Portfolio Construction with Downside Risk

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Outline

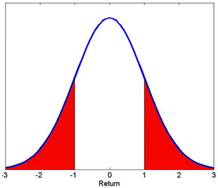
Motivation

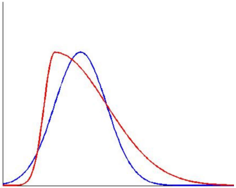
- Asset returns exhibit asymmetric return distributions
- Several asymmetric risk measures have been proposed to handle downside risk
- Portfolio optimization may well benefit from their inclusion
- We discuss several downside risk measures and empirically assess their use in a portfolio optimization study

Motivation

Volatility Issues

- popular measure for portfolio optimization
- based on the absolute deviation of returns from the mean value
- counts positive deviations as risk—contradicting intuition
- Distributions with equal mean and volatility may entail different downside risks

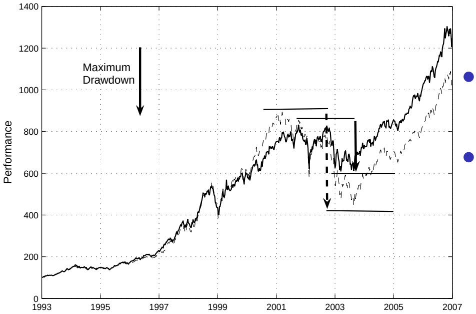




$\rightarrow$ Optimize portfolios with asymmetric risk measures instead

Maximum Drawdown

Maximum Drawdown is the most intuitive measure of downside risk: It is the maximum percentage loss in a given time period



- Independent of return distribution
- Only requires return observations

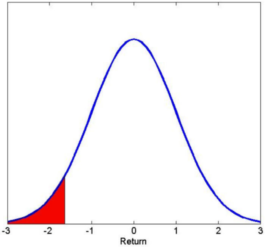
Value at Risk (VaR) and Conditional VaR

Value at Risk

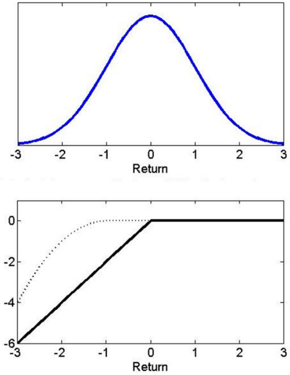
- VaR is a quantile of the return distribution
- Optimization with VaR may lead to fat-tailed return distributions

Conditional VaR:

- CVaR is a conditional expectation: What is the expected return below a given threshold?
- Estimation of CVaR requires certain amount of observations below the threshold



Lower Partial Moments



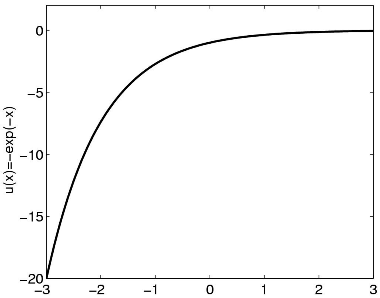
Semi-Deviation and Semi-Variance

- LPMs are a family of risk measures accounting for downside risk:

$$LPM_{\tau,k}(R) = \mathbb{E}\left((\tau - R)^k \mid R < \tau\right) \cdot P(R < \tau)$$

- Returns are perceived risky when below threshold τ, degree of risk perception dependent of k
- Semideviation: $\tau = \mu$ and $k = 1$
- Semivariance: $\tau = \mu$ and $k = 2$

Loss-oriented Utility



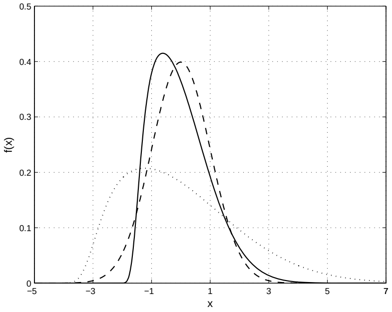
Loss Penalty

- Investor maximizes specific utility function that accounts for downside risk

$$u(x) = -\exp\{-(x - \mathbb{E}(X))\}.$$

- Adjustment for mean return to pursue risk reduction

Skewness



- Definition:

$$\gamma(R) = \frac{\mathbb{E}\left((R - \mathbb{E}R)^3\right)}{\sigma(R)^{3/2}}$$

- Right-skewed distributions appeal to downside risk-averters
- Avoid highly volatile portfolios by additionally constraining volatility

$$\hat{\gamma}(X) = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{X_t - \hat{\mu}}{\hat{\sigma}(X)}\right)^3 \text{ versus } S_B(X) = \frac{Q_3(X) + Q_1(X) - 2Q_2(X)}{Q_3(X) - Q_1(X)}$$

Portfolio Optimization with Downside Risk

- Asymmetric return distributions may severely hamper standard mean-variance optimization
- Including downside risk measures in portfolio optimization may help to construct portfolios that better suits investors' risk perception
- We consider the following portfolio optimization study:
  - A portfolio manager is benchmarked against Euro STOXX 50 with a maximum TE of 5%
  - optimizes his risk exposure according to alternative definitions of downside risk
  - has no additional incentive to exceed the benchmark's return

Portfolio Optimization with Downside Risk

Additionally the manager

- faces constraints:
  - $\boldsymbol{\mu}^T \mathbf{x} \geq \boldsymbol{\mu}^T \mathbf{x}_{BM}$
  - $\sum_{i=1}^{N} x_i = 1$ and $x_i \geq 0 \quad \forall i$
  - For skewness: $\sigma(\mathbf{x}) = \sqrt{\mathbf{x}^T C \mathbf{x}} \leq \sigma_{BM}$
- rebalances quarterly using two years' weekly returns

Manager's information set with respect to

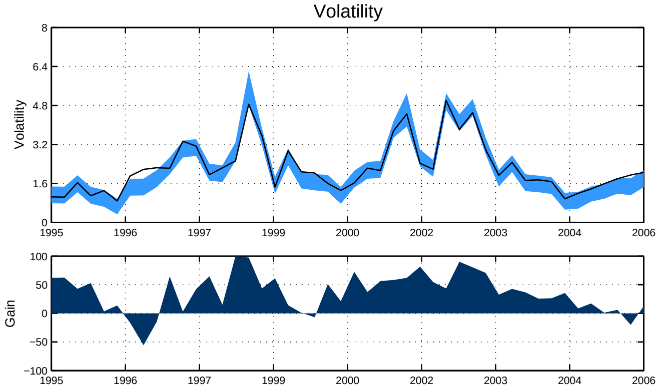
- Risk: has to be estimated from historical data
- Returns: The Manager
  1. has perfect foresight
  2. employs a naïve estimate

Minimizing Downside Risk: Perfect Foresight

	Strategy	Risk			Gain	Hit Rate
		benchmark	*optimized*	*optimal*	*average*	
1	Volatility	2.538	2.264	1.854	39.96%	**89.13%**
2	VaR	-0.460	-1.025	-2.165	33.13%	**84.78%**
3	Conditional VaR	1.702	1.095	0.553	52.83%	**91.30%**
4	Semideviation	2.106	1.877	1.345	30.01%	**73.91%**
5	Semivariance	3.679	2.868	2.095	51.16%	**84.78%**
6	Loss Penalty	33.427	17.908	7.918	60.84%	**76.09%**
7	Skewness standard	0.149	0.097	-3.437	1.46%	54.35%
8	Skewness robust	0.066	0.019	-0.919	4.74%	50.00%
9	Maximum Drawdown	0.073	0.054	0.032	46.98%	**84.78%**

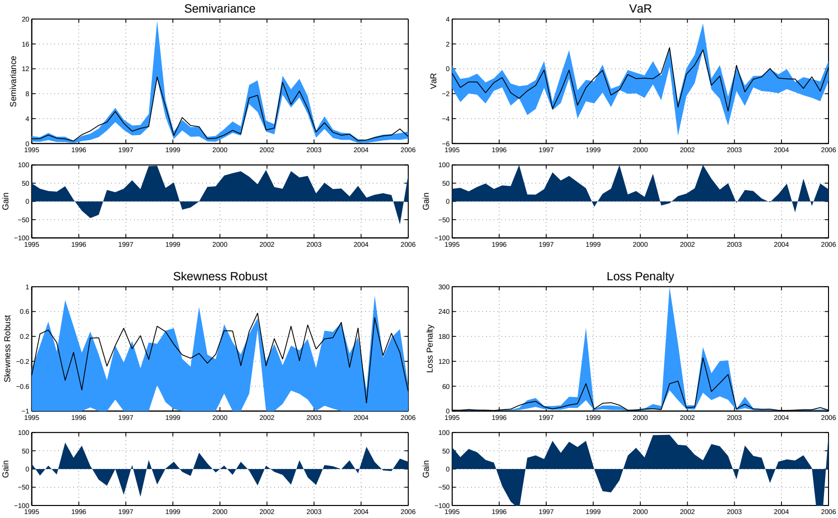
- $Gain = Mean_{\#Optimizations}\left(\frac{risk(optimized) - risk(benchmark)}{risk(optimal) - risk(benchmark)}\right)$
- $HitRate = \frac{\#\{risk(optimized) > risk(benchmark)\}}{\#optimizations}$
- except for skewness the asymmetric measures allow for minimizing downside risk

Minimizing Risk: Evolution over Time

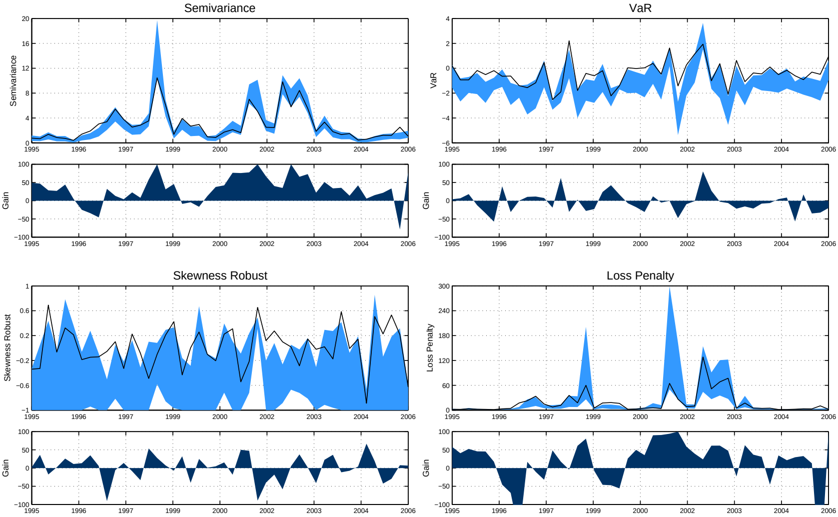


- Upper: Spread between benchmark and optimal portfolio, and risk of optimized portfolio
- Lower: Gain

Some Downside Risk Measures: Perfect Foresight



Some Downside Risk Measures: Forecasted Returns

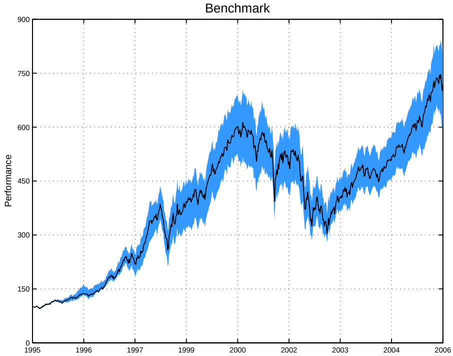


Minimizing Downside Risk: Forecasted Returns

	Strategy	Risk			Gain	Hit Rate
		benchmark	*optimized*	*optimal*	*average*	
1	Volatility	2.538	2.275	1.854	38.37%	**86.97%**
2	VaR	-0.460	-0.407	-2.165	-3.09%	39.13%
3	Conditional VaR	1.702	1.635	0.553	5.83%	58.70%
4	Semideviation	2.106	1.908	1.345	26.03%	**58.70%**
5	Semivariance	3.679	2.862	2.095	51.55%	**84.78%**
6	Loss Penalty	33.427	17.700	7.918	61.65%	**69.57%**
7	Skewness standard	0.149	0.029	-3.437	3.35%	58.70%
8	Skewness robust	0.066	0.016	-0.919	5.01%	63.04%
9	Maximum Drawdown	0.073	0.069	0.032	10.67%	**67.39%**

- The degree of risk reduction is muted as compared to perfect foresight
- Setback most severe for VaR and CVaR
- 4 measures still have significant hit rates: semivariance, semideviation, loss penalty, maximum drawdown

Performance of Downside Risk Strategies



- Spread between the highest and lowest cumulated return of downside strategies
- Solid line represents benchmark performance

- Downside risk control not necessarily at the cost of return
- Downside risk control is not hedging market risk in our setting

Mutual Tracking Errors of Strategies

	BM	Vola	VaR	CVaR	Semi- *Dev.*	Semi- *Var.*	Loss Penalty	Skewness *Std.*	Skewness *Rob.*	MDD
BM	**0.000**	4.972	6.852	7.275	4.742	5.151	5.161	5.133	5.014	6.338
Vola	5.191	**2.201**	7.889	6.852	2.796	2.109	3.303	7.038	6.062	5.827
VaR	5.200	6.833	**8.042**	6.102	7.701	8.180	8.141	8.670	7.886	7.240
CVaR	5.743	3.947	6.779	**8.127**	6.935	6.952	7.101	8.746	7.697	5.603
Semidev.	5.098	2.829	6.764	3.896	**3.131**	3.278	4.388	6.734	6.042	6.297
Semivar.	5.362	2.180	7.214	4.453	3.310	**2.112**	2.341	6.418	6.133	5.749
Loss Penalty	5.473	3.318	7.331	5.212	4.422	2.279	**1.918**	6.209	6.156	5.663
Skewness Std.	5.378	7.124	7.567	7.179	6.895	6.331	6.194	**3.866**	7.115	7.715
Skewness Rob.	5.018	6.165	7.343	6.587	6.115	6.201	6.510	6.852	**6.377**	7.349
MDD	5.982	5.540	7.461	5.618	5.919	5.474	5.441	7.104	7.288	**6.332**

- diagonale: perfect foresight versus forecasted returns
- ex-post TE slightly higher than 5%, CVaR and MDD are most different to the BM
- cluster: volatility, semideviation, and semivariance
- TE between the two skewness strategies amounts to 6.85%

Mutual Downside Risk of Strategies

	BM	Vola	VaR	CVaR	Semi- *Dev.*	Semi- *Var.*	Loss Penalty	Skewness *Std.*	Skewness *Rob.*	MDD
Panel C: Forecasted										
Vola	2.85	2.54	2.90	2.67	2.59	**2.52**	2.55	2.85	2.81	2.65
VaR	-0.53	**-0.57**	-0.41	-0.50	-0.43	-0.55	-0.50	-0.52	-0.51	-0.53
CVaR	1.71	1.53	1.81	1.63	1.57	**1.50**	1.51	1.73	1.73	1.60
Semidev.	1.04	0.94	1.08	1.00	0.96	**0.94**	0.95	1.06	1.05	1.00
Semivar.	8.13	6.42	8.38	7.13	6.70	**6.32**	6.51	8.13	7.90	6.99
Loss Penalty	291.1	70.1	689.7	113.6	107.1	60.8	**53.8**	196.5	211.3	74.3
Skewness Std.	0.00	0.11	0.08	0.03	0.01	0.11	0.10	0.01	**-0.06**	0.10
Skewness Rob.	0.07	0.13	**0.02**	0.09	0.03	0.14	0.07	0.07	0.05	0.03
MDD	50.48	50.03	46.99	46.87	49.80	48.08	**45.89**	49.63	48.93	49.17
Mean Ranking	6.89	4.56	7.78	5.89	4.89	3.44	3.67	7.00	6.00	4.89

- Semivariance is successful along 4 risk dimensions: Vola, Semideviation, Semivariance, and MDD
- Loss Penalty ranks second
- given the close relationship to semivariance volatility unsurprisingly ranks third

Conclusion

- Asymmetric risk measures capture downside risks
- Instead of volatility one should therefore stick to asymmetric measures when it comes to empirical tests
- Except for skewness all of the tested measures allow for a substantial reduction of portfolio downside risk given perfect foresight of return
- In absence of perfect foresight VaR and CVaR strategies falter while loss penalty, semivariance and maximum drawdown strategies' performance persists
- Two important prerequisites:
  - the asymmetric measure needs to be predictable
  - asset returns need to exhibit significant asymmetry