (1992), Hamilton and Susmel (1994), and Li and Lin (2003).

Two Portfolio Establishment Strategies

- A minimum variance

$$w_t^d = [h_t^w - \rho \cdot (h_t^w \cdot h_t^d)^{1/2}] / [h_t^d + h_t^w - 2 \cdot w_t^d \cdot w_t^w \cdot \rho \cdot (h_t^w \cdot h_t^d)^{1/2}]$$

$$, \qquad 0 \le w_t^d \le 1$$

$$w_t^w = 1 - w_t^d$$

- A given variance

$$w_t^d + w_t^w = 1, \qquad 0 \le w_t^d \le 1 \qquad and \qquad 0 \le w_t^w \le 1$$

$$\bar{h}_{p,t} = (w_t^d)^2 \cdot h_t^d + (w_t^w)^2 \cdot h_t^w + 2 \cdot \rho \cdot w_t^d \cdot w_t^w \cdot (h_t^d \cdot h_t^w)^{1/2}$$

$$\bar{h}_{p,t} = (1/2) \cdot (h_t^d + h_t^w)$$

Bivariate SWARCH Model

$$r_t^w = \theta_0^w + \sum_{i=1}^{i=p} \theta_i^w r_{t-i}^w + e_t^w$$

$$r_t^d = \theta_0^d + \sum_{i=1}^{i=p} \theta_i^d r_{t-i}^d + e_t^d$$

$$e_t \mid \psi_{t-1} = \begin{bmatrix} e_t^w \\ e_t^d \end{bmatrix} \mid \psi_{t-1} \sim BN(0, \quad H_t)$$

$$H_t = \begin{bmatrix} h_t^w & h_t^{w,d} \\ h_t^{w,d} & h_t^d \end{bmatrix}$$

Bivariate SWARCH Model

$$\frac{h_t^w}{g^w_{s_t^w}} = \alpha_0^w + \sum_{j=1}^{q} \alpha_{t-j}^w \frac{(e_{t-j}^w)^2}{g^w_{s_{t-j}^w}}$$

$S_t^w = 1 : h_t^w \sim ARCH(q)$

$S_t^w = 2 : h_t^w \sim g_2 \times ARCH(q)$

$$\frac{h_t^d}{g^d_{s_t^d}} = \alpha_0^d + \sum_{j=1}^{q} \alpha_{t-j}^d \frac{(e_{t-j}^d)^2}{g^d_{s_{t-j}^d}}$$

$$h_t^{w,d} = \rho_{s_t^w, s_t^d} \times (h_t^w \cdot h_t^d)^{1/2}$$

$S_t^w = 1, \quad S_t^d = 1$

$S_t^w = 2, \quad S_t^d = 1$

$S_t^w = 1, \quad S_t^d = 2$

$S_t^w = 2, \quad S_t^d = 2$

4-state System

$s_t=1$: if $s_t^w=1$ and $s_t^d=1$ -or- World=LV and Domestic=LV

$s_t=2$: if $s_t^w=2$ and $s_t^d=1$ -or- World=HV and Domestic=LV

$s_t=3$: if $s_t^w=1$ and $s_t^d=2$ -or- World=LV and Domestic=HV

$s_t=4$: if $s_t^w=2$ and $s_t^d=2$ -or- World=HV and Domestic=HV

Transition Probability Matrix

$$P = \begin{bmatrix} p_{11} & p_{21} & p_{31} & p_{41} \\ p_{12} & p_{22} & p_{32} & p_{42} \\ p_{13} & p_{23} & p_{33} & p_{43} \\ p_{14} & p_{24} & p_{34} & p_{44} \end{bmatrix}$$

Transition Probability Matrix

$$P = \begin{bmatrix} p_{11}^w \cdot p_{11}^d & p_{21}^w \cdot p_{11}^d & p_{11}^w \cdot p_{21}^d & p_{21}^w \cdot p_{21}^d \\ p_{12}^w \cdot p_{11}^d & p_{22}^w \cdot p_{11}^d & p_{12}^w \cdot p_{21}^d & p_{22}^w \cdot p_{21}^d \\ p_{11}^w \cdot p_{12}^d & p_{21}^w \cdot p_{12}^d & p_{11}^w \cdot p_{22}^d & p_{21}^w \cdot p_{22}^d \\ p_{12}^w \cdot p_{12}^d & p_{22}^w \cdot p_{12}^d & p_{12}^w \cdot p_{22}^d & p_{22}^w \cdot p_{22}^d \end{bmatrix}$$

$$p(s_t^w = 1 \mid s_{t-1}^w = 1) = p_{11}^w, \quad p(s_t^w = 2 \mid s_{t-1}^w = 1) = p_{12}^w$$

$$p(s_t^w = 2 \mid s_{t-1}^w = 2) = p_{22}^w, \quad p(s_t^w = 1 \mid s_{t-1}^w = 2) = p_{21}^w$$

$$p(s_t^d = 1 \mid s_{t-1}^d = 1) = p_{11}^d, \quad p(s_t^d = 2 \mid s_{t-1}^d = 1) = p_{12}^d$$

$$p(s_t^d = 2 \mid s_{t-1}^d = 2) = p_{22}^d, \quad p(s_t^d = 1 \mid s_{t-1}^d = 2) = p_{21}^d$$

4X4 Probability Matrix

- Hamilton and Lin (1996) is one of the first to address a system involving two state variables
- The connection between the regression/expansion phrase of the U.S. real output (s^+_t) and the HV/LV phase of the U.S. stock market (s^*_t).
- They proposed the restricted specification in which the two state variables, s^+_t and s^*_t, are driven by the same fundamentals but are not in phase together.
- In particular, they hypothesized participants in stock markets are forward-looking and the restriction of $s^+_t = s^*_{t-1}$ is thus imposed on their model.

4X4 Probability Matrix

- A domestic-global stock portfolio in which the domestic and global market assets are proxied by the world and individual country-based index, respectively
- The world market index is considered to serve as a synthetic holding of global assets, and the idiosyncratic risk pertaining to individual country-based index is understood to decrease to an arbitrarily low level

4X4 Probability Matrix

- Variance-switching process of global and domestic market returns is here seen as being subject to the distinct processes of the switching of volatility states characterizing each of these types of market component.
- The volatility state variable for global market returns (s_t^w) is expected to be independent of that for individual country-based market returns (s_τ^d) in the case of all t and τ.

Empirical Results and Interpretations

- Two levels of observation
  - The world index maintained by Morgan Stanley Capital International (MSCI)
  - The individual G7 country
- 7 (7X1) domestic-global portfolios
- Weekly data (Wednesday to Wednesday)
- January 1980 to May 2007. The entire sample consists of 1,426 observations

Estimation Results of Bivariate

Table 2 Parameter Estimates of the Bivariate GARCI

	Canada-World	France-World	Germany-World	Ita
World Market Eq.				
θ^w_0	0.2294 (0.0435)***	0.2365 (0.0440)***	0.2018 (0.0446)***	0.210
θ^w_1	-0.0100 (0.0140)	-0.0214 (0.0246)	-0.0062 (0.0413)	-0.0
α^w_0	0.2055 (0.0469)***	0.1592 (0.0358)***	0.2520 (0.0520)***	0.192
α^w_1	0.0940 (0.0138)***	0.0861 (0.0119)***	0.0906 (0.0132)***	0.101
β^w_1	0.8518 (0.0203)***	0.8729 (0.0155)***	0.8405 (0.0203)***	0.849
Domestic Market Eq.				
θ^d_0	0.2138 (0.0567)***	0.3097 (0.0665)***	0.2334 (0.0647)***	0.199
θ^d_1	0.0751 (0.0235)***	-0.0618 (0.0252)***	0.0100 (0.0226)	0.02
α^d_0	1.2060 (0.2175)***	0.5840 (0.1159)***	0.4261 (0.0915)***	0.299
α^d_1	0.1431 (0.0222)***	0.1278 (0.0177)***	0.1001 (0.0145)***	0.086
β^d_1	0.6505 (0.0475)***	0.8070 (0.0235)***	0.8491 (0.0199)***	0.890
Correlation				
ρ	0.6833 (0.0142)***	0.6523 (0.0153)***	0.6637 (0.0149)***	0.513
Log-lik.	-5652.1071	-5938.0385	-5904.8489	-(
AIC value	-5663.1071	-5949.0385	-5915.8489	-(
Schwarz value	-5692.0516	-5977.9830	-5944.7934	-(

Notes:

Estimation Results of Bivariate SWARCH

Table 3 Parameter Estimates of the Bivariate SWAR

	Canada-World	France-World	Germany-World
World Market Eq.			
p^{w}_{11}	0.9856 (0.0053)***	0.9822 (0.0060)***	0.9905 (0.0043)***
p^{w}_{22}	0.9643 (0.0140)***	0.9507 (0.0184)***	0.9834 (0.0062)***
θ^{w}_{0}	0.2106 (0.0441)***	0.2250 (0.0475)***	0.2166 (0.0437)***
θ^{w}_{1}	-0.0097 (0.0169)	-0.0176 (0.0298)	-0.0089 (0.0039)
α^{w}_{0}	2.0808 (0.1326)***	2.1850 (0.1225)***	1.9625 (0.1193)***
α^{w}_{1}	0.0582 (0.0268)**	0.0324 (0.0237)	0.0683 (0.0232)***
g^{w}_{2}	3.0975 (0.2875)***	3.1460 (0.2887)***	2.7738 (0.2466)***
Domestic Market Eq.			
p^{d}_{11}	0.9864 (0.0054)***	0.9675 (0.0110)***	0.9891(0.0055)***
p^{d}_{22}	0.9619 (0.0148)***	0.9241 (0.0238)***	0.9754 (0.0091)***
θ^{d}_{0}	0.1893 (0.0544)***	0.2790 (0.0701)***	0.2390 (0.0680)***
θ^{d}_{1}	0.0673 (0.0219)***	-0.0622 (0.0297)**	0.0126 (0.0182)
α^{d}_{0}	3.4797 (0.1926)***	4.6742 (0.3360)***	4.6673 (0.2746)***
α^{d}_{1}	0.0477 (0.0225)**	0.0745 (0.0267)***	0.0517 (0.0188)***
g^{d}_{2}	2.9862 (0.2749)***	3.1235 (0.3103)***	2.9275 (0.2358)***

Four-state Correlations

Correlations			
$\rho_{1,1}$	0.7135 (0.0235)***	0.7905 (0.0190)***	0.726
$\rho_{2,1}$	0.6306 (0.0632)***	0.5539 (0.0619)***	0.507
$\rho_{1,2}$	0.6204 (0.0460)***	0.2929 (0.0691)***	0.430
$\rho_{2,2}$	0.7648 (0.0395)***	0.8488 (0.0309)***	0.843
Log-lik.	-5588.1415	-5841.6290	-5
AIC value	-5606.1415	-5859.6290	-5
Schwarz value	-5653.5052	-5906.9927	-5
LR for $\rho_{1,1}=\rho_{2,1}=\rho_{1,2}=\rho_{2,2}$	5.6104	74.9768***	43

Notes:

1. Please refer to Eqs. 9 to 20 for the specifications of the bivariate SWARCH model used in thi

2. To test the null hypothesis of identical correlations, this bivariate SWARCH model is firs estimated assuming the existence of a single constant correlation ($\rho_{1,1}=\rho_{2,1}=\rho_{1,2}=\rho_{2,2}=\rho$), which used to carry out a likelihood ratio test, $LR=-2[L(H_0)- L(H_A)]$. In terms of the null hypothesis, analysis make it possible to reject the null hypothesis at a 1% level of significance in most cases

Four-state Correlations

- Volatility-correlation relationship
  - High volatility + high correlation
  - “HV-HV” , “LV-LV” , “HV-LV” and “LV-HV”
- The maximum correlation estimate corresponds to the “HV-HV” state combination in 5 of 7 cases

“HV-HV” State

- “Stock-to-bond” asset reallocation process
- Redirection of capital flows from stock markets to non-stock markets, such as bond markets
- Across-market-hedging strategy

“LV-HV” and “HV-LV” States

- “Stock-to-stock” asset reallocation process
- Within-market-hedging strategy
- Consequently, stock prices on both the domestic and global markets move in opposite directions, thus reducing the magnitude of co-movements between these different markets.

Asset Allocation Effectiveness: In-sample Tests

- Two-step procedure
  - Variance and correlation estimations as well as the probabilities of specific volatility regime
  - The optimal portfolio loadings at any specific point in time are identified as the average loadings in different states of volatility using the probabilities of specific volatility regime

Asset Allocation Effectiveness: In-sample Tests

Figures

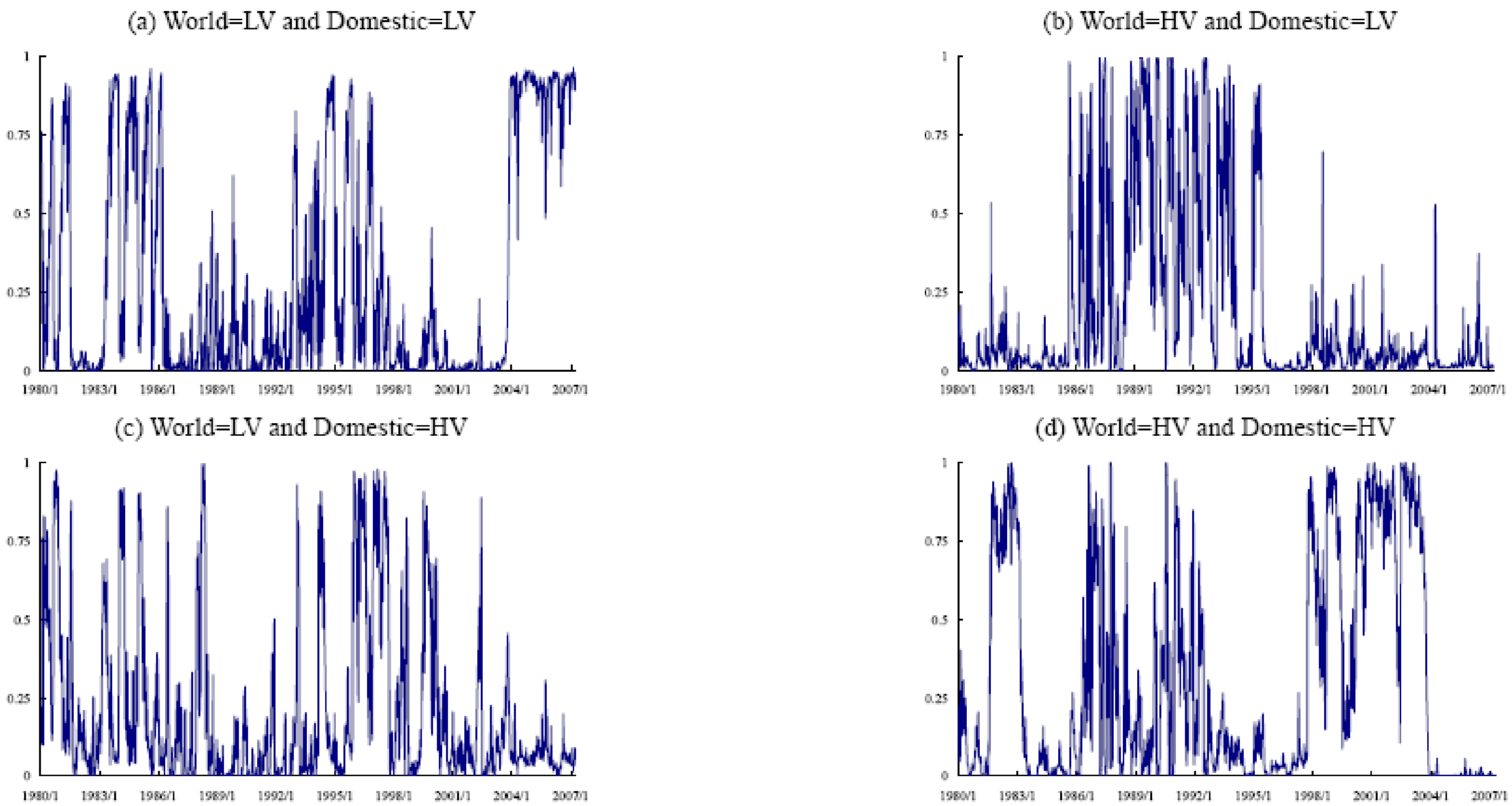
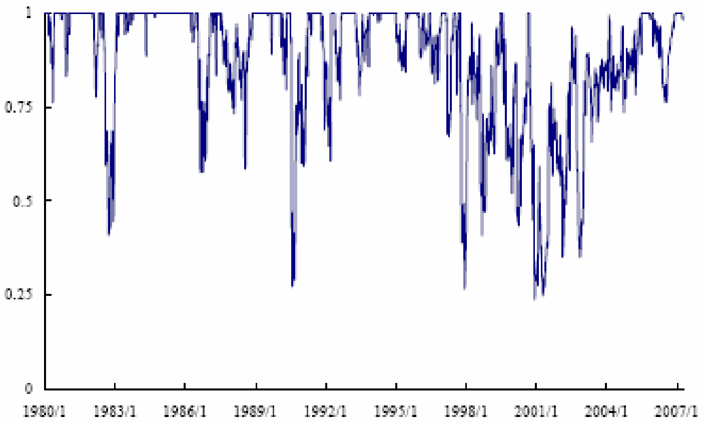


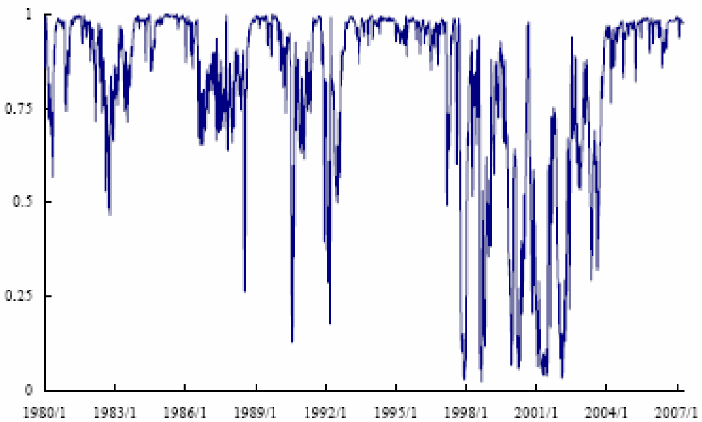
Figure 1 SWARCH-based Estimates of the Filtering Probabilities of Specific Volatility State Combinations for the Illustrative Case of U.S.-World

A Minimum Variance

(a) Optimal Domestic-Global Loading on Global Assets by GARCH Model

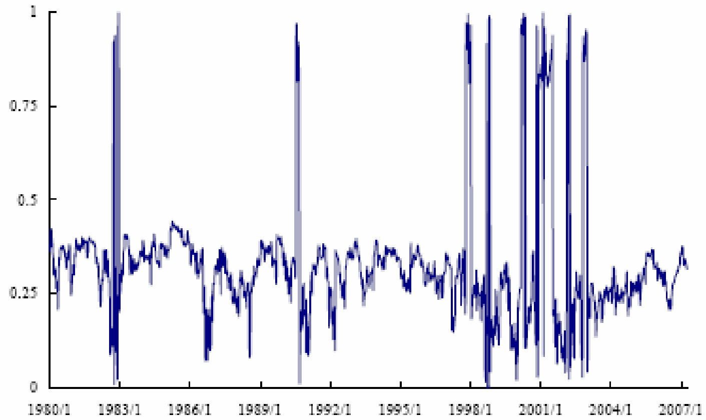


(b) Optimal Domestic-Global Loading on Global Assets by SWARCH Model

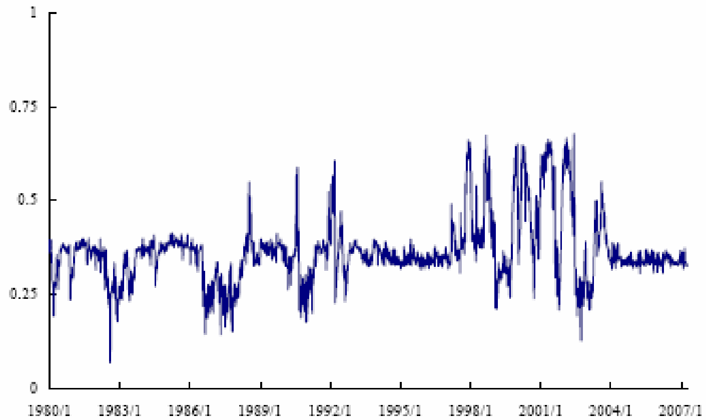


A Given Variance.

(a) Optimal Domestic-Global Loading on Global Assets by GARCH Model



(b) Optimal Domestic-Global Loading on Global Assets by SWARCH Model



Asset Allocation Effectiveness: In-sample Tests

Table 4 In-sample Asset Allocation Effectiveness for Domestic-Global Portfolio with Minimum Variance Strategy: GARCH vs. SWARCH

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Canada-World	0.1759	0.1604	1.7373*
France-World	0.1685	0.1774	-0.9015
Germany-World	0.1605	0.1591	0.1411
Italy-World	0.1589	0.1682	-1.1579
Japan-World	0.1627	0.1586	0.4412
U.K.-World	0.1572	0.1667	-1.3622
U.S.-World	0.1656	0.1730	-0.7340

(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Canada-World	3.5525	3.1963	6.8612***
France-World	3.8200	3.3374	4.4507***
Germany-World	3.7466	3.4071	3.8310***
Italy-World	3.6771	3.3796	6.6110***
Japan-World	3.6181	3.3743	4.7607***
U.K.-World	3.6735	3.4329	6.6308***
U.S.-World	3.6949	3.5091	5.0932***

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
Canada-World	0.6599	0.6344	-3.86%
France-World	0.6096	0.6866	12.63%
Germany-World	0.5863	0.6095	3.96%
Italy-World	0.5859	0.6470	10.43%
Japan-World	0.6048	0.6105	0.94%
U.K.-World	0.5800	0.6362	9.69%
U.S.-World	0.6092	0.6530	7.19%

Notes:

1. The ***, ** and * denote the 1%, 2.5% and 5% levels of significance, respectively.
2. See the appendix for details of the statistical test of whether the variances of portfolio returns vary significantly between the bivariate GARCH and SWARCH models.
3. Results show that modeling domestic-global correlations and corresponding variances as a state-varying phenomenon is both statistically significant and strategically effective. Moreover, reductions in risk, rather than increases in mean returns, are responsible for the benefits stemming such improved effectiveness.

Table 5 In-sample Asset Allocation Effectiveness for Domestic-Global Portfolio with Given Variance Strategy: GARCH vs. SWARCH

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Canada-World	0.1428	0.1615	-1.8084*
France-World	0.1829	0.1815	0.1155
Germany-World	0.1701	0.1891	-1.2705
Italy-World	0.1809	0.1793	0.1278
Japan-World	0.1572	0.1631	-0.4556
U.K.-World	0.1710	0.1667	0.5554
U.S.-World	0.1733	0.1736	-0.0292

(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Canada-World	4.7409	4.6042	2.3144**
France-World	5.9286	5.6457	3.2433***
Germany-World	5.9508	5.6461	3.1335***
Italy-World	6.7184	6.4862	3.9689***
Japan-World	6.2040	5.8621	5.3391***
U.K.-World	4.8611	4.8370	0.6808
U.S.-World	4.1866	4.0719	2.6627***

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
Canada-World	0.4637	0.5322	14.76%
France-World	0.5312	0.5401	1.69%
Germany-World	0.4931	0.5627	14.13%
Italy-World	0.4935	0.4978	0.87%
Japan-World	0.4463	0.4763	6.74%
U.K.-World	0.5484	0.5360	-2.27%
U.S.-World	0.5989	0.6083	1.57%

Notes:

Asset Allocation Effectiveness: Out-of-Sample Test

- The final 200 weekly observations of the sample (representing approximately 4-year's worth of data) are omitted from the initial sample, and formed the sequential inputs of the rolling estimation
- $\left\{ r_{t-i}^{w}, r_{t-i}^{d} \right\}_{i=1}^{1,226}$

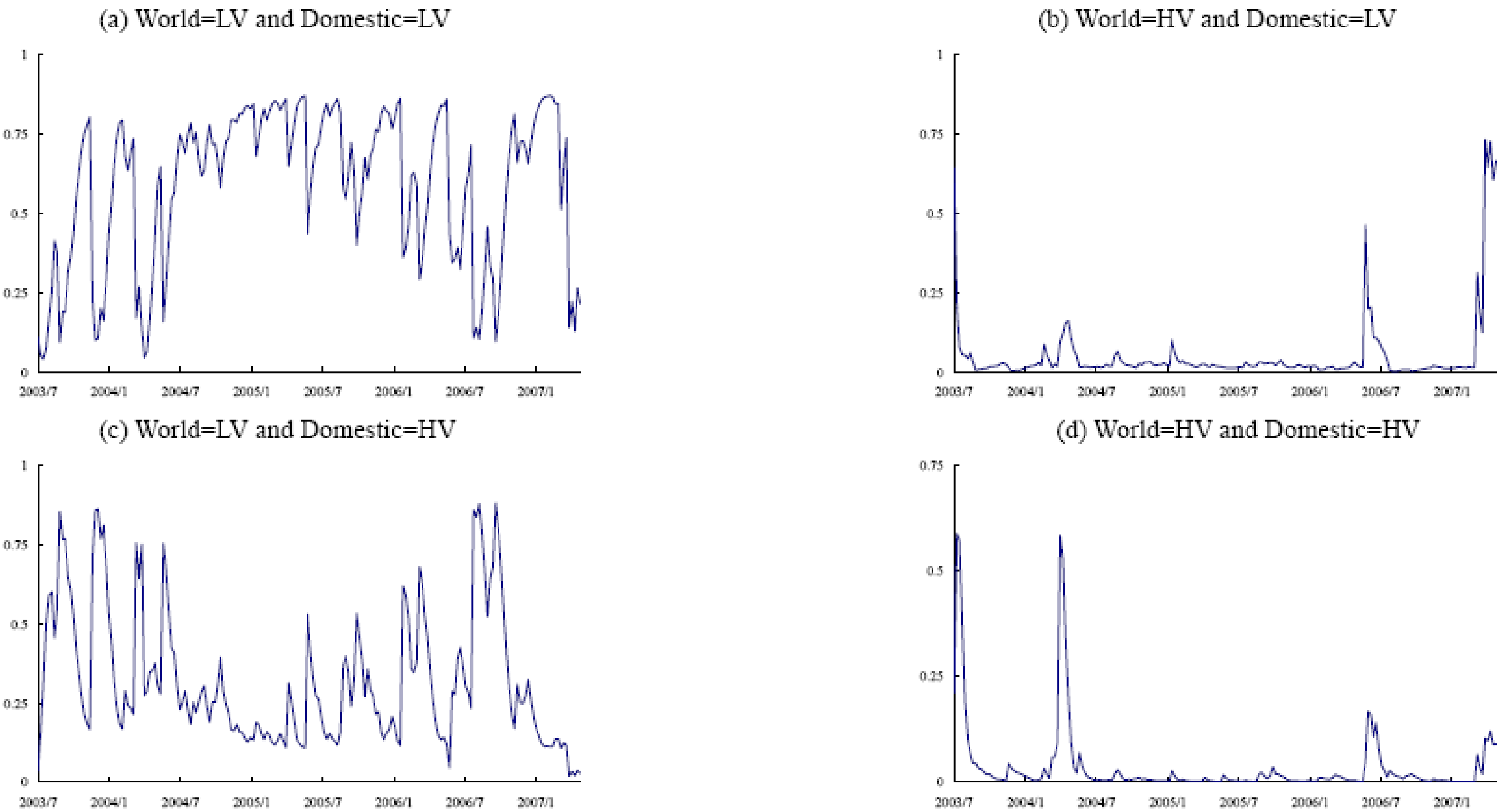


Figure 4 Rolling Regime Probability Forecasts Derived on the Basis of the SWARCH Model in the Illustrative Case of Japan-World

Asset Allocation Effectiveness: Out-of-Sample Test

Table 6 Out-of-Sample Allocation Effectiveness for Domestic-Global Portfolio with Minimum Variance Strategy: GARCH vs. SWARCH

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Japan-World	0.2928	0.2804	0.7842
U.K.-World	0.2871	0.2879	-0.0528
U.S.-World	0.2699	0.2652	0.8802

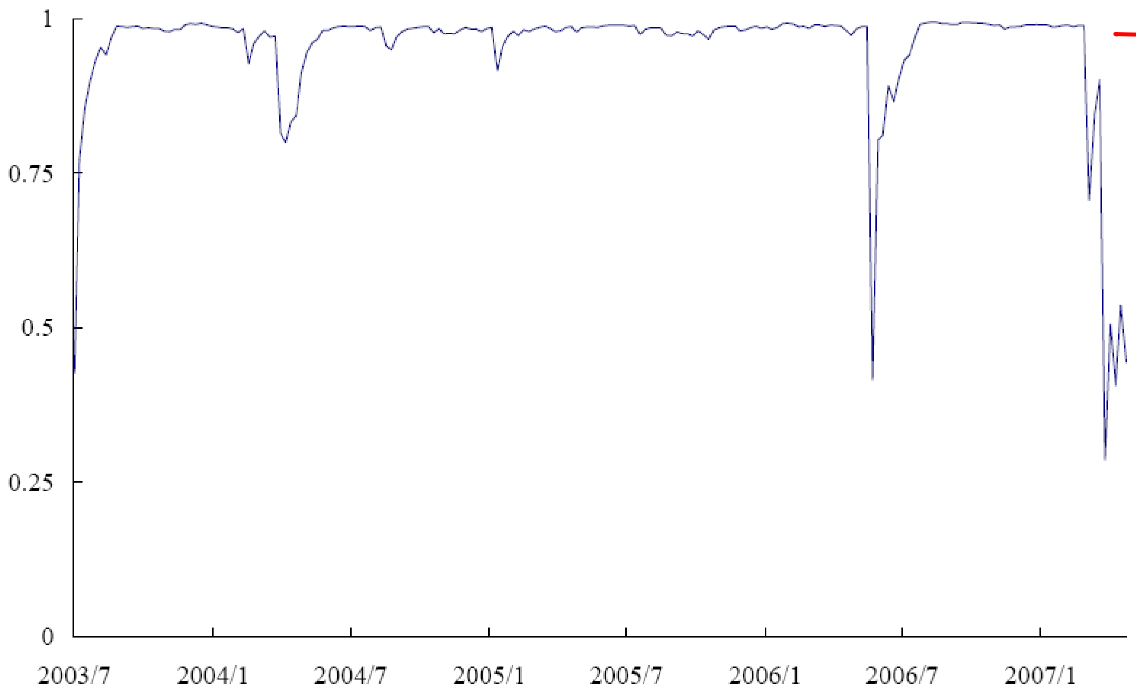
(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Japan-World	1.9137	2.0784	-1.9526*
U.K.-World	1.6786	1.6898	-0.4813
U.S.-World	1.6670	1.6585	1.0273

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
Japan-World	1.4967	1.3751	-8.12%
U.K.-World	1.5667	1.5663	-0.03%
U.S.-World	1.4784	1.4563	-1.50%

(a) Portfolio Loading on Global Assets for a Minimum Variance Portfolio



→ Corner solution.

(b) Portfolio Loading on Global Assets for a Given Variance Portfolio

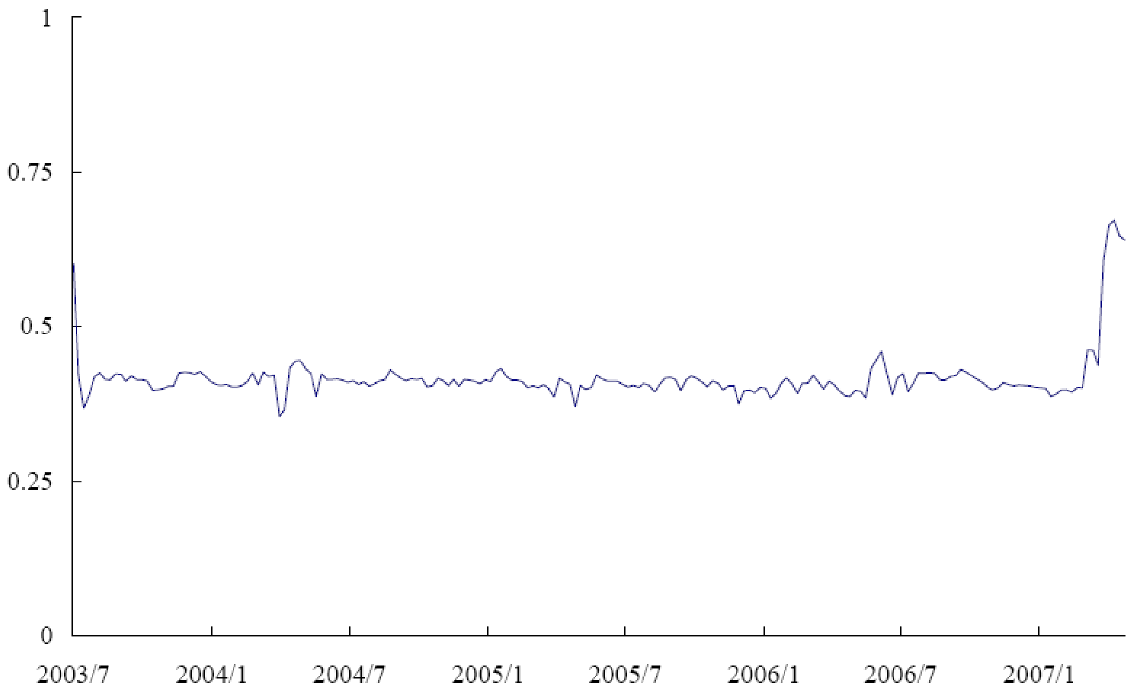


Table 7 Out-of-Sample Allocation Effectiveness for Domestic-Global Portfolio with Given Variance Strategy: GARCH vs. SWARCH

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Japan-World	0.3176	0.3291	-0.7411
U.K.-World	0.2908	0.2930	-0.1585
U.S.-World	0.2253	0.2488	-1.9252*

(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
Japan-World	3.8164	3.4714	3.6839***
U.K.-World	2.0415	1.9246	3.7503***
U.S.-World	1.6979	1.6946	0.4943

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
Japan-World	1.1495	1.2491	8.67%
U.K.-World	1.4392	1.4936	3.78%
U.S.-World	1.2225	1.3512	10.53%

Robustness Test: An Alternative Measurement for the World Market Index

- The MSCI world index is weighted by market capitalization and a specific country market with high capitalization, such as the U.S. market, is thus assigned with an excessive loading
- Mechanical relation between the MSCI world index and the largest market
- Taking the case of U.S.-World as example, the return rates in world stock market is proxied by the average value of all the G7 stock market index returns except for the U.S. market.

Table 8 Parameter Estimates of the Bivariate SWARCH Model: A Robustness Test b
Market Index

	Canada-World	France-World	Germany-World	Italy-World
World Market Eq.				
p^{w}_{11}	0.9871 (0.0050)***	0.9830 (0.0080)***	0.9902 (0.0045)***	0.9954 (0.0024)***
p^{w}_{22}	0.9600 (0.0160)***	0.9546 (0.0240)***	0.9733 (0.0074)***	0.9875 (0.0064)***
θ^{w}_{0}	0.2243 (0.0486)***	0.2271 (0.0534)***	0.2407 (0.0430)***	0.2358 (0.0494)***
θ^{w}_{1}	-0.0173 (0.0169)	-0.0150 (0.0692)	0.0068 (0.0132)	-0.0022 (0.0225)
α^{w}_{0}	2.5873 (0.1573)***	2.5432 (0.1603)***	2.5483 (0.1386)***	2.4535 (0.1241)***
α^{w}_{1}	0.0592 (0.0253)***	0.0358 (0.0221)	0.0756 (0.0210)***	0.0806 (0.0243)***
g^{w}_{2}	2.9833 (0.3017)***	2.3146 (0.2381)***	1.9376 (0.1503)***	2.9841 (0.2552)***
Domestic Market Eq.				
p^{d}_{11}	0.9911 (0.0038)***	0.9685 (0.0143)***	0.9810 (0.0058)***	0.9993 (0.0011)***
p^{d}_{22}	0.9737 (0.0118)***	0.9508 (0.0169)***	0.9607 (0.0128)***	0.9990 (0.0008)***
θ^{d}_{0}	0.1774 (0.0548)***	0.2929 (0.0704)***	0.2981 (0.0575)***	0.2549 (0.0700)***
θ^{d}_{1}	0.0500 (0.0238)**	-0.0876 (0.0510)*	-0.0092 (0.0491)	0.0140 (0.0080)*
α^{d}_{0}	3.4521 (0.1887)***	4.6087 (0.2892)***	4.8404 (0.2411)***	3.2859 (0.2771)***
α^{d}_{1}	0.0554 (0.0276)**	0.0985 (0.0277)***	0.0646 (0.0215)***	0.0574 (0.0228)***
g^{d}_{2}	3.2845 (0.3296)***	2.5840 (0.2233)***	2.9349 (0.1820)***	3.5163 (0.3166)***

Correlations				
$\rho_{1,1}$	0.5987 (0.0255)***	0.8530 (0.0207)***	0.7686 (0.0162)***	0.8534 (0.0184)**
$\rho_{2,1}$	0.5498 (0.0526)***	0.7373 (0.0543)***	0.6790 (0.0316)***	0.5045 (0.0082)**
$\rho_{1,2}$	0.5252 (0.0449)***	0.3818 (0.0890)***	0.3467 (0.0575)***	0.3769 (0.0293)**
$\rho_{2,2}$	0.6834 (0.0465)***	0.8616 (0.0222)***	0.9099 (0.0117)***	0.8233 (0.0169)**
Log-lik.	-5864.1057	-5787.2125	-5829.6654	-6234.7536
AIC value	-5882.1057	-5805.2125	-5847.6654	-6252.7536
Schwarz value	-5929.4694	-5852.5762	-5895.0291	-6300.1173
LR for $\rho_{1,1}=\rho_{2,1}=\rho_{1,2}=\rho_{2,2}$	4.9094	104.0244***	84.5838***	114.4782***

Notes:

1. To avoid the mechanical correlation between the MSCI world index and the largest market represented within the index, namely the U index of the respective domestic market that is being analyzed and a robustness test is examined below. Taking the case of U.S.-World a: of all the G7 stock market index returns except for the U.S. market.

2. Using the new measurement of the world index in which the domestic market index that is analyzed is excluded considerably decreas the case of U.S.-World as an example, the 4-state correlation estimates, as shown in Table 12, are $\rho_{1,1}$=0.6699 ("LV-LV"), $\rho_{2,1}$=0.3010 (these values are lower than the corresponding values in Table 3: 0.9079 ($\rho_{1,1}$), 0.5254 ($\rho_{2,1}$), 0.8835 ($\rho_{1,2}$) and 0.9102 ($\rho_{2,2}$)

3. Other notations are consistent with Table 3.

Table 9 R/R Ratio Performance of GARCH-based versus SWARCH-based Portfolios: A Robustness Test by Using an Alternative Measure for the World Market Index

(a) Minimum Variance Strategy

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
Canada-World	0.5883	0.6192	5.25%
France-World	0.5947	0.6405	7.71%
Germany-World	0.5885	0.6107	3.78%
Italy-World	0.5774	0.5955	3.14%
Japan-World	0.5844	0.6280	7.46%
U.K.-World	0.5522	0.6055	9.65%
U.S.-World	0.5193	0.5789	11.47%

(b) Given Variance Strategy

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
Canada-World	0.4260	0.4970	16.66%
France-World	0.5075	0.5168	1.83%
Germany-World	0.5025	0.5293	5.35%
Italy-World	0.4729	0.4793	1.37%
Japan-World	0.4589	0.4390	-4.34%
U.K.-World	0.5222	0.5231	0.18%
U.S.-World	0.6355	0.7329	15.33%

Conclusions and Future Research Directions

- Volatility-correlation dynamics and 4-state system
- The domestic-world correlations are significantly higher (lower) in “HV-HV” and “LV-LV” (“HV-LV” and “LV-HV”) states
- An effective strategy for the allocation of assets
  - due to reductions in risk, rather than increases in mean returns
  - Out-of-sample testing shows the relative performance of the SWARCH model to be less promising, especially where corner solutions are concerned

Conclusions and Future Research Directions

- A domestic-global equity portfolio
  - a mature-emerging portfolio
- State-varying loading implied by the SWARCH model and time-varying loading obtained from the GARCH model
  - Comparisons of these approaches with other models
- Two variance targets
  - A minimum and given variances
  - Other targets