Can Noise Create the Size and Value Effects?

Robert Arnott, Research Affiliates
Jason Hsu, Research Affiliates & UCLA
Jun Liu and Harry Markowitz, UCSD

EFM-EDHEC 2008 Symposium on Risk and Asset Management

April, 2008

Outline

- Introduction/Setup
- Time-Series Expected Return Conditional on Size and Value
- The Cross-Section of Expected Return
- Related Studies
- Conclusion

The Contribution of This Paper

We assume that the market price of a stock may be different from its fundamental value by a noise (based on the model of Blume and Stambaugh (1983)).

1. Time-series wise, we show that the conditional expected return of a stock is high when its market capitalization or price-dividend ratio is low.

2. Cross-section wise, we show that stocks with low market caps or low price-dividend ratios have high expected return.

   Furthermore, the cross-section of expected returns is similar to that observed in the data.

Thus, noise can create size and value effects.

The Main Assumptions

The main assumptions are

- The price P_t of a stock is different from its "intrinsic value" V_t by a random noise.

$$P_t = V_t \frac{e^{\Delta_t}}{\mathrm{E}[e^{\Delta_t}]}.$$

- V_t is independent of Δ_t.
- V_t has all the nice properties expected by economists. For example, return computed from its is dictated by APT. Thus return computed using V_t has no size and value effects.
- Δ_t is mean-reversion, which captures temporary nature of the noise.

Noise in the Literature

- market microstructure: Blume and Stambaugh (1983) study unconditional expected return.
- fundamental: Black (1986) argues that noise is essential for trades in markets.
- alternative to efficient market: Summers (1986) argues that it is difficult to detect the noise in price using the traditional econometric techniques.
- term structure: stochastic singularity.
- behavioral: over-or under-reaction.

Technical Assumptions

To quantify the effects of the noise, we will make additional technical assumptions about the properties of V_t and Δ_t.

Notation: $p_t = \ln P_t$, $v_t = \ln V_t$, $d_t = \ln D_t$, where D_t is the dividend.

1. v_t is a white noise.

   Expected return is independent of v_t.

2. Δ_t follows an AR(1) process: $\Delta_{t+1} = \rho \Delta_t + \sigma_{\epsilon_\Delta} \epsilon_{\Delta t+1}$.

   AR(1) is one of the many possible mean-reversion processes.

3. The price-dividend ratio satisfies:

$$p_{t+1} - d_{t+1} = (1 - \rho_x)\bar{x} + \rho_x (p_t - d_t) + \sigma_{\epsilon_x} \epsilon_{xt+1}.$$

Intuition for Conditional Expected Return

- If the noise is known to be negative, a stock is cheaper than its intrinsic value, thus its expected return should be high.
- In reality, we do not observe the noise. But we can infer the noise from the price. If the price is low, it is likely that the noise is negative, thus higher returns on average.

The price or price ratios are signals of the noise.

“Time-Series Value Effect”

- The expected return of a stock is high when its price-dividend ratio is high: the expected return conditional on the price-dividend ratio is inversely proportional to a power of the price-divided ratio.

  Note that this proposition is about the time-series property of a stock return.

- The “time-series size effect” is really the dividend-yield predictability.

Expected Return Conditional on Price-Dividend Ratio

Proposition 4 *Suppose that Assumptions 1, 2, and 3 hold. Furthermore, assume that the distribution of* (Δ_t, x_t) *is their unconditional distribution. Then the expected return conditional on* X_t *is*

$$
\begin{aligned}
& \mathrm{E}\left[\frac{P_{t+1}+D_{t+1}}{P_t}|X_t\right] \\
= \quad & e^{\mu+\frac{1}{2}\sigma_r^2}\left(e^{\frac{\sigma_{\epsilon_\Delta}^2}{1+\rho}}\frac{X_t^{-(1-\rho)\gamma_2}}{\mathrm{E}\Big[X_t^{-(1-\rho)\gamma_2}\Big]}+e^{-\bar{x}_v+\frac{\sigma_{\epsilon_x}^2}{2(1-\rho_x^2)}+\frac{\sigma_{\epsilon_\Delta}^2}{1-\rho^2}}\frac{X_t^{-(1-\rho_x)\gamma_2-\rho_x}}{\mathrm{E}\Big[X_t^{-(1-\rho_x)\gamma_2-\rho_x}\Big]}\right),
\end{aligned}
$$

where $\gamma_2 = \frac{(1-\rho_x^2)\sigma_{\epsilon_\Delta}^2}{(1-\rho_x^2)\sigma_{\epsilon_\Delta}^2+(1-\rho^2)\sigma_{\epsilon_x}^2}$.

“Time-Series Size/Value Effect”

In our model, the size and value effects arise from the same source.

One might wonder if the size effect is the same as the value effect in our model: the value effect will disappear if we control for the size effect, and vice versa.

Neither turns out to be true. This is because the price is a noisy signal of the noise–the price does not determine the noise; the price-dividend ratio provide an additional signal about the noise.

For example, if both the price and the price-dividend ratio are low, it is more likely that the noise is negative than if only of them is low.

Cross-Sectional Variation

We assume that the cross-sectional difference in expected returns is generated completely from different realization of noise.

1. Prior to realization of random noise, all stocks have identical distribution (same beta, volatility, etc) and thus identical expected return.
2. After realization of random noise which is independent across stocks, they have different distributions and thus different expected returns.

Our model is parsimonious and restrictive.

Table 1: Summary of Parameters

μ	σ_r	σ_{ϵ_Δ}	ρ	$\bar{x}_v$	ρ_x	σ_{ϵ_x}
3%	30%	6%	0.5	4	0.9	10%

Note that

- If the variation is generated using unconditional expected return, we need to assign 100 parameters for 100 portfolios.
- We only need to assign 7 parameters. More restrictive.

Table 2: Expected Annual Returns Conditional on Size and Value Deciles

	Dividend-to-Price Ratio										
	All	1	2	3	4	5	6	7	8	9	10
All	10.08	7.52	8.50	9.03	9.45	9.84	10.22	10.62	11.08	11.68	12.89
ME-1	11.63	9.08	10.04	10.56	10.98	11.36	11.73	12.13	12.58	13.17	14.36
ME-2	11.00	8.49	9.44	9.95	10.37	10.74	11.11	11.51	11.95	12.53	13.71
ME-3	10.67	8.18	9.13	9.64	10.05	10.43	10.80	11.19	11.63	12.21	13.39
ME-4	10.42	7.94	8.88	9.39	9.80	10.18	10.55	10.94	11.38	11.95	13.13
ME-5	10.19	7.72	8.66	9.17	9.58	9.95	10.32	10.71	11.15	11.73	12.90
ME-6	9.97	7.51	8.45	8.95	9.36	9.74	10.11	10.49	10.93	11.51	12.68
ME-7	9.74	7.29	8.23	8.73	9.14	9.51	9.88	10.27	10.71	11.28	12.45
ME-8	9.49	7.04	7.98	8.49	8.89	9.27	9.63	10.02	10.46	11.03	12.20
ME-9	9.17	6.74	7.67	8.18	8.58	8.95	9.32	9.71	10.14	10.71	11.87
ME-10	8.56	6.13	7.07	7.57	7.98	8.35	8.72	9.10	9.54	10.11	11.27

Effects of Noise on Risks

Is the higher conditional expected return due to higher conditional (systematic) risk?

Assuming a riskfree return and single factor model (such as CAPM), we can determine the systematic risk.

Table 3: Beta Conditional on Size and Value Deciles

	Dividend-to-Price Ratio										
	All	1	2	3	4	5	6	7	8	9	10
All	1.005	0.971	0.984	0.991	0.997	1.002	1.007	1.012	1.018	1.025	1.040
ME-1	1.019	0.984	0.998	1.005	1.011	1.016	1.021	1.026	1.032	1.040	1.054
ME-2	1.013	0.979	0.992	1.000	1.005	1.010	1.015	1.021	1.026	1.034	1.048
ME-3	1.010	0.976	0.990	0.997	1.002	1.007	1.012	1.018	1.023	1.031	1.045
ME-4	1.008	0.974	0.987	0.994	1.000	1.005	1.010	1.015	1.021	1.028	1.043
ME-5	1.006	0.972	0.985	0.992	0.998	1.003	1.008	1.013	1.019	1.026	1.041
ME-6	1.004	0.970	0.983	0.990	0.996	1.001	1.006	1.011	1.017	1.024	1.039
ME-7	1.002	0.968	0.981	0.988	0.994	0.999	1.004	1.009	1.015	1.022	1.037
ME-8	1.000	0.966	0.979	0.986	0.992	0.997	1.002	1.007	1.013	1.020	1.034
ME-9	0.997	0.963	0.976	0.983	0.989	0.994	0.999	1.004	1.010	1.017	1.031
ME-10	0.991	0.957	0.971	0.978	0.984	0.989	0.993	0.999	1.004	1.012	1.026

Effects of Noise on Alpha

As expected, the conditional alpha is positive for small and value stocks and negative for large and growth stocks.

Table 4: Alpha Conditional on Size and Value Deciles

	Dividend-to-Price Ratio										
	All	1	2	3	4	5	6	7	8	9	10
All	0.24	-1.43	-0.76	-0.41	-0.13	0.12	0.36	0.61	0.89	1.24	1.93
ME-1	1.67	-0.03	0.65	1.01	1.29	1.55	1.79	2.05	2.33	2.69	3.40
ME-2	1.09	-0.59	0.08	0.44	0.71	0.96	1.21	1.45	1.74	2.10	2.79
ME-3	0.79	-0.89	-0.21	0.14	0.41	0.66	0.91	1.16	1.44	1.79	2.48
ME-4	0.55	-1.12	-0.45	-0.10	0.18	0.43	0.67	0.92	1.20	1.55	2.24
ME-5	0.34	-1.33	-0.66	-0.31	-0.03	0.22	0.46	0.71	0.99	1.34	2.03
ME-6	0.14	-1.53	-0.86	-0.51	-0.24	0.01	0.25	0.50	0.78	1.13	1.82
ME-7	-0.08	-1.74	-1.07	-0.72	-0.45	-0.20	0.04	0.29	0.57	0.92	1.60
ME-8	-0.31	-1.97	-1.30	-0.96	-0.68	-0.44	-0.20	0.05	0.33	0.68	1.37
ME-9	-0.61	-2.26	-1.60	-1.25	-0.98	-0.73	-0.49	-0.24	0.03	0.38	1.07
ME-10	-1.18	-2.84	-2.17	-1.82	-1.55	-1.30	-1.06	-0.81	-0.54	-0.18	0.50

Other Sources of Cross-Sectional Variation

In reality, prior distribution of stocks are different;

1. different β and σ;
2. different adjustment cost;
3. consumption-based: non-IID growth, durable versus non-durable, etc

As such, there will be more cross-sectional variations in expected returns.

Comparison with Blume and Stambaugh

Blume and Stambaugh	**Our Paper**
microstructure	non-microstructure
small=high noise σ	small=low market cap
unconditional expected return	conditional expected return
parameter variation	noise variation
no variation if ex ante IID	variation observed in the data

Comparison with Berk (1996, 1997)

Berk	**Our Paper**
price is right	price is wrong
conditioning on price has no effect	conditioning on price has effect
unconditional expected return	conditional expected return
parameter variation	noise variation
no variation if ex ante I.D.	variation observed in the data

Comparison with NREE Models

NREE Models	**Our Paper**
price=value	price=value+noise
noise in demand	noise in price
price is endogenous	price is exogenous
risk premium is $\propto$ risk	risk premium is not $\propto$ risk
$\alpha = 0$	$\alpha \neq 0$

After all, price noise is proposed by Summers (1986) as an alternative to rational expectation models.

Conclusions

It is plausible that the theoretical value from economic theories is different from the market price.

We show that pricing errors can generate significant variations in expected returns. In particular,

- For a given stock, conditional expected returns decreases with price and price-dividend ratio;
- Cross-section wise, expected returns conditional on size an value deciles calibrated from the data are similar from their empirical counterparts documented in Fama and French (1992).

Noise can create size and value effect!