# Estimating Asset Correlations From Stock Prices or Default Rates – Which Method is Superior?

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The views herein are our own and do not necessarily reflect those of the Deutsche Bundesbank

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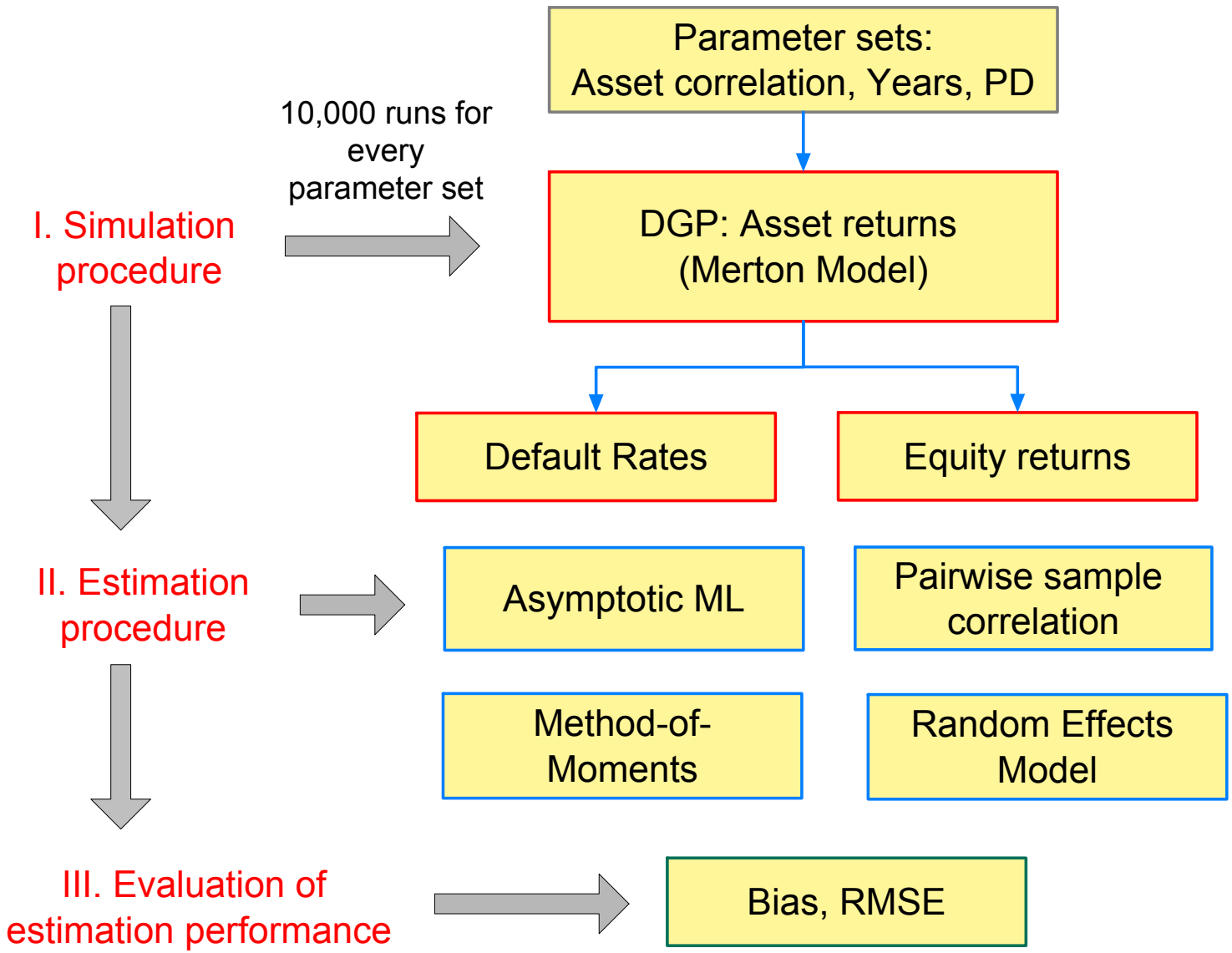
# Motivation

- Solve empirical puzzle: estimates of asset correlations from stock returns generally exceed estimates from default rates
- Asset correlation estimates based on default rates
  - Roesch (FMPM, 2003): 0.5% – 3%
  - Dietsch, Petey (JBF, 2004): 0.1% – 6%
- Asset correlation estimates based on stock returns
  - Lopez (WP, 2002): 10% – 26%
- Potential explanations
  - Sample mismatches
  - Model errors
  - Different small sample properties of estimators

# Contribution

- Clinical simulation study ensures consistency between
  - Asset returns and
  - Default rates
- Performance evaluation of different estimators
  - Error distribution
  - Bias, Root Mean Squared Error (RMSE)
- Wide range of parameters of data generation process (DGP) increases representativeness
  - Number of years
  - Asset correlation
  - Default probability
- First attempt to measure impact of model errors

# Simulation Setup



# Data Generation Process (DGP)

- Generate asset returns based on Merton (1974)

$$\Delta V_{i,t} = V_{i,t}(r+\sigma\lambda)\Delta t + V_{i,t}\,\sigma\sqrt{\Delta t}\,W_{i,t} + V_{i,t}\frac{1}{2}\sigma^2\Delta t(W_{i,t}^2-1)$$

$$\text{with } W_{i,t} = \sqrt{\rho}X_t + \sqrt{1-\rho}B_{i,t}.$$

- Default occurs if $(V_{i,t+h} < D)$ with $D$ calibrated from

$$PD = \Phi\left(\frac{\log\left(\frac{D}{V_{i,0}}\right) - (\mu - \frac{\sigma^2}{2})h}{\sigma\sqrt{h}}\right)$$

- Robustness check: asset correlation follows Vasicek process

$$\Delta\rho_t = \kappa(\rho - \rho_t)\Delta t + \sigma_\rho\sqrt{\Delta t}\,Z_t \text{ with } Z_t \sim \mathcal{N}(0,1)$$

# Estimation From Default Rates

1. Asymptotic Maximum Likelihood (AML) method
   - Fast, analytic solution
   - Assumes number of borrowers $N \to \infty$ and number of years $Y \to \infty$
2. Method-of-Moments (MM) method
   - Match mean default rate and sample variance
   - Assumes $N \to \infty$ and $Y \to \infty$

# Estimation From Asset Returns

1. Pairwise sample correlation of stock returns
2. Random Effects (RE) estimator
   - Asset values estimated by iterative (ML-equivalent) "KMV" method (See Duan, Gauthier, Simonato (2004))
   - RE model

$$\Delta \log[V_{i,t}] = \alpha_i + \sigma_X \tilde{X}_t + \sigma_U \tilde{U}_{i,t}$$

where $\widetilde{X}_t, \widetilde{U}_{i,t} \sim \mathcal{N}(0,1),\ i \in \{1,\ldots,N\}, t \in \{1,\ldots,Y\}$ *iid*.

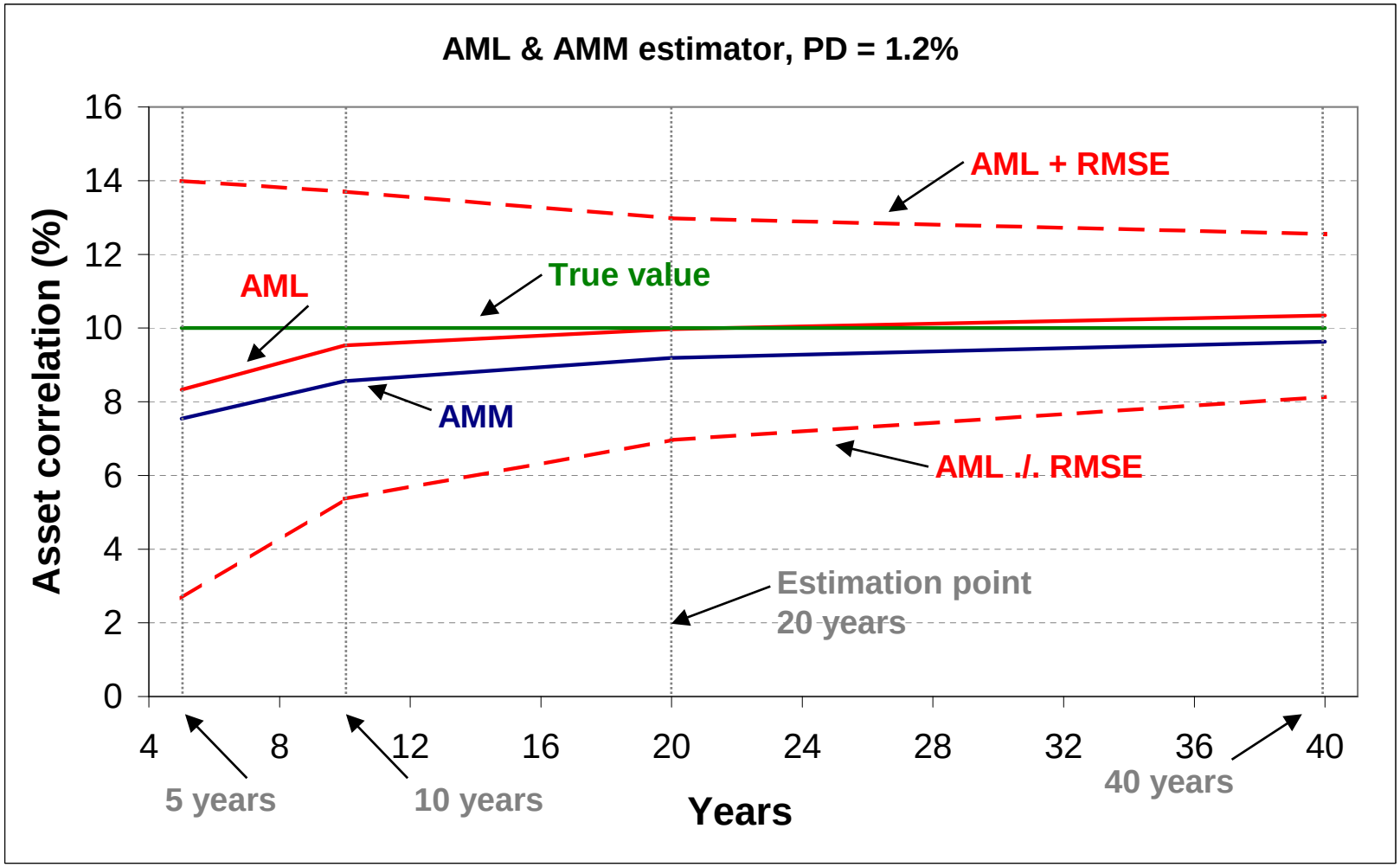
   - Estimated parameters: $\alpha_i, \sigma_X, \sigma_U$
   - Asset correlation $\hat{\rho}$ estimates given by

$$\hat{\rho} = \frac{\hat{\sigma}_X^2}{\hat{\sigma}_X^2 + \hat{\sigma}_B^2}.$$

# DGP Parameter Settings

| **Parameter** | | **Values** | | | | **Number** |
|---|---|---|---|---|---|---|
| Asset correlation | $\rho$ | 10% | 25% | | | 2 |
| Number of years | $Y$ | 5 | 10 | 20 | 40 | 4 |
| Default probability | $PD$ | 0.5% | 1.2% | 2% | | 3 |
| **Combinations** | | | | | | 24 |

# Estimates From Default Rates



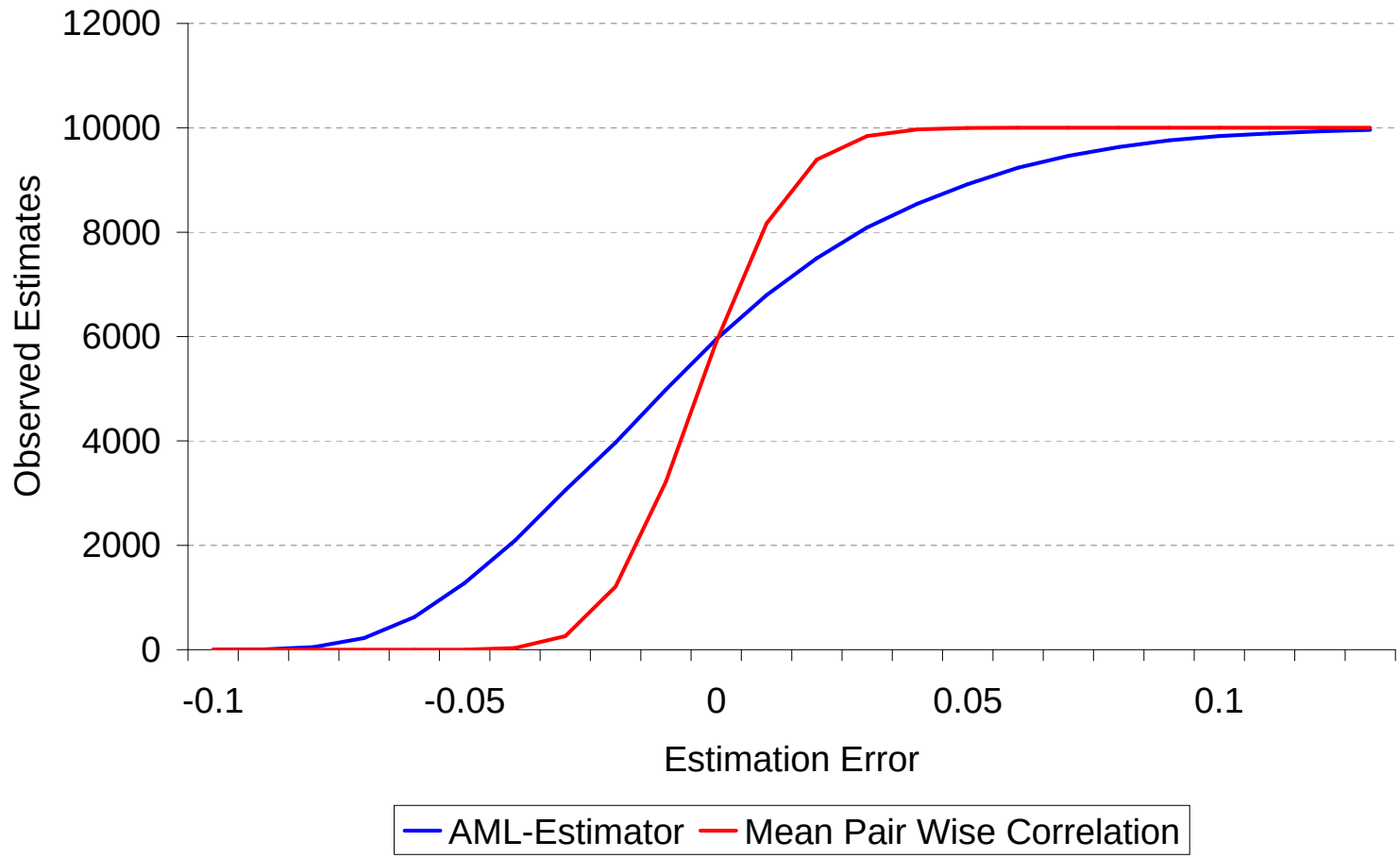
# Default Rates vs. Asset Returns – Bias

| **Bias** ($PD = 1.2\%$) | **Default Rates** 20 Years | | **Asset Returns** | |
|---|---|---|---|---|
| | **AML** | **MM** | **Pairwise Correl.** | **Random Effects** |
| Constant Asset Correlation | | | | |
| 10% | 0.03 | −0.81 | −0.28 | −0.06 |
| 25% | −2.28 | −4.98 | −0.59 | −0.17 |
| Stochastic Asset Correlation | | | | |
| mean 10% | 0.16 | −0.80 | −0.14 | – |
| mean 25% | −2.19 | −4.95 | −0.60 | – |

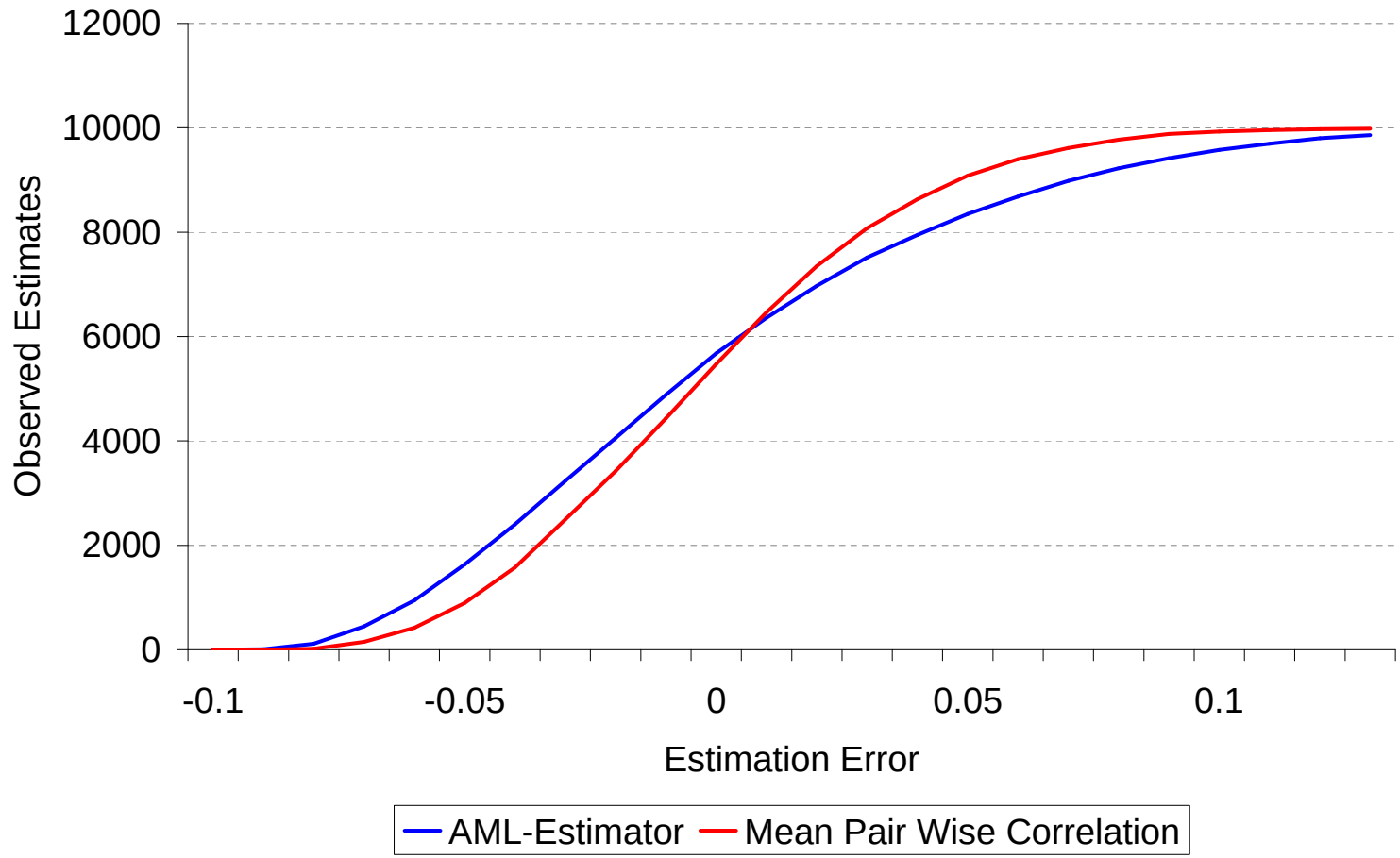
# Default Rates vs. Asset Returns – RMSE

| **RMSE** ($PD = 1.2\%$) | **Default Rates** 20 Years | | **Asset Returns** | |
|---|---|---|---|---|
| | **AML** | **MM** | **Pairwise Correl.** | **Random Effects** |
| Constant Asset Correlation | | | | |
| 10% | 3.01 | 3.70 | 1.49 | 1.48 |
| 25% | 5.52 | 9.51 | 2.85 | 2.79 |
| Stochastic Asset Correlation | | | | |
| mean 10% | 3.79 | 4.62 | 3.70 | – |
| mean 25% | 5.62 | 9.98 | 4.55 | – |

# Error Distribution – Fix Asset Correlations



# Error Distribution – Stochastic Asset Correlations



# Summary

- Correlation estimates from stock returns substantially less biased and less dispersed than estimates from default rates
- Sample correlation of equity returns provides reasonable results compared with estimation from RE model of inferred asset returns
- Higher estimates of asset correlations based on stock returns than those based on default rates are *partly* explained by differences in the performance of the estimation methods
- Results are robust against DGP perturbation by stochastic asset correlation process