

# **Total Volatility and the Cross Section of Expected Returns**

joint work with Lionel Martellini

# Outline of Presentation

- Motivation
- Literature Review
- Three sets of results:
  1. Statistical Results
  2. Asset Pricing Implications
  3. Portfolio Performance

## ***Introduction***

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- The basic question: Does total volatility have explanatory power for the cross section of expected stock returns.
- If so is total volatility risk as opposed to aggregate volatility risk priced in the cross section.
- What are the implications for portfolio performance.

## ***Motivation***

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- There is thus evidence both theoretical and empirical to suggest that in the presence of market imperfections, expected return should be positively related to both systematic and idiosyncratic risk.
- We thus consider total volatility which is the sum of systematic and idiosyncratic volatility and has the advantage of being a model free quantity .
- We first examine the cross sectional evidence and then focus on the asset pricing implications. Finally, we carry out a portfolio performance exercise.

## ***Literature Review***

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- The basic risk return implication is at the aggregate level (Merton (1973)) and says that the total return on the market is positively related to the volatility of the market.
- Several studies starting with Campbell, Lettau, Malkiel and Yu (2001) have examined the risk return relationship at the individual stock level .
- In the absence of of perfect markets and rational behaviour economic arguments both rational and behavioural suggest that stocks with high idiosyncratic volatility should earn higher returns.

## ***Literature Review-Theoretical***

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- Merton (1987) suggests that in an information segmented market firms with higher firm-specific variances require higher average returns to compensate investors for holding imperfectly diversified portfolios.
- Malkiel and Yu (2002) that the inability to hold the market portfolio will force investors to care about total risk in addition to market risk .
- Barberis and Huang (2001) study an equilibrium model of loss-averse investors and find that firms with higher idiosyncratic volatility should earn higher returns

## ***Literature Review-Time Series Empirical Evidence***

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- The ICAPM relationship (Merton (1973)) between aggregate risk and return has tested in a number of studies.
- French, Schwert and Stambaugh (1973) and Campbell and Hentschel (1987) find a positive albeit insignificant relationship between aggregate risk and return.
- Glosten et al (1993) , Harvey (2001) and Turner et al (1989) find a positive or negative relationship depending on the method used.
- Ghysels, Santa Clara and Valkanov (2005) using forecasts of monthly volatility using daily data find a positive and significant relationship.

## ***Literature Review-Cross Sectional Empirical Evidence***

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- Early tests of the CAPM (Douglas (1969) and Lintner (1965)) found that the residuals from the market model were significant in explaining returns.
- Fama and Macbeth (1973) point out statistical pitfalls but Lehmann (1990) confirms the results after careful econometric analysis.
- Tinic and West (1986) find that adding idiosyncratic volatility relative to the CAPM, to the CAPM beta leads to better return prediction.
- Ang et al (2004) find that aggregate volatility risk is a priced factor in the cross section of stock returns, with a negative price of volatility risk. They also find that stocks with high idiosyncratic volatility have lower expected returns.

## ***Preview of Results***

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- We find that stocks with higher total volatility have a significantly higher return than those with low total volatility.
- Total volatility has explanatory power for the cross section of expected returns and that this is not driven by systematic volatility.
- Total volatility risk appears to be priced particularly over the 1990-2004 period with the price of total volatility risk being positive.
- Using total volatility as the estimate of mean return improves the out of sample performance of the tangency portfolio. Also Markowitz strategies can beat the equally weighted portfolio on the deciles sorted by volatility.

## ***Volatility Premiums***

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- We take individual firm data from 1975-2004 from CRSP (around 700 firms discarding non-survivors).
- Sort firms into quintiles every month based on total volatility based on the previous 60 months.
- Calculate the difference in mean return (equally weighted and value weighted) between the top quintile (highest total vol) and the bottom quintile (lowest total vol, omitting non-survivors) and test if this is statistically different from zero. Call this 5-1 premium.

## ***Volatility Premiums***

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- The Equally Weighted 5-1 premium is 0.74% per month with a p-value of 1.37%.

### ***Split Sample-Original Sample***

#### ***Period 1 1975-1990***

- The Equally Weighted 5-1 premium is 0.38% per month with a p-value of 20%.

#### ***Period 2 1990-2004***

- The Equally Weighted 5-1 premium is 1.10% per month with a p-value of 1.24%.

## ***Volatility Premiums-Systematic Fama French Volatility***

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- The Equally Weighted 5-1 premium is 0.68% per month with a p-value of 3.52%.

### ***Split Sample-Original Sample***

#### ***Period 1 1975-1990***

- The Equally Weighted 5-1 premium is 0.20% per month with a p-value of 35%.

#### ***Period 2 1990-2004***

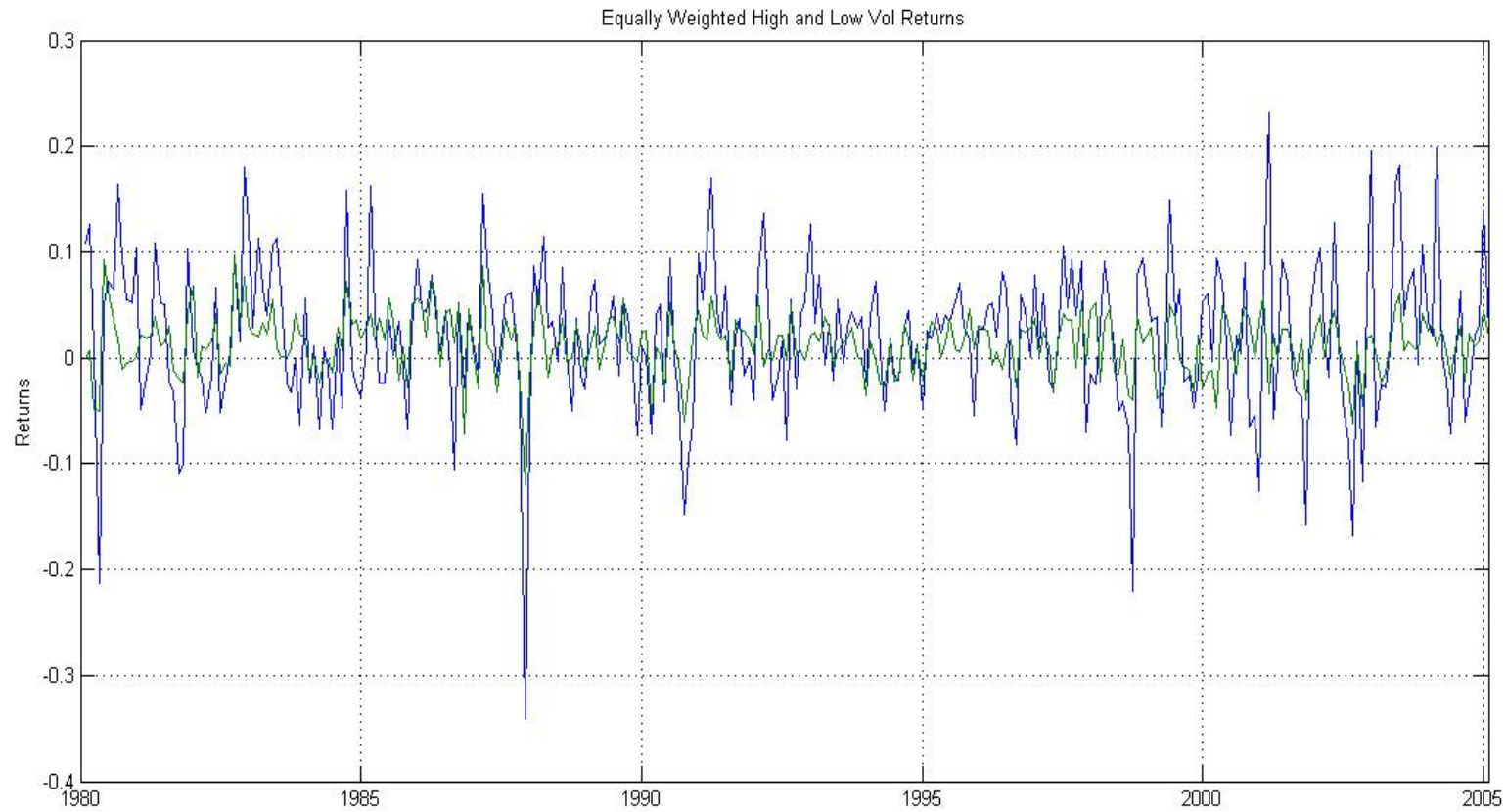
- The Equally Weighted 5-1 premium is 1.16% per month with a p-value of 1.66%.

## ***Volatility Premiums-What drives it?***

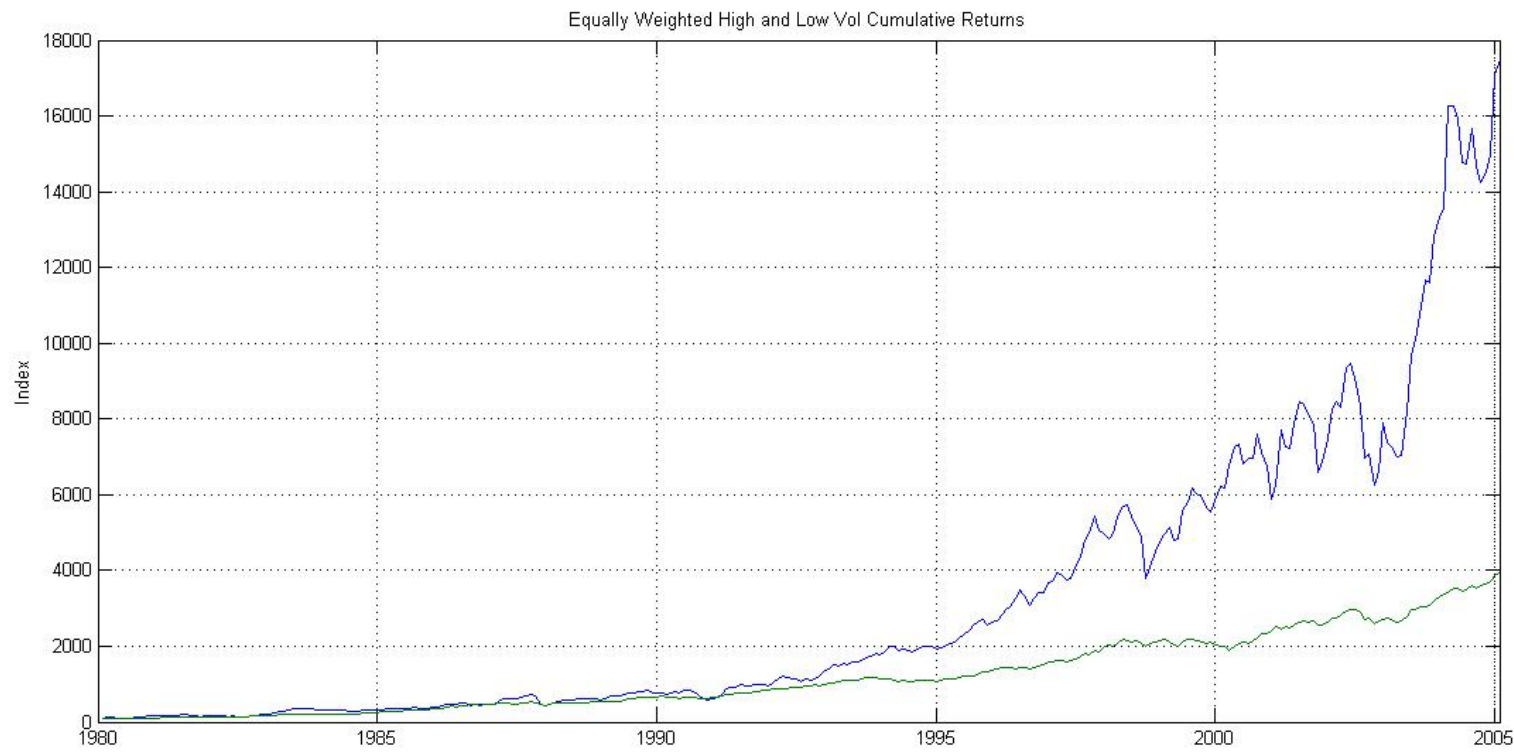
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- If the Fama-French model were a true asset pricing model then the premiums based on total and systematic volatility should be the same.
- Also the difference in average returns on stocks with high systematic volatility and low total volatility and low systematic volatility and high total volatility should be close to the premium with systematic volatility.
- This premium is 0.13% considerably lower than the 0.68% premium for systematic volatility.
- Conversely the premium between stocks with high total volatility and low systematic volatility and low total volatility and high systematic volatility is 0.48% with a p-value of 4.4%

# Equally Weighted Top and Bottom Vol Returns



# Equally Weighted Top and Bottom Vol Cumulative Returns



## ***Attribute Controlled Volatility Premiums***

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- Size Controlled
  - Equally Weighted 1.00% (0.14%)
- Book-to-Market Controlled
  - Equally Weighted 0.79% (1.71%)
- Momentum Controlled
  - Equally Weighted 1.06% (0.01%)

## ***Volatility Premiums Across Attribute Deciles***

	Equally Weighted	
	Premium	p-value
<b>SIZE</b>		
Group 1(Largest)	<b>0.81%</b>	<b>0.17%</b>
Group 2	<b>0.88%</b>	<b>0.03%</b>
Group 3	<b>0.95%</b>	<b>0.01%</b>
Group 4	<b>1.11%</b>	<b>0.02%</b>
Group 5(Smallest)	<b>1.23%</b>	<b>0.01%</b>
<b>BOOK TO MARKET</b>		
Group 1(High BM)	<b>0.02%</b>	<b>80%</b>
Group 1	<b>0.55%</b>	<b>1.15%</b>
Group 3	<b>0.75%</b>	<b>1.87%</b>
Group 4	<b>1.36%</b>	<b>0.00%</b>
Group 5(Low BM)	<b>1.91%</b>	<b>0.00%</b>

## ***Volatility Premiums Across Attribute Deciles***

Equally Weighted

Premium

p-value

### **Momentum**

Group 1(Winners)	<b>2.61%</b>	<b>0</b>
Group 2	<b>0.63%</b>	<b>0.23%</b>
Group 3	<b>0.81%</b>	<b>0.28%</b>
Group 4	<b>0.92%</b>	<b>0.38%</b>
Group 5(Losers)	<b>0.34%</b>	<b>27.07%</b>

## ***Cross Sectional Explanatory Power***

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- We now run cross sectional regressions and study the significance of the slope coefficients following Fama-Macbeth and using a moving window of 60 months.
- We first use total volatility as an explanatory factor and then add the Fama-French systematic volatility as an additional explanatory factor
- The slope coefficient using total volatility alone is 0.102 with a p-value of .1%
- Adding the Fama-French systematic volatility actually increases both the slope coefficient to 0.124 with a p-value of .02% with the Fama-French systematic volatility having a small and insignificant negative slope coefficient (-0.041).

## ***Cross Sectional Explanatory Power***

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- This indicates that the cross sectional explanatory power of total volatility is not driven entirely by Fama-French systematic volatility.
- The cross sectional explanatory power is stronger over the 1990-2004 period with the slope coefficient being 0.136 as compared to .052 (not significant) over the 1980-1990 period.

## ***Asset Pricing Implications***

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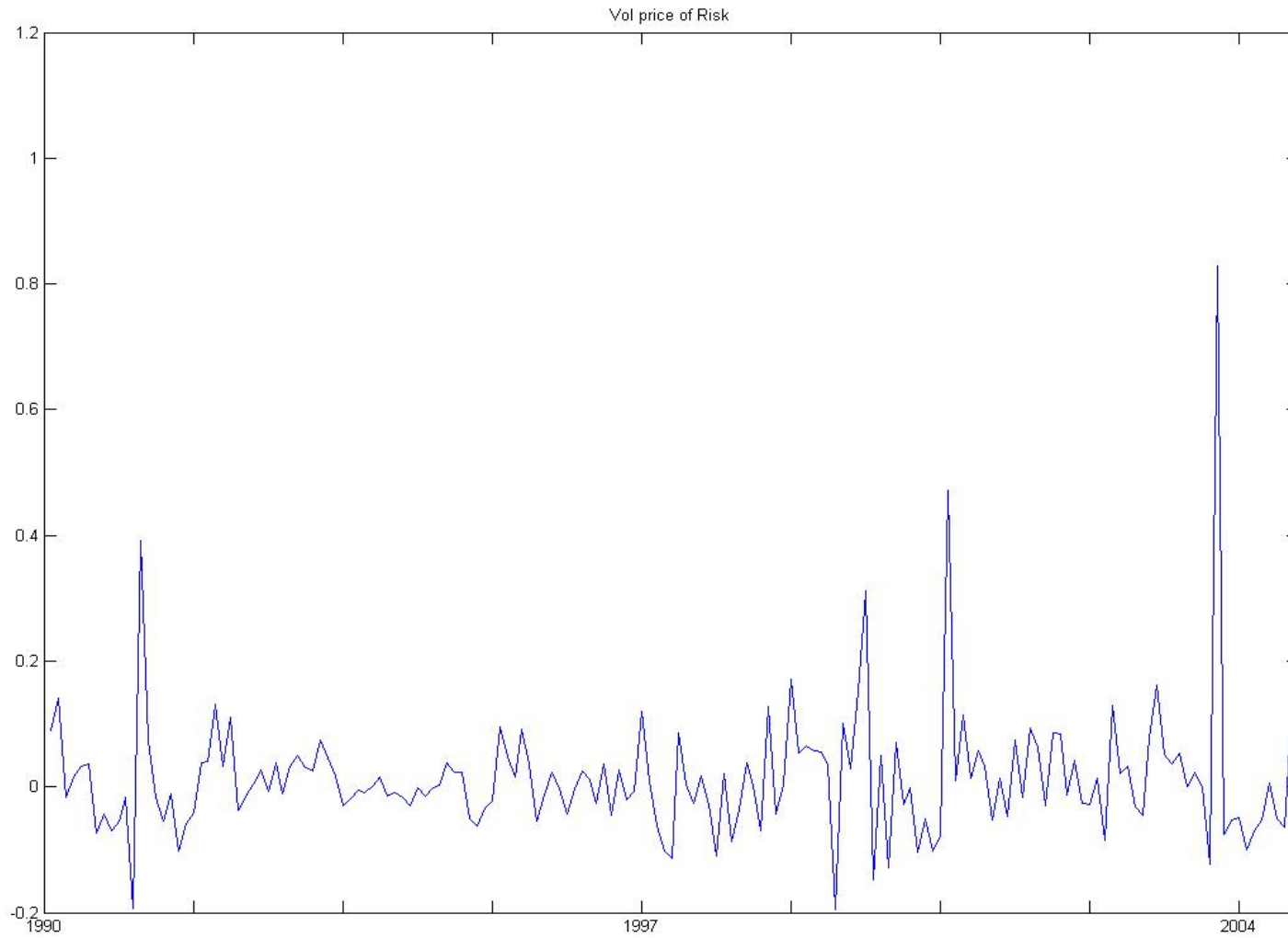
- Following the Fama-French procedure we consider the factor mimicking portfolio for volatility as the difference in return between the high volatility and the low volatility portfolio
- We first analyze whether it has any additional explanatory power over the standard CAPM for the size, value and momentum effects.
- For the smallest size decile and winner decile the CAPM alone explains 53% and 60% of the variation while adding the volatility factor increases the R square to 73% and 67% respectively. The beta for the volatility factor is significant for the size portfolios.

## ***Asset Pricing Implications***

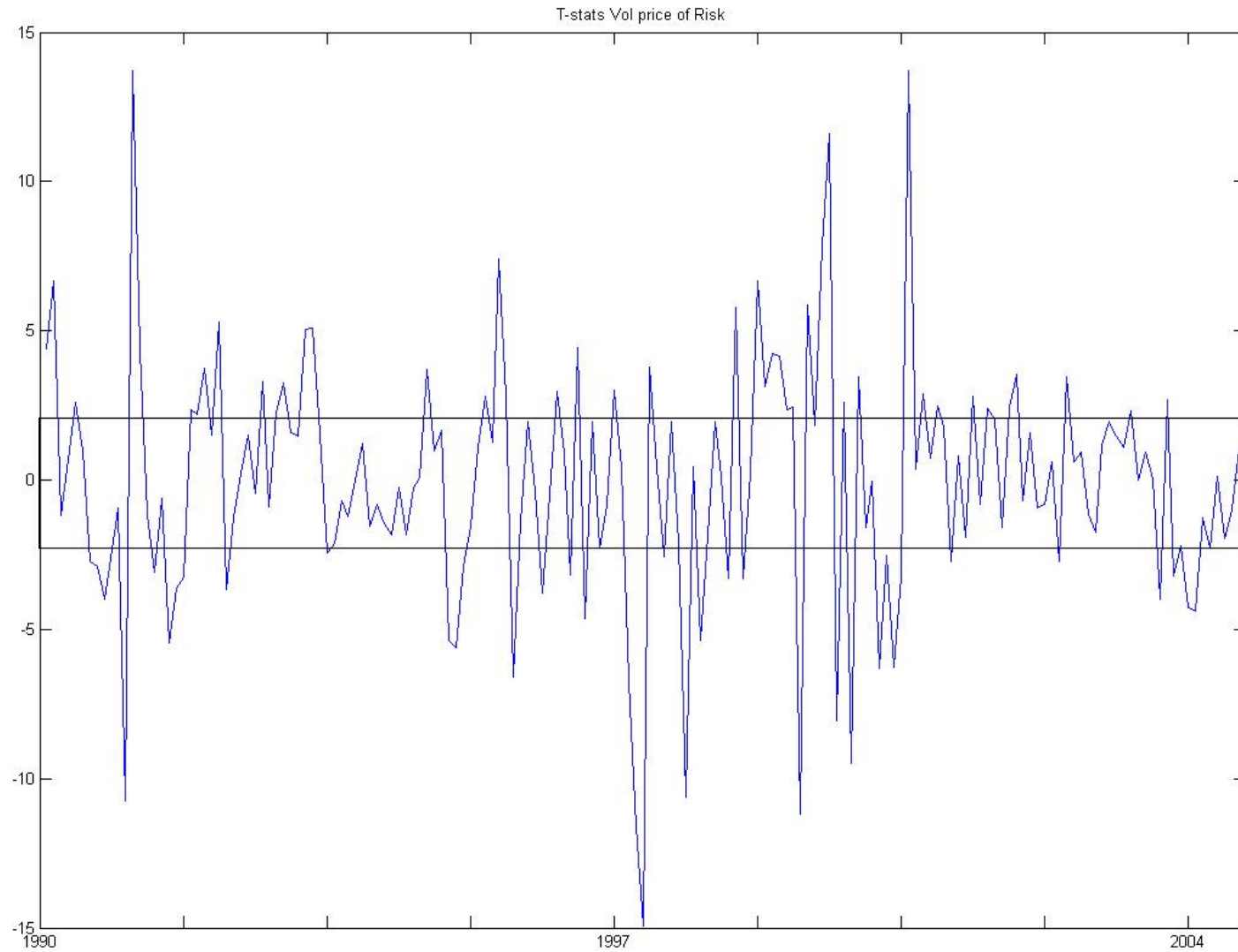
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- We estimate the factor price of risk for the volatility factor jointly with the market factor via a Fama-Macbeth regression on individual stocks again using a rolling window of 60 months to estimate betas.
- Over the entire sample period the volatility price of risk is .68% per month with a t-stat of 1.58 (p-value 5.8%).
- Over the 1990-2004 period the volatility price of risk is 1.42% per month with a t-stat of 1.75 (p-value 4%).
- There is considerable time variation in the factor price of risk and its t-stat.

# Vol Factor Price of Risk 90-04



# T-stats Vol Factor Price of Risk 90-04



## ***Portfolio Performance***

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- We now examine the implications of our cross sectional findings for portfolio performance
- Two new ideas-First using volatility as a proxy for mean return in Markowitz optimized portfolios-see if this improves the out of sample performance of the tangency portfolio (the Vol portfolio)
- Second consider the decile portfolios sorted on total volatility as a new set of base assets and analyze the performance of Markowitz portfolios on these assets.
- We consider out of sample portfolio strategies that are held for one month as in DeMiguel et al (2006).

# Portfolios

- Tangency portfolio (MSR)

$$\frac{\Sigma^{-1}\mu}{1'\Sigma^{-1}\mu}$$

Global Minimum Variance  
(GMV)

$$\frac{\Sigma^{-1}1}{1'\Sigma^{-1}1}$$

- Vol Portfolio

$$\frac{\Sigma^{-1}\sigma}{1'\Sigma^{-1}\sigma}$$

- Fama-French mean portfolio  
(MSRFF)

$$\frac{\Sigma^{-1}\mu_{FF}}{1'\Sigma^{-1}\mu_{FF}}$$

- 1/N

## ***Portfolio Performance***

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	Monthly		Monthly
	Volatility	Mean	Sharpe Ratio
<hr/>			
<b>Volatility Deciles</b>			
Vol	<b>3.86%</b>	<b>1.35%</b>	<b>0.2493</b>
GMV	<b>3.31%</b>	<b>1.21%</b>	<b>0.2493</b>
MSRS	<b>4.86%</b>	<b>1.34%</b>	<b>0.1954</b>
MSRFF	<b>3.36%</b>	<b>1.07%</b>	<b>0.2028</b>
EW	<b>4.29%</b>	<b>1.35%</b>	<b>0.2247</b>

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## ***Portfolio Performance***

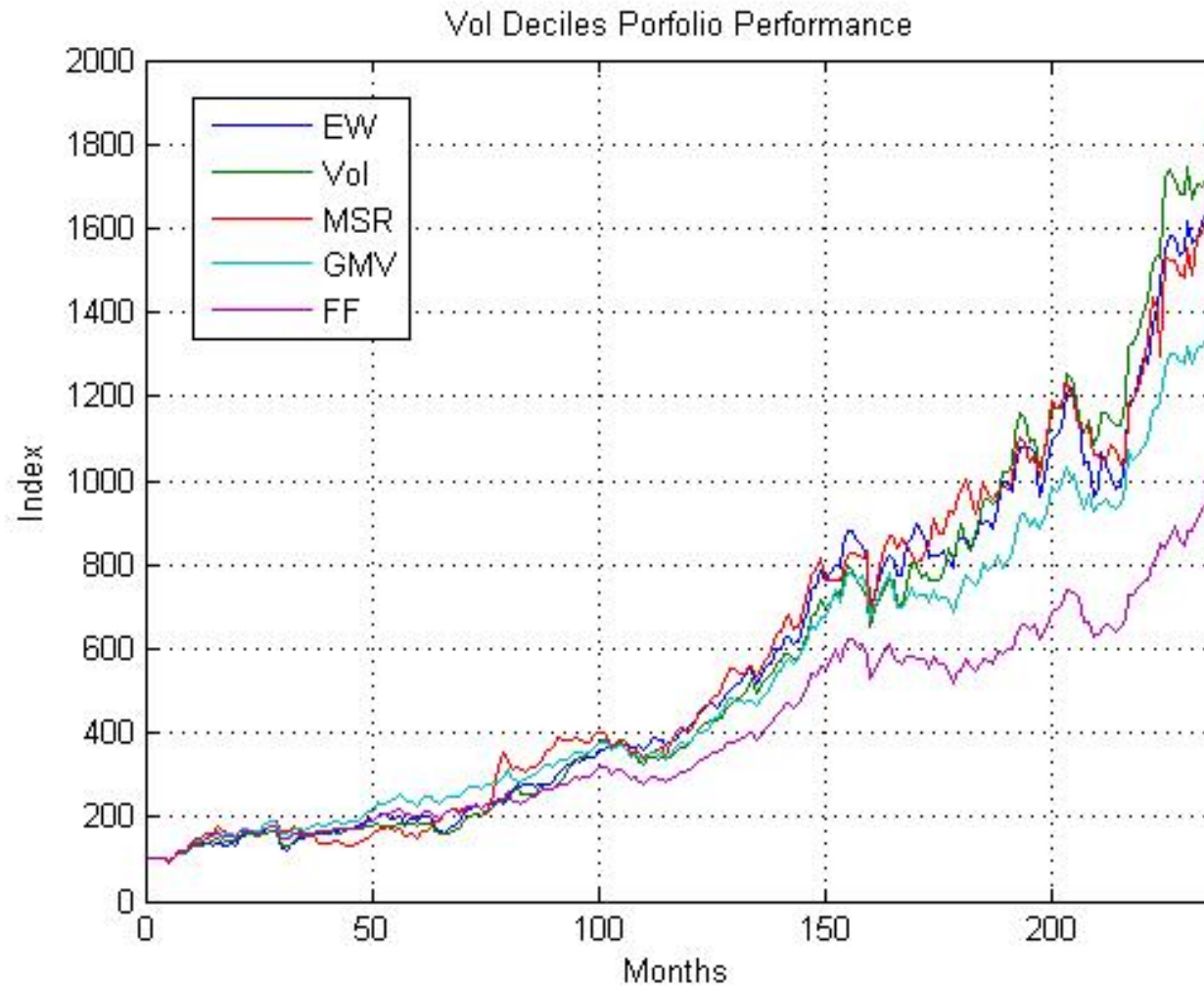
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	Monthly		Monthly
	Volatility	Mean	Sharpe Ratio
<hr/>			
<b>Industry Deciles</b>			
Vol	<b>1.00%</b>	<b>4.03%</b>	<b>0.1762</b>
GMV	<b>1.05%</b>	<b>3.77%</b>	<b>0.2002</b>
MSRS	<b>0.75%</b>	<b>7.05%</b>	<b>0.0848</b>
MSRFF	<b>0.96%</b>	<b>4.15%</b>	<b>0.1909</b>
EW	<b>1.47%</b>	<b>4.05%</b>	<b>0.1791</b>

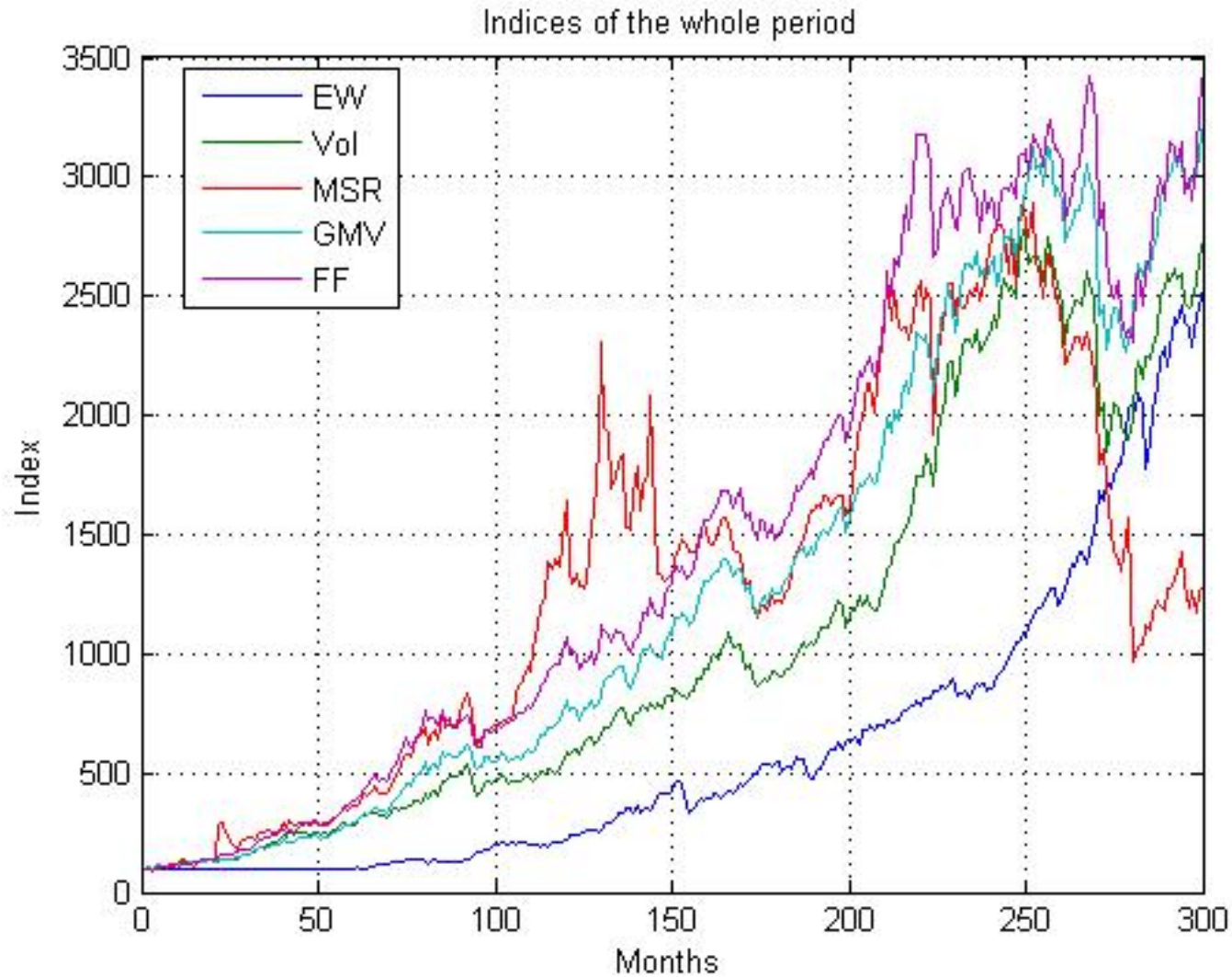
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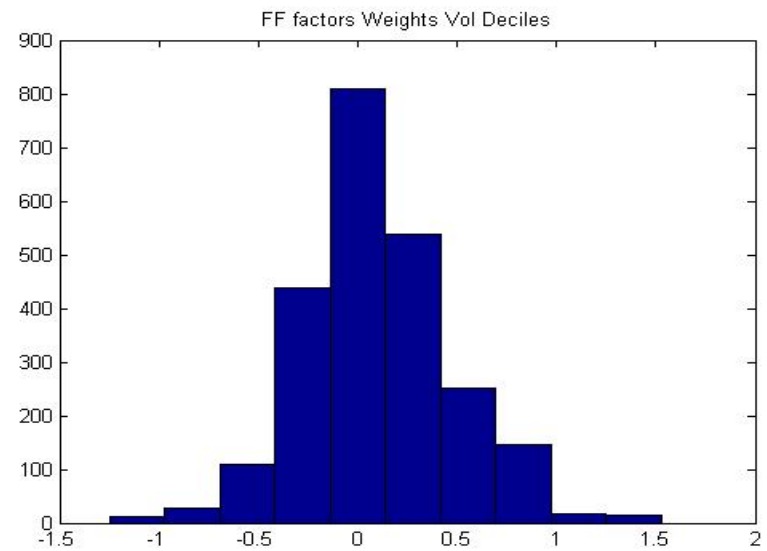
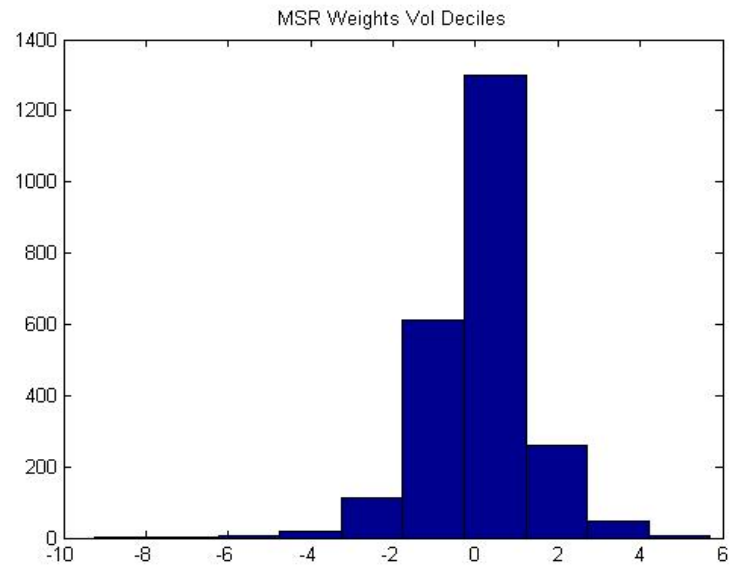
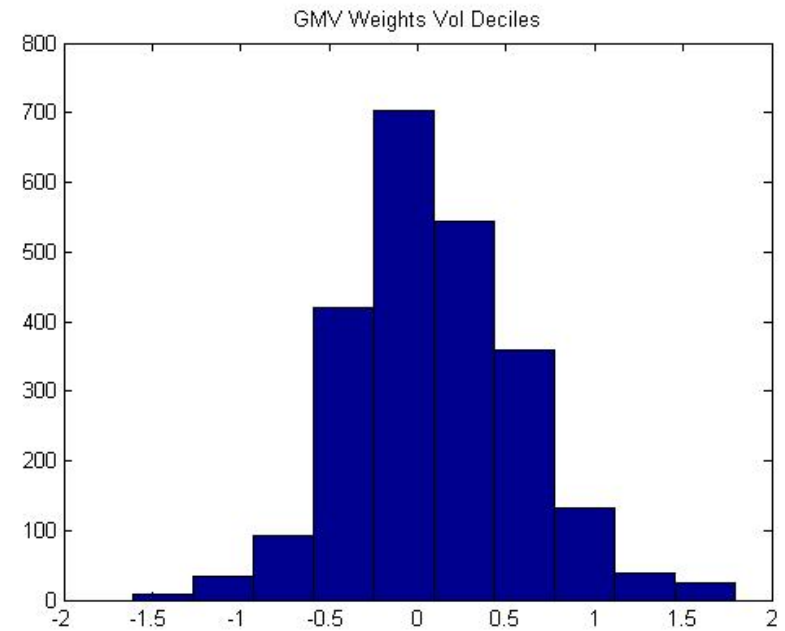
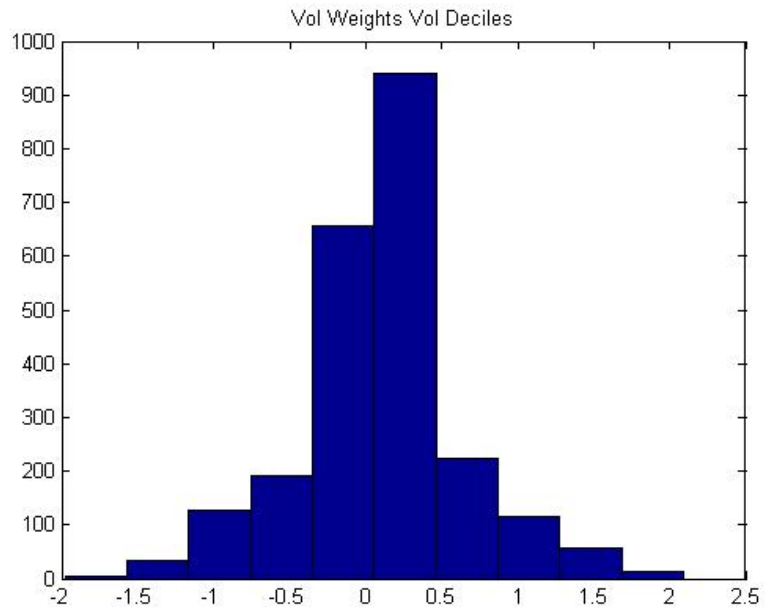
# Portfolios Performance on Vol Deciles (out-of-sample)



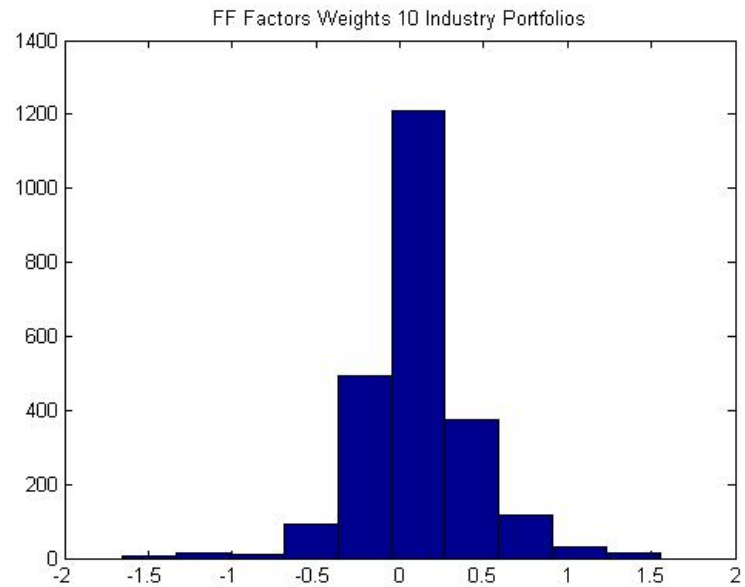
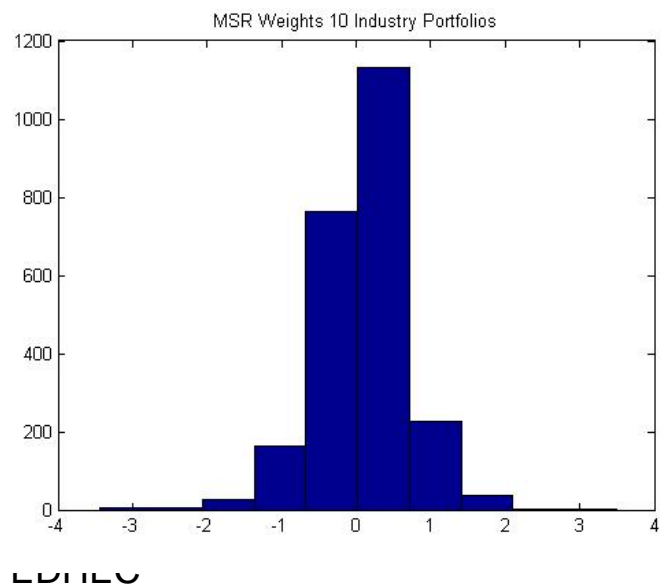
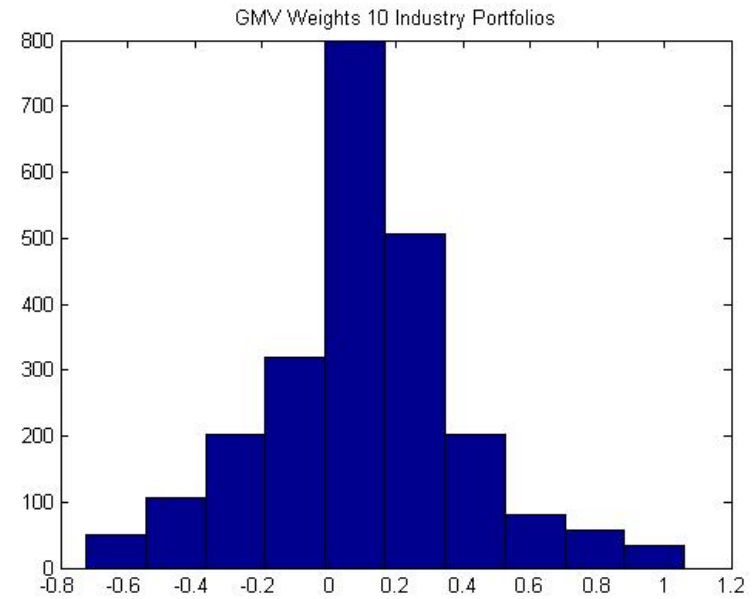
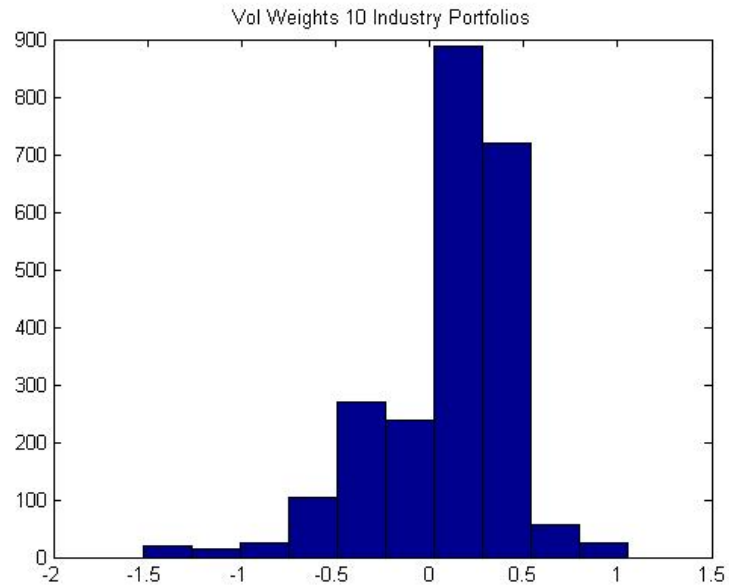
# Portfolios Performance on 10 Industry portfolios (out-of-sample)



# Portfolio Weights Vol Deciles



# Portfolio Weights on 10 Industry Portfolios



## ***Conclusion***

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- We find that total volatility, a model free quantity, is a good proxy for total return. Stocks with higher total volatility earn significantly higher return than those with low total volatility.
- A cross sectional regression shows that total volatility has explanatory power for the cross section of expected returns and that this is not driven by systematic volatility.
- A factor mimicking portfolio for total volatility appears to be priced particularly over the 1990-2004 period with the factor price of total volatility risk being positive.
- Using total volatility as the estimate of mean return improves the out of sample performance of the tangency portfolio.

## ***Portfolio Performance***

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	Monthly		Monthly
	Volatility	Mean	Sharpe Ratio
<hr/>			
<b>100 Stocks GMV</b>			
Const Corr	<b>4.14%</b>	<b>1.11%</b>	<b>0.1727</b>
Quintiles	<b>3.33%</b>	<b>0.62%</b>	<b>0.1875</b>
EW	<b>3.65%</b>	<b>0.68%</b>	<b>0.1869</b>

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