The Value Premium and Time-Varying Idiosyncratic Risk

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Explanations for the value premium

- From the behavioural point of view
  - Investors' judgement biases cause market mis-pricing
  - Haugen (1995), LSV (1994), LLSV (1997)

- From the rational point of view
  - The value premium is compensation for risk
  - It reflects a misspecification of CAPM, not a sign of market inefficiency
  - FF (1993, 1995, 1996), Chen and Zhang (1998)

Explanations for the value premium

- The static CAPM fails to capture the post-1963 value premium
  - FF (2005), Ang and Chen (2007)

- Conditional versions of the CAPM perform better than the traditional version at explaining the cross-sectional variation in expected return but the post 1963-value premium is still a puzzle
  - Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Adrian and Franzoni (2005), FF (2005), Petkova and Zhang (2005), Lewellen and Nagel (2006), Ang and Chen (2007)

- Analysis of the post-1963 value premium within the family of GARCH(1,1) models; i.e., we allow
  - for more than one factor,
  - for the variance of returns to be time-dependent,
  - for heteroscedasticity and autocorrelation in the error terms of the market model

Methodology

- Model 1: Static CAPM $r_{Pt} = \alpha + \beta\left(R_{Mt} - R_{ft}\right) + \varepsilon_{Pt}$

- Model 2: CAPM with GARCH(1,1) process (Bollerslev, 1986)

$$r_{Pt} = \alpha + \beta\left(R_{Mt} - R_{ft}\right) + \varepsilon_{Pt}$$
$$\sigma_{Pt}^2 = \omega + \gamma\varepsilon_{Pt-1}^2 + \theta\sigma_{Pt-1}^2$$

- Model 3: CAPM with GJR-GARCH(1,1) process (Golsten, Jagannathan and Runkle, 1993)

$$r_{Pt} = \alpha + \beta\left(R_{Mt} - R_{ft}\right) + \varepsilon_{Pt}$$
$$\sigma_{Pt}^2 = \omega + \gamma\varepsilon_{Pt-1}^2 + \eta I_{t-1}\varepsilon_{Pt-1}^2 + \theta\sigma_{Pt-1}^2, \quad I_{t-1} = 1 \;\; if \;\; \varepsilon_{Pt} < 0$$

Methodology

- Models 4 and 5: CAPM with a GARCH(1,1)-M process (Bollerslev, Engle and Wooldridge, 1988) with or without GJR term

Model 4

$$r_{Pt} = \alpha + \beta\left(R_{Mt} - R_{ft}\right) + \delta\sigma_{Pt} + \varepsilon_{Pt}$$
$$\sigma_{Pt}^2 = \omega + \gamma\varepsilon_{Pt-1}^2 + \theta\sigma_{Pt-1}^2$$

Model 5

$$r_{Pt} = \alpha + \beta\left(R_{Mt} - R_{ft}\right) + \delta\sigma_{Pt} + \varepsilon_{Pt}$$
$$\sigma_{Pt}^2 = \omega + \gamma\varepsilon_{Pt-1}^2 + \eta I_{t-1}\varepsilon_{Pt-1}^2 + \theta\sigma_{Pt-1}^2$$

- Models 6 and 7: Nelson (1991) and Hentschel (1995) use variance of returns in mean equation

Model 6

$$r_{Pt} = \alpha + \beta\left(R_{Mt} - R_{ft}\right) + \nu\sigma_{Pt}^2 + \varepsilon_{Pt}$$
$$\sigma_{Pt}^2 = \omega + \gamma\varepsilon_{Pt-1}^2 + \theta\sigma_{Pt-1}^2$$

Model 7

$$r_{Pt} = \alpha + \beta\left(R_{Mt} - R_{ft}\right) + \nu\sigma_{Pt}^2 + \varepsilon_{Pt}$$
$$\sigma_{Pt}^2 = \omega + \gamma\varepsilon_{Pt-1}^2 + \eta I_{t-1}\varepsilon_{Pt-1}^2 + \theta\sigma_{Pt-1}^2$$

Data

- US data from French's website
  - Value = Decile with highest B/M, C/P or E/P
  - Growth = Decile with lowest B/M, C/P or E/P
  - HML = Fama and French (1993)
  - Sample: July 1926 – June 2006, split into pre and post July 1963

- UK data from Nagel's website
  - Value = Portfolio with 40% highest B/M
  - Growth = Portfolio with 40% lowest B/M
  - Sample: Jan 1963 – Dec 2001

Data

	B/M Portfolio			C/P Portfolio			E/P Portfolio		
	High	*Low*	*HML*	*High*	*Low*	*HML*	*High*	*Low*	*HML*
Panel A: 7/1926-6/2006									
Mean (%)	1.40	0.87	0.54						
	(4.63)	(4.65)	(2.49)						
Std Dev (%)	9.40	5.77	6.69						
Panel B: 7/1926-6/1963									
Mean (%)	1.43	0.93	0.50						
	(2.39)	(3.05)	(1.23)						
Std Dev (%)	12.57	6.39	8.55						
Panel C: 7/1963-6/2006									
Mean (%)	1.39	0.81	0.57	1.33	0.84	0.49	1.42	0.82	0.60
	(5.90)	(3.58)	(2.88)	(6.03)	(3.42)	(2.58)	(6.12)	(3.24)	(2.96)
Std Dev (%)	5.34	5.18	4.52	5.01	5.58	4.30	5.27	5.74	4.60

In line with FF (1992, 1993, 2005), Davis, Fama and French (2000) and Ang and Chen (2007)

The static CAPM

	B/M Portfolio			C/P Portfolio			E/P Portfolio		
	High	*Low*	*HML*	*High*	*Low*	*HML*	*High*	*Low*	*HML*
Panel A: 7/1926-6/2006									
α (%)	0.17	-0.09	0.25						
	(1.11)	(-1.45)	(1.36)						
β_M	1.45	1.00	0.44						
	(17.06)	(58.62)	(4.51)						
R^2	0.70	0.90	0.13						
$LM-ARCH(5)$	23.01	26.43	23.84						
	[0.00]	[0.00]	[0.00]						
Panel B: 7/1926-6/1963									
α (%)	-0.14	0.00	-0.13						
	(-0.51)	(-0.04)	(-0.43)						
β_M	1.70	0.96	0.74						
	(15.63)	(47.28)	(5.95)						
R^2	0.76	0.94	0.31						
$LM-ARCH(5)$	4.51	8.45	3.58						
	[0.21]	[0.13]	[0.31]						
Panel C: 7/1963-6/2006									
α (%)	0.46	-0.16	0.62	0.41	-0.18	0.59	0.48	-0.21	0.69
	(3.31)	(-1.87)	(3.15)	(3.48)	(-1.97)	(3.22)	(3.73)	(-2.04)	(3.45)
β_M	0.98	1.09	-0.11	0.96	1.18	-0.22	1.00	1.20	-0.19
	(21.10)	(44.64)	(-1.69)	(22.34)	(46.60)	(-3.45)	(22.51)	(39.84)	(-2.84)
R^2	0.65	0.86	0.01	0.71	0.86	0.05	0.70	0.84	0.03
$LM-ARCH(5)$	27.24	10.94	19.26	27.82	12.70	26.07	82.72	25.91	70.15
	[0.00]	[0.05]	[0.00]	[0.00]	[0.03]	[0.00]	[0.00]	[0.00]	[0.00]

The CAPM with a GARCH(1,1) framework

Model	High B/M (Value)						Low B/M (Growth)						HML (Value Premium)					
	2	3	4	5	6	7	2	3	4	5	6	7	2	3	4	5	6	7
Panel A: Mean Equation																		
α (%)	0.46	0.41	-0.49	-0.48	0.08	0.07	-0.12	-0.11	0.45	0.30	0.27	0.18	0.52	0.46	-1.51	-1.39	-0.45	-0.44
	(3.51)	(3.34)	(-1.12)	(-1.11)	(0.37)	(0.31)	(-1.45)	(-1.37)	(1.58)	(1.20)	(1.43)	(1.08)	(2.97)	(2.60)	(-1.81)	(-1.76)	(-1.07)	(-1.12)
β_M	0.96	0.96	0.96	0.96	0.96	0.96	1.09	1.10	1.09	1.09	1.09	1.09	-0.07	-0.08	-0.07	-0.08	-0.07	-0.08
	(28.04)	(28.26)	(28.20)	(28.40)	(27.88)	(28.14)	(48.91)	(46.54)	(49.36)	(47.11)	(49.42)	(47.41)	(-1.50)	(-1.64)	(-1.45)	(-1.60)	(-1.44)	(-1.59)
δ	—	—	0.35	0.33	—	—	—	—	-0.32	-0.23	—	—	—	—	0.50	0.46	—	—
			(2.16)	(2.07)					(-2.04)	(-1.69)					(2.46)	(2.36)		
ν	—	—	—	—	4.68	4.29	—	—	—	—	-11.30	-8.64	—	—	—	—	5.63	5.28
					(1.88)	(1.79)					(-2.21)	(-1.83)					(2.48)	(2.46)
Panel B: Conditional Variance Equation																		
ω (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
	(1.48)	(1.33)	(1.55)	(1.54)	(1.55)	(1.54)	(1.95)	(1.68)	(1.81)	(1.73)	(2.57)	(2.31)	(1.66)	(1.36)	(1.70)	(1.70)	(1.71)	(1.72)
γ	0.13	0.10	0.13	0.10	0.13	0.10	0.04	0.04	0.04	0.04	0.04	0.04	0.10	0.06	0.09	0.06	0.10	0.06
	(3.53)	(2.43)	(3.77)	(2.61)	(3.65)	(2.60)	(2.45)	(2.42)	(2.73)	(2.58)	(2.72)	(2.64)	(3.43)	(2.18)	(3.57)	(2.29)	(3.57)	(2.37)
η	—	0.07	—	0.06	—	0.06	—	-0.05	—	-0.04	—	-0.04	—	0.07	—	0.05	—	0.05
		(1.01)		(0.95)		(0.96)		(-1.98)		(-1.84)		(-1.78)		(1.58)		(1.31)		(1.34)
θ	0.85	0.85	0.85	0.85	0.85	0.85	0.94	0.96	0.94	0.96	0.94	0.96	0.87	0.88	0.88	0.89	0.88	0.89
	(18.84)	(19.39)	(21.93)	(22.49)	(20.81)	(21.27)	(44.35)	(59.51)	(43.52)	(59.18)	(51.25)	(66.59)	(22.33)	(26.70)	(25.93)	(30.41)	(26.27)	(31.22)
AIC	-4.177	-4.178	-4.181	-4.181	-4.179	-4.180	-5.040	-5.046	-5.043	-5.045	-5.045	-5.046	-3.433	-3.435	-3.440	-3.440	-3.441	-3.441

$$r_{Pt} = \alpha + \beta_M \left(R_{Mt} - R_{ft}\right) + \delta\sigma_{Pt} + \nu\sigma_{Pt}^2 + \varepsilon_{Pt}$$

$$\sigma_{Pt}^2 = \omega + \gamma\varepsilon_{Pt-1}^2 + \eta I_{t-1}\varepsilon_{Pt-1}^2 + \theta\sigma_{Pt-1}^2, \quad I_{t-1} = 1 \ if \ \ \varepsilon_{Pt} < 0$$

Robustness tests

- The results are robust to
  - the criterion used to define value (B/M, E/P or C/P)
  - the inclusion of SMB in the mean equation
  - the country under review (US or UK)

- Conclusion: Time-varying total risk, as measured by the GARCH(1,1)-M term, captures the value premium

Interpretation of the results

- Conditional APT world with two risk factors

$$R_{Pt} - R_{ft} = \beta_{Mt}\left(R_{Mt} - R_{ft}\right) + \beta_{1t}\lambda_{1t}$$

  - λ_{1t} = Time-varying risk premium associated with F_{1t}
  - F_{1t} = Non-specified risk factor with zero-mean and zero correlation to M
  - β_{1t} = Time t sensitivity of P to risk factor F_1

- If we run a conditional CAPM regression, $R_{Pt} - R_{ft} = \alpha_t + \beta_{Mt}\left(R_{Mt} - R_{ft}\right) + \varepsilon_{Pt}$

$$\alpha_t = \beta_{1t}\lambda_{1t} = \frac{\operatorname{cov}\left(r_{Pt}, F_{1t}\right)}{\operatorname{var}\left(F_{1t}\right)}\lambda_{1t} = \frac{\rho_{PF_1,t}\,\boxed{\sigma_{Pt}}}{\sigma_{F_1 t}}\lambda_{1t}$$

- A mis-specified conditional CAPM leads to the presence of an alpha that depends on the conditional volatility of the returns of portfolio *P*

- This is presumably what the GARCH(1,1)-M term captures