TEV Sensitivity to Views in Black-Litterman Model

Maria Debora Braga
University of Valle d'Aosta
SDA Bocconi School of Management, Italy

Francesco Paolo Natale
Banca IMI, Italy

Introduction: The Black-Litterman Model

Black-Litterman (1992) proposed a Bayesian model to combine equilibrium returns implied in the market and specialized views in order to derive the posterior distribution of expected returns

$$R_{BL} = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \cdot \left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right]$$

R_{BL} : vector of blended expected returns ($n \times 1$);
τ : scalar for weighting the variance-covariance matrix;
Σ : variance-covariance matrix of historical returns ($n \times n$);
P : matrix for the n assets involved in the k views ($k \times n$);
Ω : diagonal covariance matrix of error terms for the stated views ($k \times k$). This matrix is usually derived from the level of confidence in the views declared by the specialists;
Π : vector of implied equilibrium returns ($n \times 1$);
Q : vector with the q_j (for $j = 1, \ldots, k$) views as entries ($k \times 1$)

Literature Review

- He G., Litterman R., Goldman Sachs Investment Management Series, 1999
- Satchell S., Scowcroft A., JAM, 2000
- Idzorek T., WP Zephyr Associates Inc., 2002, 2004
- Herold U., JPM, 2003
- Meucci A., Risk, 2006
- Martellini L., Ziemann V., JPM, 2007
- Jones R., Lim T., Zangari P.J., JPM, 2007
- *Fusai G. Meucci A., Risk, 2003*

Research Question

Most of the previous papers deal with the issue of parameters estimation. Only Fusai and Meucci investigate a different issue. They propose a test based on Mahalanobis distance and its Chi-Square probability distribution in order to indentify the boldest view.

Could we obtain a measure of risk contribution of each view to a familiar active risk measure?

We extend the approach of Fusai and Meucci by proposing a new measure of the marginal risk contribution to the ex-ante active risk (TEV). Our measure:

- is not probability-based;
- directly relates the views to active risk
- exploits the most familiar, among practitioners, measure of active risk

TEV Sensitivity to Views

The optimization process without constraints

$$\max_{w^*_{BL}} \left(w_{BL}^T \cdot R_{BL} - \lambda \left(w_{BL}^T \cdot \Sigma \cdot w_{BL} \right)^{\frac{1}{2}} \right)$$

Recall that:

$$TEV = \left(w_{ACT}^T \cdot \Sigma \cdot w_{ACT} \right)^{1/2}$$

$$\frac{\partial TEV}{\partial q} = \left| \begin{array}{ll} \displaystyle\sum_{i=1}^{n} \frac{\partial TEV}{\partial w_{ACT;i}} \cdot \frac{\partial w_{ACT;i}}{\partial q_j} & (j=1) \\ \dots\dots\dots\dots & \dots\dots \\ \dots\dots\dots\dots & \dots\dots \\ \displaystyle\sum_{i=1}^{n} \frac{\partial TEV}{\partial w_{ACT;i}} \cdot \frac{\partial w_{ACT;i}}{\partial q_j} & (j=k) \end{array} \right| = \frac{\partial \sqrt{w_{ACT}^T \cdot \Sigma \cdot w_{ACT}}}{\partial w_{ACT}} \cdot \frac{\partial w_{ACT}}{\partial q}$$

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TEV Sensitivity to Views

1 $$\frac{\partial \sqrt{w_{ACT}^T \cdot \Sigma \cdot w_{ACT}}}{\partial w_{ACT}} = \nabla TEV$$

$n \times 1$ gradient of the TEV with respect to the active weights

2 $$\frac{\partial w_{ACT}}{\partial q} = \frac{1}{\lambda} \Sigma^{-1} \left[\left(\tau \Sigma^{-1} \right) + P^T \Omega^{-1} P \right]^{-1} P^T \Omega^{-1}$$

k gradients of each active weight with respect to each view

$$\frac{\partial TEV}{\partial q} = \left[\frac{\mathbf{1}}{\lambda} \Sigma^{-\mathbf{1}} \left[\left(\tau \Sigma^{-1} \right) + P^T \Omega^{-1} P \right]^{-1} P^T \Omega^{-1} \right]^T \cdot \frac{\Sigma \cdot w_{ACT}}{\sqrt{w_{ACT}^T \cdot \Sigma \cdot w_{ACT}}}$$

Empirical Investigation: data and parameters

Sectors	Oil & Gas	Chemicals	Automobiles & Parts	Banks	Basic Resources	Construction & Materials	Financial Services	Food & Beverages	Industrial Goods & Services
Labels	***OIL***	***CHEM***	***AUTO***	***BANK***	***BRES***	***CONS***	***FISV***	***FBEV***	***INDS***
Mean	0,0773	0,0695	0,0551	0,1126	0,1239	0,1133	0,1084	0,0774	0,0545
Std dev	0,1927	0,2133	0,2640	0,2294	0,2673	0,1969	0,2274	0,1567	0,2290
Market Cap.	0,0826	0,0255	0,0260	0,2023	0,0395	0,0320	0,0349	0,0527	0,0738
Sectors	Insurance	Media	Healthcare	Retail	Technology	Telecommu-nication	Utilities	Persolnal & Household Goods	Travel & Leisure
Labels	***INSU***	***MED***	***HEAL***	***RTL***	***TECH***	***TELE***	***UTIL***	***HOUS***	***TRAV***
Mean	0,0562	0,0033	0,0716	0,0399	0,0448	0,0625	0,1000	0,0855	0,0523
Std dev	0,2814	0,2769	0,1657	0,1622	0,4025	0,2677	0,1461	0,1851	0,2237
Market Cap.	0,0660	0,0258	0,0699	0,0370	0,0296	0,0603	0,0798	0,0457	0,0168

Market cap. at 31 December 2006

The table shows our investment universe built on 18 asset classes constituing the DJ Stoxx Europe 600. The sample period is 1/1997 - 1/2007 of monthly euro-denominated returns. The annual risk free rate is assumed to be 3%.

Empirical Investigation: data and parameters

Risk aversion coefficient (Best-Grauer, 1985) = 1.6118

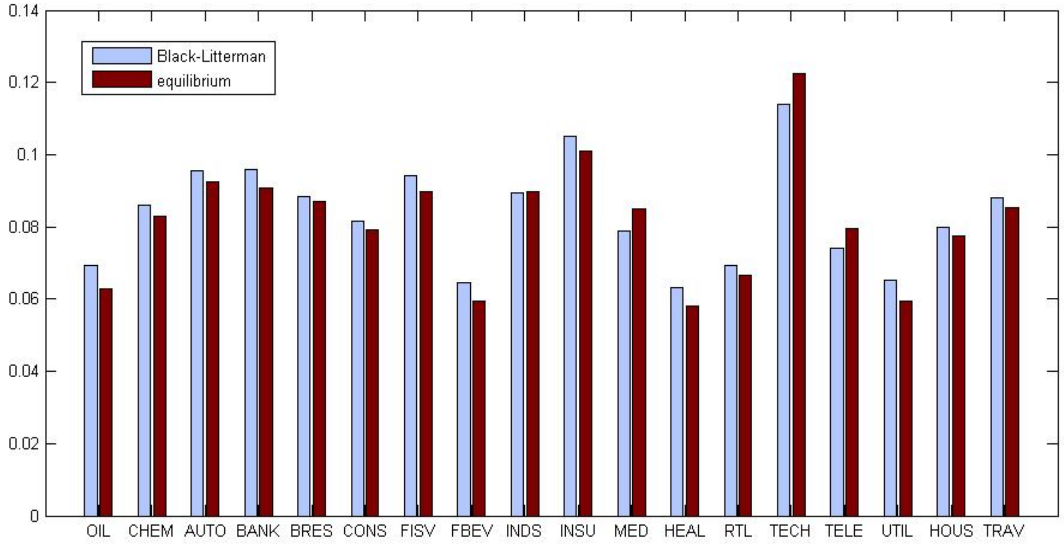
Views:

- **View 1:** Utilities (UTIL) will have an absolute return of 7% per year with a Confidence Level 80% of experiencing a final return in the range [6%; 8%];

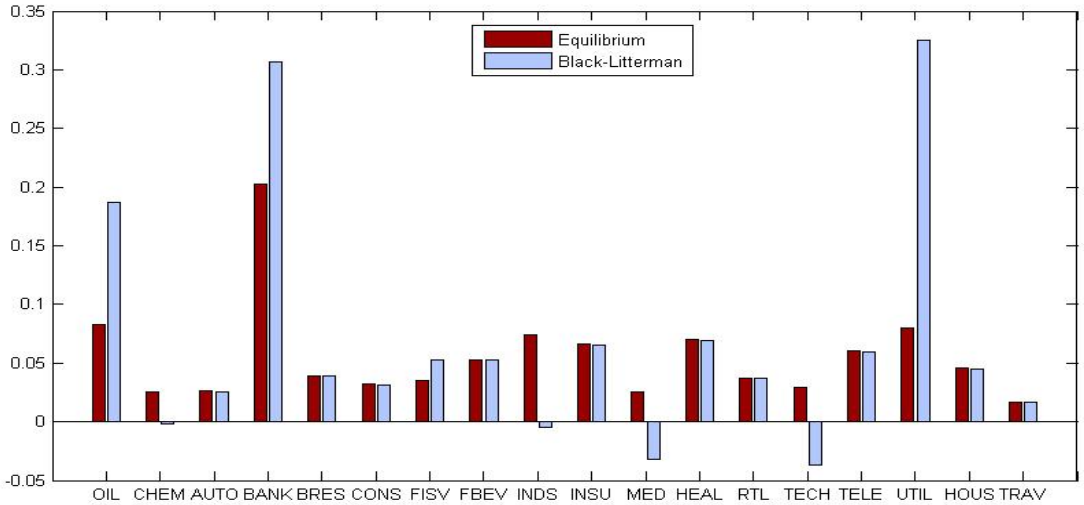
- **View 2:** Chemical (CHEM) and Industrial Goods (INDS) will outperform Oil & Gas (OIL) of 1% per year with a Confidence Level equals to 90% to register a final relative return in the interval [0%; 2%];

- **View 3:** Banks (BANK) and Financial Services (FISV) will outperform Media (MED) and Technology (TECH) of 2.5% per year with a Confidence Level 70% of recording a final relative return in the range [1.5%; 3.5%].

Empirical Investigation:BL returns and weights



Equilibrium and Black-Litterman returns



Equilibrium and Black-Litterman portfolio weights

Ex-ante TEV = 4.71%

Measure of marginal contributions [$\partial TEV/\partial q$]

	$\partial TEV/\partial w_{ACT;i}$		$\partial w_{ACT}/\partial q_1$	$\partial w_{ACT}/\partial q_2$	$\partial w_{ACT}/\partial q_3$
OIL	0.0247	*OIL*	6.3980	-83.7992	-7.1934
CHEM	0.0095	*CHEM*	-1.6456	21.5532	1.8501
AUTO	0.0097	*AUTO*	0.0000	0.0000	0.0000
BANK	0.0174	*BANK*	1.3883	6.1359	33.3300
BRES	0.0032	*BRES*	0.0000	0.0000	0.0000
CONS	0.0078	*CONS*	0.0000	0.0000	0.0000
FISV	0.0147	*FISV*	0.2392	1.0574	5.7439
FBEV	0.0191	*FBEV*	0.0000	0.0000	0.0000
INDS	-0.0036	*INDS*	-4.7524	62.2460	5.3432
INSU	0.0125	*INSU*	0.0000	0.0000	0.0000
MED	-0.0259	*MED*	-0.7584	-3.3521	-18.2084
HEAL	0.0186	*HEAL*	0.0000	0.0000	0.0000
RTL	0.0086	*RTL*	0.0000	0.0000	0.0000
TECH	-0.0352	*TECH*	-0.8691	-3.8413	-20.8654
TELE	-0.0224	*TELE*	0.0000	0.0000	0.0000
UTIL	0.0208	*UTIL*	68.8961	-6.3980	1.6275
HOUS	0.0064	*HOUS*	0.0000	0.0000	0.0000
TRAV	0.0085	*TRAV*	0.0000	0.0000	0.0000

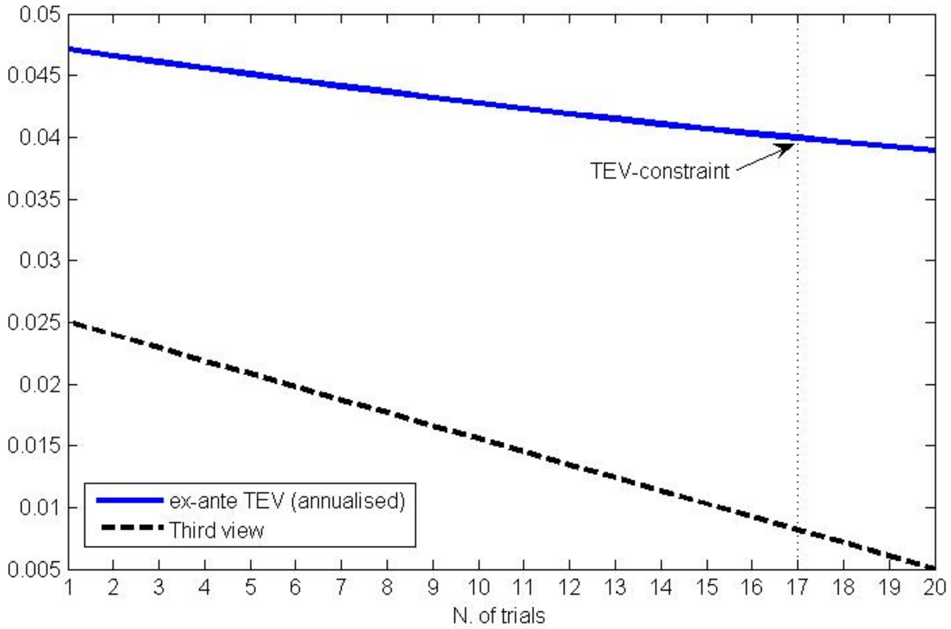
$$\left[\partial TEV/\partial q_1 = 1.6695;\ \partial TEV/\partial q_2 = -1.8741;\ \partial TEV/\partial q_3 = 1.7243\right]$$

Relevance of our sensitivity measure

The proposed sensitivity measure provides the asset managers with a method for revising the portfolio consistently with a given TEV constraint.

Suppose we have a TEV constraint of 4% per year, our measure is helpful to identify which view is mostly contributing to the TEV

A goal-attainment procedure is required in order to get the desired TEV threshold by repeatedly changing the view

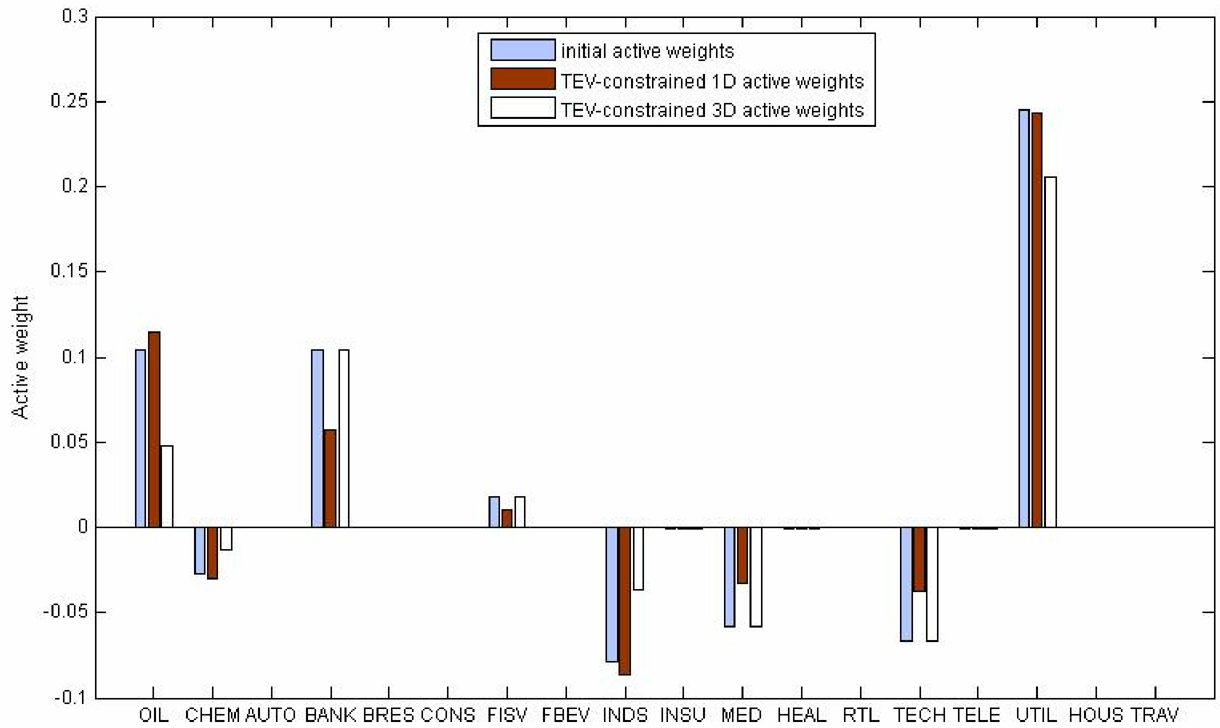


TEV - sensitivity to changes in q_3

Relevancy of our sensitivity measure

The extension to the multidimensional case is also possible

We test all triples of views able to achieve our TEV – constraint. The criterion to chose the best triple is the minimization of the multidimensional Minkowski distance (care should be taken in the choice fo the right span).



Conclusions

- We have proposed a probability-free measure of marginal contribution of each view to active risk (TEV)
- Our measure suggests the portfolio manager an objective way to refine the view in order to comply with a TEV constraint
- The calculation of the marginal contributions ($\partial TEV/\partial q$) can significantly enhance the interpretation and understanding of sources of active risk inherent in the active portfolio

Further developments

- Analyze the topic of TEV sensitivity to views with no-short selling and budget constraints in the optimization.
- Propose an optimal mechanism to connect the incentive fees both to the active return and the marginal contribution to ex-ante TEV generated by each view.

Further developments

On the empirical investigation side:

➢Apply active portfolio decisions derived from the Black Litterman model to set periodically a tactical portfolio as alternative to a strategic portfolio (it is the market neutral portfolio, i.e. the portfolio from which equilibrium returns are obtained)

➢Apply momentum strategies as generating process for the views (this means views and their confidence levels are generated according to past data)

➢ Extend the use of our marginal contribution measure in order to periodically validate or correct views should they imply a violation of the ex-ante active risk threshold.

➢ Measure the possible benefits from correcting the boldest view in terms of active risk contribution on the risk-adjusted performance of the portfolio.