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This chair, led by Lionel Martellini, scientific director of EDHEC-Risk Institute, is examining dynamic allocation strategies in asset liability management in order to formulate an integrated ALM model.

This study paper is an attempt to analyse the valuation of pension liabilities, regarded as defaultable claims issued by the sponsor company to workers and pensioners, in the context of a structural intertemporal capital structure model with contingent contributions. Our model has important policy implications in that it provides a first step towards a much-needed methodological framework for the design of firm-specific regulatory constraints and accounting valuation principles. It also has a number of implications in terms of investment decisions at the pension fund level, and funding decisions at the sponsor company level.

I would like to thank the co-authors, Lionel Martellini and Vincent Milhau, for the quality of their analysis and research. We hope that you will find the paper valuable and will continue to monitor and contribute to our research in this area.

We would also like to extend our warm thanks to our partners at BNP Paribas Investment Partners for their collaboration on the project and their commitment to this research chair.

Wishing you a pleasant and informative read,

Noël Amenc
Professor of Finance
Director of EDHEC-Risk Institute
About the Authors

**Lionel Martellini** is professor of finance at EDHEC Business School and scientific director of EDHEC-Risk Institute. He has graduate degrees in economics, statistics, and mathematics, as well as a PhD in finance from the University of California at Berkeley. Lionel is a member of the editorial board of the *Journal of Portfolio Management* and the *Journal of Alternative Investments*. An expert in quantitative asset management and derivatives valuation, Lionel has published widely in academic and practitioner journals and has co-authored textbooks on alternative investment strategies and fixed-income securities.

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Abstract
Abstract

Correctly assessing the value of an undefunded pension plan with a weak sponsor company is a real challenge given that no comprehensive model is currently available for the joint quantitative analysis of capital structure choices, pension fund allocation decisions and their impact on rational pricing of liability streams. This paper is an attempt to fill this gap by analysing the valuation of pension liabilities regarded as defaultable claims issued by the sponsor company to workers and pensioners in the context of an integrated model of capital structure. Our results show that leverage decisions have a strong impact on the fair value of pension liabilities, and conversely that the presence of a pension plan decreases the optimal leverage ratio. We also find that interior optimal values may exist for allocation decisions. In an extension to a dynamic setting, we find that risk-controlled strategies allow the pension fund to take more risks, which has a positive effect on equity value, while protecting pensioners. Our model has important policy implications in that it provides a first step towards a much-needed methodological framework for the design of firm-specific regulatory constraints and accounting valuation principles.
Executive Summary
One of the main risks for pension plan participants, indeed the only source of risk in a defined-benefit plan with unconditional liability payments, is that of sponsor bankruptcy when the pension plan is underfunded. In an attempt to address this risk, the legal, accounting, and fiscal environments of corporate pension funds have undergone great changes; these changes have led to closer scrutiny of the valuation of pension liabilities and of the impact of both the market and credit risk components on the value of pension obligations. Correctly assessing the value of an underfunded pension plan with a weak sponsor is nonetheless a great challenge, as there is no comprehensive model for the joint quantitative analysis of capital structure choices, pension fund allocation decisions, and their impact on rational pricing of liability streams. In fact, international accounting standards SFAS 87.44 and IAS19.78 recommend that pension obligations be valued at a discount rate equal to the market yield on AA corporate bonds, the same rate for all firms. Although using a market rate is arguably an improvement on using a constant rate (including a credit spread component or not) independently of market conditions, using the same market rate to discount all pension liabilities regardless of the sponsor’s credit rating, pension funding situation, and asset allocation policy is unlikely to lead to an accurate assessment by stakeholders of the impact of specific default risk on the value of pension obligations. Our paper broadens this approach by analysing the valuation of pension liabilities regarded as defaultable claims issued by the sponsor to workers and pensioners in the context of an integrated model of capital structure.

It focuses on the interaction of allocation decisions made by the pension fund and the valuation of these liabilities, thereby extending the literature on capital structure and that on defaultable bond pricing to account for the presence of a pension plan.

Our base-case model is a stylised representation of the relationships between the stakeholders of a company with a pension plan, including shareholders of the sponsor company, bondholders, and beneficiaries of the pension fund (workers and pensioners). This model assumes separate balance sheets for the sponsor and the pension plan, and it can be summarised as follows. The sponsor issues a debt with face value $D$ and also issues pension claims, perceived as a collateralised form of debt held by workers and pensioners, with face value $L$. The initial capital of the firm is allocated to funding investment projects (company asset value denoted by $V$) and to funding the pension plan (pension asset value denoted by $A$). The pension fund allocates a fraction $\omega$ of the initial endowment to some performance-seeking portfolio (PSP) and a fraction $1-\omega$ to some liability-hedging portfolio (LHP). If the assets of the pension fund $A$ are insufficient to deliver the promised pension payment $L$, the sponsor makes a contribution equal to the deficit $L - A$. If the sponsor is unable to make this contribution, default is triggered. If the pension fund enjoys a surplus, equity-holders receive a fraction $\gamma$ of this surplus, which can be used to pay back bondholders. If debt cannot be fully repaid, bankruptcy is also triggered. When default has not been triggered, equity-holders are left with the remaining assets of the pension fund and the sponsor, plus their access to surpluses. Otherwise, they
receive nothing. Also incorporated are tax
effects, bankruptcy costs, and contributions
triggered by the presence of regulatory
funding ratio constraints (see exhibit 1). As
a variant of the base-case model, we study
a regime of internal funding, in which the
assets and the liabilities of the pension fund
are held on the sponsor’s balance sheet.
Internal funding results in a decrease in the
seniority of pensioners and a simultaneous
increase in the seniority of bondholders,
so it is always preferred by bondholders.
Whether it is preferred or not by equity-
holders to external funding depends on
the surplus-sharing rule enforced under
external funding: equity-holders will prefer
external funding only if they are given
access to surpluses.
Under standard assumptions regarding
the dynamics followed by all variables
of interest, including the return on the
performance-seeking portfolio and the
return on the real assets held by the firm,
one can use option-pricing theory to find
the rational value of the claims held by all
stakeholders and analyse the impact on
the value of these claims of funding and
leverage decisions made by the sponsor,
as well as asset allocation decisions made
by the pension fund. The main ingredients
of the model are the size of the pension
fund relative to the assets of the sponsor
(L/V), the size of the pension assets relative
to the pension liabilities (the funding ratio
A/L), and the size of the outstanding
debt relative to the assets of the sponsor

Executive Summary

company \((D/V)\). Other important parameters are those defining the allocation strategy of the pension fund, as well as the correlation between the return on pension assets and the return on sponsor assets (see exhibit 2).¹

Our findings have two main implications: macro (with a number of possible policy recommendations for pension fund regulators), and micro (with a number of strategy recommendations for pension fund managers).

1. Macro/Policy Implications
Our results have important potential policy implications. In particular, they raise a series of questions related to current practices in terms of liability cash-flow valuation, in which accounting regulations recommend an arbitrary market rate identical for all firms. They also raise questions about prudential regulation, as they provide new formal insight into the impact of the presence of short-term funding ratio constraints or pension insurance.

(a) Accounting regulations
Our model suggests that valuation principles for liability streams should account for differences in financial health and capital structure decisions made by the sponsor, as well as differences in asset allocation policy made by the pension fund. Our analysis also suggests that any minimum funding requirements should be a function of these items as well because they are based on estimates of fair pension liability value. We first analyse the structural relationship between the value of the assets of the firm and the value of the claims issued by the sponsor to bondholders and to liability-holders, in the context of a formal capital structure model. Explicitly recognising that bondholders and liability-holders hold (relatively complex) payoffs that are functions of both pension fund and firm assets allows us to obtain a wealth of new insight into the interaction of capital structure decisions and the valuation of pension claims, as well as into the interaction of asset allocation decisions and the sponsor’s credit rating. We show that the existence of a pension plan has a strong impact on capital structure decisions, with the optimal leverage ratio a decreasing function of promised pension payments. This finding is consistent with the intuition that suggests that a firm without a pension fund would optimally take on more debt than an otherwise identical firm sponsoring a pension plan, since the latter firm has already issued a form of debt by committing to a payment to retired employees. We also find that the presence of a pension fund has a substantial impact on debt value and credit ratings (see exhibit 3).

These results suggest that a fair assessment of a firm’s creditworthiness can be done only when analysts and rating agencies have an integrated view of the firm’s financial situation, a view that also takes into account the pension fund’s situation and its funding status as well as its allocation strategy. Conversely, capital structure decisions have a substantial impact on the fair value of pension claims; the creditworthiness of liabilities is a decreasing function of the leverage ratio (see exhibit 4).

One important corollary is that the regulatory valuation leads to overestimates of the fair value of liabilities for highly leveraged firms, whereas it leads to underestimates for firms with little debt.

¹ Institutional elements such as those governing the surplus-sharing rule, the tax rate, and bankruptcy costs will also have an impact on the numerical results.
Executive Summary

outstanding, and one of the contributions of the model is to enable a quantitative estimate of the magnitude of the over- or underestimate as a function of parameter values. On the whole, our research has important policy implications in that it takes a first step towards a much-needed methodological framework for the design of firm-specific regulatory constraints and liability valuation principles; it also calls for the emergence of a scheme-specific rule for pricing pension insurance.
Executive Summary

Although providing firm-specific regulatory constraints and accounting valuation principles might seem a formidable challenge, our paper presents an analytical model that could be used to achieve such objectives. It should be emphasised that a move towards an endogenous discount rate rationally taking into account credit risk in liability streams would lead to counter-cyclical regulation, in the sense that sponsors in bad shape will face a loosening of the funding ratio constraints since a pension fund will report an improvement in the surplus or funding ratio in the event of a decline in the credit quality of pension liabilities. It can be argued that reporting a gain from a decline in credit quality is potentially misleading and can mask a deteriorating situation. On the other hand, it can also be argued that a loosening of the funding ratio constraints in difficult situations might help maintain the sustainability of the defined-benefit pension system. Of course, in a rational expectation model, employees of the sponsor would observe the deterioration in the present value of their pension benefits and bargain for increases in wages to compensate for the loss, an option that would not be available to pensioners.\(^2\)

(b) Prudential regulations

In this paper, we extend the analysis of funding and investment decisions made by the pension fund, and their interaction with leverage and contribution decisions made by the sponsor, to an intertemporal setting; previous literature has, for the most part, considered simple static models. Extending the analysis to the dynamic setting is of critical relevance because it allows us to examine a number of such empirically relevant features as the short-term minimum funding ratio constraints imposed by regulators in most developed countries (with the notable exception of the United States and the United Kingdom). Such short-term funding constraints, whose presence cannot be accounted for in a static setting, have a great impact on the valuation of pension liabilities, and on shareholder and total firm value, by forcing the sponsor to make early additional contributions when the poor performance of the pension

\(^2\) One possible way to mitigate this problem would be to introduce minimum regulatory levels for the fair liability value, in addition to minimum levels for the fair funding ratio.

Exhibit 5: Impact of short-term constraints on stakeholders’ welfare

These figures perform comparative static analysis of the fraction of surpluses attributed to equity-holders when the pension plan is terminated. The firm is assumed to be positively correlated with the market, and three values of the initial regulatory funding ratio are considered (70%, 100%, and 130%). Parameters are fixed at their base-case values (see table 1 page 107).
fund portfolio makes such additional contributions necessary (see exhibit 5). As expected, we find that pensioners benefit unambiguously from the enforcement of minimum funding requirements, with the greatest benefits to be expected when the initial funding ratio is low, which is when pension protection is most critically needed. The situation is more ambiguous for equity-holders. On the one hand, it can be argued that early contributions make insolvency of the pension fund less likely. On the other hand, they may also prove ex post to have been unnecessary if strong stock market performance can enable the fund to recover without calling for exceptional contributions. On the whole, we find that equity-holders will benefit from the introduction of short-term constraints only if their access to the pension fund surpluses is sufficiently high, in which case they can expect unnecessary contributions to be returned to them at the terminal date. In any case, the marginal cost of introducing short-term constraints is relatively modest for equity-holders, which suggests that such prudential regulations could be a welcome ingredient in pension fund management. 3

2. Micro/Strategy Implications
We also obtain novel results regarding optimal funding and investment decisions.

(a) Investment decisions
When the correlation of the firm value process and the stock index return process is positive, we find that the fair value of promised payments to bondholders and pensioners is a decreasing function of the allocation to risky assets by the pension fund. This is a clear case of asset substitution, since a larger allocation to risky assets leads to an increase in the total riskiness of the total assets held by the firm (financial assets held off the balance sheet through the pension funds and real assets held directly on the balance sheet), which is the underlying state variable the value of such claims is based on. When the correlation is negative, however, greater allocation to risky assets may result in diversification benefits. This competition between the asset-substitution effect and the diversification effect, which has never been analysed in the related literature, leads to an interior optimal solution to the problem of maximising total firm value (and to that of maximising pensioner value), at least for reasonably low funding ratios. On the whole, there is clear evidence of conflicts of interest between stakeholders, between shareholders and pensioners, in particular. If it is assumed that they do not have access to any pension fund surplus, risk taking is detrimental from the pensioners’ perspective, because it involves increasing the likelihood of partial recovery of pension claims, whereas the upside potential of the performance-seeking assets allows shareholders to reduce the burden of contributions needed to meet expected pension payments (see exhibit 6).

These conflicts of interest could be mitigated by granting pensioners some access to the surplus, thereby allowing plan beneficiaries to benefit from the increase in expected performance implied by more aggressive investment strategies. Practically, a form of surplus sharing can be introduced by indexing pension benefits to inflation when the funding status of the pension plan permits it. Such conditional indexation rules exist in the Netherlands. More generally, our results have implications for the optimal

3 - The presence of a time-varying mean-reverting equity risk premium would probably lead to an increase in the cost of short-term constraints for equity-holders.
design of pension plans, since they provide evidence in favour of more subtle surplus-sharing rules, rules that could include, say, hybrid retirement plans or contribution holidays for defined-benefit plans, or both, and would allow equity-holders to reduce the contribution burden, all while protecting the interests of pensioners. We also find that an effective way to align the interests of shareholders and pensioners without any complex adjustment to the pension plan structure is to enlarge the set of admissible investment strategies so as to include such dynamic risk-controlled strategies as constant proportion portfolio insurance (CPPI), or to extend them to a pension management context sometimes referred to as contingent immunisation or dynamic liability-driven investment (LDI). In fact, implementing risk-controlled strategies that attempt to ensure a funding ratio above 100% allows shareholders (limited) access to the expected upside of risky assets, while ensuring that pensioners will not be hurt by the increase in risk (see exhibit 7).

Exhibit 6: Impact of allocation decisions on stakeholder welfare
These figures perform comparative static analysis of the allocation to the risky asset, when the firm is positively correlated with the market. The pension fund is fully funded in the regulatory sense at the initial date. Parameters are fixed at their base-case values (see table 1, page 107).

Exhibit 7: Impact of allocation decisions with risk-controlled strategies
These figures perform comparative static analysis of the multiplier of the constant-proportion portfolio insurance (CPPI) strategy when the firm is positively correlated with the market. The initial funding ratio is 130%, so as to create a positive risk budget. Parameters are fixed at their base-case values (see table 1, page 107). The vertical line identifies the base case, where the initial weight allocated to the stock is 50%.
(b) Funding decisions
We first find that pensioners always benefit from increases in funding to the pension plan because the plan assets are used as collateral for the pension claims. This increase in the value of liabilities takes place at a decreasing rate: when the pension plan is already very well funded, marginal increases in asset value generate only marginal improvements in pensioner welfare. The impact on equity-holders is, in principle, positive as well, at least when shareholders have full access to the pension fund surpluses and when the initial funding ratio is sufficiently high: increases in funding lead to increases in tax benefits, and therefore to higher shareholder value; when shareholders have full access to pension surpluses, the tax benefits come with no opportunity costs. For large pension claims, an increase in funding starting from low funding ratios can, on the other hand, be detrimental to shareholders: when it is highly unlikely that pension assets will ever be in excess of pension liabilities, one additional dollar invested in the pension fund is certain to be lost for shareholders, who would sooner have it invested in the firm. We also analyse the value of the bonds as a function of the allocation to the pension fund. As expected, a firm with a pension fund in surplus has a better rating than a firm with a pension fund in deficit, all else equal, and the impact of pension funding decisions is substantial. These results are consistent with empirical findings that have documented the positive influence of pension funding on credit ratings. Moreover, we find that the introduction of a pension benefit guarantee corporation would have a great impact on optimal funding decisions. We likewise analyse the determinants of the difference between the fair and regulatory values of pension insurance, which, for reasonable parameter values, can be substantial.

Although the base-case model assumes that promised pension benefits are fixed in nominal terms, it can be extended to account for indexation to inflation. The impact of allocation and funding decisions is found to be the same as in the case of nominal promised payments, but, unsurprisingly, pensioners benefit from indexation, even if it is only conditional. Moreover, as mentioned above, a conditional indexation rule has the advantage over unconditional indexation that it helps to align the interests of pensioners and shareholders regarding the riskiness of the strategy used by the pension fund.

Our work can be extended in a number of other directions. First, the model could be extended to account for the indexation, in many countries, of promised pension payments to wage rather than to price inflation. In terms of allocation strategies, we have tested fixed-mix strategies as well as basic risk-controlled strategies and found that the benefits of moving away from static allocation strategies to even the simplest form of dynamic risk-controlled strategies were substantial for both shareholders and pensioners. It would be useful to try to test more sophisticated forms of welfare-improving strategies in a more general dynamic context, including strategies with a floor given as a function of the (regulatory and/or fair) value of the liability portfolio, strategies with a performance cap in addition to floors, which can allow for a decrease in the cost of downside risk protection, as well as strategies involving corporate bonds in
the liability-hedging portfolio. Moreover, one would also like to test risk-controlled strategies that encompass state variables related to the sponsor. The intuition is that one could further decrease the cost of downside protection by relaxing risk constraints when the sponsor is financially healthy and thus make up for eventual deficits through increased contributions, while focusing on hedging away the states of the world characterised by a joint occurrence of poorly performing pension assets and poor financial health on the sponsor’s part.
1. Introduction
1. Introduction

In recent decades, the legal, regulatory, accounting, and fiscal environments of corporate pension funds have undergone great changes; put together, these changes have led to significantly heightened scrutiny of the valuation of pension liabilities, with a focus on greater transparency of the impact of both market and credit risk components on pension commitments. In the United States, the greatest change in the pension environment was the passage of the Employee Retirement Income Security Act (ERISA) in 1974, which has led, among other things, to corporate pension liabilities becoming parts of corporate liabilities. Although a significant change, implying that beneficiaries could use firm assets to cover any deficit with respect to accrued liabilities in the event of termination of a pension plan, the impact of including pension liabilities in sponsor companies’ balance sheets and income statements was somewhat softened by having pension liabilities reported at their historical value. As a result, the impact of changes in market and credit risk factors on the value of the pension obligation could not be accurately assessed by beneficiaries or by shareholders of the sponsor. More recently, the move towards fair valuation in global accounting standards has generated renewed interest in a more dynamic measurement of pension liabilities, with a focus on the impact not only of market risk (interest rate and inflation risks) but also of credit risk. In 2006, the FASB issued SFAS 157, which states (in paragraph 15): “The reporting entity shall consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value”. Obviously, this argument makes compelling sense from a financial economics standpoint. Its implications in the specific context of pension liability valuation, however, are yet to be fully recognised. In fact, international accounting standards SFAS 87.44 and IAS19.78 recommend that pension obligations be valued on the basis of a discount rate equal to the market yield on AA corporate bonds, the same rate for all firms. Although using a market rate is arguably an improvement on using a constant rate (including a credit spread component or not) independently of market conditions, using the same market rate to discount all pension liabilities, regardless of the sponsor’s credit rating, pension funding situations, and asset allocation policy is unlikely to lead to accurate stakeholder assessments of the impact of specific default risk on the value of pension obligations.

This is a severe problem, as can perhaps best be evidenced by the failure of pension fund regulations (both accounting and prudential) in developed countries to protect beneficiaries from events combining pension fund underfunding and sponsor default. One of the main risks for plan participants, actually the only source of uncertainty for a defined-benefit plan with unconditional liability payments, is precisely that of sponsor bankruptcy when the pension plan is underfunded. Although this risk is somewhat mitigated by the presence of a nation-wide pension guarantee fund such as the Pension Benefit Guaranty Corporation in the US or the Pension Protection Fund in the UK, benefits usually fall sharply when transferred to such a guarantee fund. For example, when United Airlines filed for bankruptcy (chapter 11), a controversial filing that allowed it to cancel its pension obligations and transfer these...
obligations to the Pension Benefit Guaranty Corporation (PBGC), employees lost $3.2 billion because pension benefits are not fully insured by the PBGC. This loss meant anywhere from 20% to more than 50% of pension rights (Amenc, Martellini, and Sender 2009). One additional concern is the presence of systemic risk for the pension fund industry if many plan sponsors simultaneously default on their pension obligations. Regulatory bodies in charge of pension fund supervision, as well as pension fund beneficiaries and pension fund managers, thus have great incentives to monitor changes in the joint probability distribution of default risk from the sponsor and underfunding of the pension fund, as well as to assess the implications in terms of changes in the fair value of liability obligations.

Correctly assessing the value of an underfunded pension plan with a weak sponsor is, however, a real challenge. Although a large body of literature on pension economics has generated a wealth of useful qualitative insight, there is no realistic comprehensive asset/liability management model for the joint quantitative analysis of capital structure choices, pension fund allocation decisions, and the rational pricing of liability streams. The development of such a model, and its application to an empirical analysis of the mispricing of liability streams, is, as it happens, the focus of our paper.

In this context, we contribute to the literature on pension economics by analysing the valuation of pension liabilities regarded as defaultable claims issued by the sponsor to workers and pensioners in the context of a structural intertemporal capital structure model with contingent contributions. Our paper focuses on the interaction of the allocation decisions of the pension plan and the valuation of these liabilities, thereby extending the literatures on capital structure (Leland 1994; Leland and Toft 1996) and defaultable bond pricing (Merton 1974; Black and Cox 1976; Longstaff and Schwartz 1995 and many other subsequent papers) to account for the presence of a pension plan.

On the one hand, several relatively recent papers have addressed optimal portfolio decisions in the presence of liability constraints, including Merton (1993), Sundaresan and Zapatero (1997), Rudolf and Ziemba (2004), van Binsbergen and Brandt (2007) or Martellini and Milhau (2010). For the most part, however, these papers have considered the pension fund problem in isolation, without taking into account the relationship with the sponsor. As a result, the liability value process is exogenously specified in all of these and other related papers. This is a serious limitation since it fails to recognise that pension liabilities are a particular claim on the assets of the firms, and, more precisely, that they are a collateralised form of debt owned by the workers and pensioners of the sponsor, a debt for which the assets of the pension plans are the collateral, in exchange for which the company receives the present value of lower wage demands. In particular, what is missing from such analyses is the taking into account of credit risk in liability streams and how it is affected by capital structure decisions made by the sponsor and by asset allocation decisions made by the pension fund. On the other hand, a series of early papers has looked at the pension fund problem from a corporate finance perspective (Sharpe 1976; Treynor 1977;
1. Introduction

Black 1980; Tepper 1981; Harrison and Sharpe 1983; Bicksler and Chen 1985).5 Those papers, however, were cast in a highly stylised one-period model (one notable exception being Harrison and Sharpe [1983]), with constant interest rates, and in the absence of proper modelling of intermediate contributions from the sponsor to the pension plan and with no formal analysis of capital structure decisions, asset allocation decisions, or the rational value of liability streams.6 As a result of the highly stylised nature of those early models, the predictions they generate cannot be used to properly assess the value of default risk in pension obligations from a purely quantitative standpoint. In fact, most of those papers produce only extreme solutions to the optimal pension funding and investment policy problems involving either funding as little as possible and using allocation decisions to maximise default risk to take advantage of the insurance provided by the nation-wide pension guarantee fund if there is one (Sharpe 1976), or funding to the greatest extent and investing fully in safe liability-matching assets to capture the preferential treatment of pension plans under current tax law (Black 1980; Tepper 1981). In a related effort, Harrison and Sharpe (1983) also obtain extreme solutions in pension funding and investment strategies while simultaneously taking into account the tax and insurance effects, depending on whether the insurance or the tax effect dominates. One exception is Bicksler and Chen (1985), arguably the paper most closely related to ours, which shows that there could be interior optimal policies depending on the relative strengths of the tax and the insurance effects in the presence of such frictions as pension termination costs and progressive and asymmetric corporate income tax structures. We provide several extensions to this paper and the related literature.

We first explicitly analyse the structural relationship between the value of the assets of the firm and the value of the claims issued by the sponsor to bondholders and to liability-holders, in the context of a formal capital structure model. Explicitly recognising that bondholders and liability-holders hold (relatively complex) payoffs that are functions of the pension fund assets but also firm assets allows us to obtain a wealth of new insight into the interaction of capital structure decisions and the valuation of pension claims, as well as into the interaction of asset allocation decisions and the sponsor’s credit rating. We first show that the existence of a pension plan has a great impact on capital structure decisions, with the optimal leverage ratio a decreasing function of promised pension payments. This finding is consistent with the intuition that suggests that a firm without a pension fund would optimally take on more debt than an otherwise identical firm sponsoring a pension plan, since the latter firm has already issued a form of debt by committing to a payment to retired employees. Conversely, capital structure decisions have a substantial impact on the fair value of pension claims, with a pension credit spread that increases approximately proportionally to the leverage ratio. One important corollary is that the regulatory valuation leads to overestimates of the fair value of liabilities for highly leveraged firms, whereas it leads to underestimates of the liability for firms with little debt outstanding, and one of the contributions of the model is to enable a quantitative estimate of the magnitude of the over- or

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5 - See also Bodie (1990b), Scherer (2005), and Kocken (2009).
6 - See Hu (1992) for an integrated analysis of pension funding and corporate financing strategies in the presence of default risk, albeit in a static setting.
underestimate as a function of parameter values. We also obtain novel results regarding optimal funding and investment decisions. In particular, we obtain interior solutions for some parameter values even in the absence of pension termination costs and progressive and asymmetric corporate income tax structures.

More specifically, when the correlation of the value of the firm process and the stock index return process is positive, we find that the fair value of promised payments to bondholders and pensioners decreases following an increase in the allocation to risky assets by the pension fund. This is a clear case of asset substitution, since larger allocation to risky assets leads to an increase in the total riskiness of the total assets held by the firm (financial assets held off the balance sheet through the pension funds and real assets directly held on the balance sheet), which is the underlying state variable on which is based the value of such claims. When the correlation is negative, on the other hand, greater allocation to risky assets may result in diversification benefits. This competition between the asset-substitution effect and the diversification effect, which has not been analysed in previous papers, leads (at least for reasonably low funding ratios) to an interior optimal solution to the pension investment strategy problem, both when the objective is related to maximising total firm value and when it is related to maximising pensioner value. On the whole, there is clear evidence of conflicts of interest between stakeholders, between shareholders and pensioners, in particular. From the pensioners’ perspective, risk taking is detrimental because it involves increasing the likelihood of partial recovery of pension claims. On the other hand, the upside potential of performance-seeking assets allows shareholders to reduce the burden of contributions needed to meet expected pension payments. These conflicts of interest could be mitigated by granting pensioners some access to the surplus, thereby allowing plan beneficiaries to benefit from the increase in expected performance implied by more aggressive investment strategies. Practically, a form of surplus sharing can be introduced by indexing pension benefits to inflation when the funding status of the pension plan permits. Such conditional indexation rules exist in the Netherlands. We also find that another way to align the interests of shareholders and pensioners is to enlarge the set of admissible investment strategies so as to include such dynamic risk-controlled strategies as constant proportion portfolio insurance (CPPI) (Black and Jones 1987; Black and Perold 1992), or extending them to a pension management context sometimes referred to as contingent immunisation (Bodie 1990a) or dynamic liability-driven investment (LDI) (Martellini and Milhau (2010)). In fact, we demonstrate that risk-controlled strategies that attempt to ensure a funding ratio above 100% allow shareholders (limited) access to the upside performance of risky assets, while ensuring that pensioners will not be hurt by the resulting increase in risk. In terms of funding decisions, we first find that pensioners always benefit from increases in funding to the pension plan because the plan assets are used as collateral for the pension claims. This increase in the value of liabilities takes place at a decreasing rate: when the pension plan is already very well funded, marginal increases in asset value generate only marginal improvement in pensioner welfare. The impact on equity-
holders is, in principle, positive as well, at least when shareholders have full access to the pension fund surpluses and when the initial funding ratio is sufficiently high: increases in funding lead to increases in benefits from the tax advantage and therefore to higher shareholder value; when shareholders have full access to pension surpluses, the tax benefits come with no opportunity costs, which would be a factor in the event of partial or zero access by shareholders to pension surpluses. For large pension claims, an increase in funding starting from low funding levels can, on the other hand, be detrimental to shareholders: when it is highly unlikely that pension assets will ever be in excess of pension liabilities, one additional dollar invested in the pension fund is certain to be lost for shareholders, who would sooner have it invested in the firm. We also analyse the value of the bonds as a function of the allocation to the pension fund. As expected, a firm with a pension fund in surplus has a better rating than a firm with a pension fund in deficit, all else equal, and the impact of pension funding decisions is substantial. These results are consistent with the empirical findings by Martin and Henderson (1983) or Carroll and Niehaus (1998), who show that pension funding has a positive influence on credit ratings. We also analyse the determinants of the difference between the fair and regulatory value of pension insurance, which, for reasonable parameter values, can be substantial.

In an initial series of extensions to the base-case model, we introduce a pension benefit guarantee corporation and find that the presence of this corporation would have a great impact on optimal funding decisions. In our model, maximising the total value in the presence of the guarantee corporation with respect to corporate and pension fund policies and maximising the difference between the fair and the regulatory values of the pension put are not equivalent objectives because of the presence of frictions (tax shields and bankruptcy costs) that were not present in Sharpe (1976), for example. For large promised payments to pensioners, and given our base-case parameter values, we obtain that funding the pension plan to the maximum is the optimal solution from a perspective of maximising firm value. In a second series of extensions, we compare the values of the claims written on the assets of the firm and of the pension fund under external funding, in which pension fund assets and liabilities are held off the balance sheet of the sponsor, and internal funding, where these assets and liabilities are fully integrated into the balance sheet of the sponsor. We find that internal funding is detrimental to pensioners, who are less senior than under external funding, whereas equity-holders prefer external to internal funding only if the surplus-sharing rule favours them rather than pensioners.

From a methodological standpoint, one additional contribution of the paper is to extend the analysis of funding and investment decisions made by the pension fund, and their interaction with leverage and contribution decisions made by the sponsor, to an intertemporal setting, while Bicksler and Chen (1985) and most related papers have considered only a simple static model. Extending the analysis to the dynamic setting turns out to be of critical relevance, because it allows us to analyse such empirically relevant features.
as state-dependent contribution policies or the short-term minimum funding ratio constraints imposed by regulators in most developed countries (with the notable exception of the United States and the United Kingdom). By forcing the sponsor to make early additional contributions when the poor performance of the pension fund portfolio makes such additional contributions necessary, these short-term funding constraints, whose presence cannot be accounted for in a static setting, have a dramatic impact on the valuation of pension liabilities, as well as on shareholder and total firm value. Unsurprisingly, we find that pensioners benefit unambiguously from the enforcement of minimum funding requirements, with the greatest benefits to be expected when the initial funding ratio is low, which is when pension protection is most critically needed. The situation is more ambiguous for equity-holders. On the one hand, it can be argued that early contributions make insolvency of the pension fund less likely. On the other, if strong stock market performance enables the fund to recover without calling for exceptional contributions, they may also prove, after the fact, to have been unnecessary. On the whole, we find that equity-holders will benefit from the introduction of short-term constraints only if their access to pension fund surpluses is sufficiently easy, in which case they can expect unnecessary contributions to be returned to them at the terminal date.

In addition to these contributions, when addressing the question of pricing the equity of sponsors with underfunded plans, we shed new light on the empirical findings in Franzoni and Marin (2006), who document systematic mispricing of US companies sponsoring defined-benefit (DB) pension plans (see also Picconi [2006], who concludes that even analysts appear to misinterpret readily available information about firms’ pension earnings and funding status). In other words, although previous empirical research documented the presence of a relationship between pension fund funding status on the one hand and equity and debt prices on the other (Jin, Merton, and Bodie [2006] find empirical evidence that equity risk reflects pension risk despite opaque accounting rules and Carroll and Niehaus [1998] find empirical evidence of a positive relationship between the funding of DB plans and debt ratings), there is reason to believe that the market incorrectly assesses the price of claims on the firm’s assets in the presence of a pension fund.

It is our hope that our paper can contribute to mitigating this mispricing problem. As noted in Rauh (2009), corporations with defined-benefit pension plans in fact face conflicting motives when it comes to managing the uncertainty of the cash flows. On the one hand, the theory of asset substitution (Jensen and Meckling 1976) suggests that increasing the volatility of the assets once the debt is in place increases the value of shareholders’ equity holdings. Although there may be a variety of debt covenants that might prevent managers from increasing the volatility of the real assets of the firms, increasing the equity allocation in the pension fund is a straightforward means of increasing the volatility of the financial assets held indirectly, through the pension fund, by the firm. Our model allows us to analyse this question through a formal quantitative analysis of the impact on stakeholders

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(sharholders, bondholders, and pensioners) of an increase in the allocation to equity in the pension plan. On the other hand, reducing the volatility of the assets in place would allow the firm to reduce the likelihood of bankruptcy and the associated costs (Smith and Stulz 1985) or the likelihood of being unable to undertake profitable investment projects (Mayers and Smith 1987, Froot, Scharfstein, and Stein 1993). Here again, the investment policy chosen by the pension fund can be used to reduce the volatility of the surplus (by investing in liability-matching instruments) or even by taking into account the presence of the assets of the firm (by shorting financial assets that would be positively correlated with the firm’s real assets). The empirical analysis done by Rauh (2009) suggests that, on average, risk management incentives to avoid costly bankruptcy dominate risk shifting (asset-substitution motives) in pension fund investing. Our paper provides a normative framework for the quantitative analysis of the comparative impact of risk-management and asset-substitution motives on shareholder wealth.

This paper also has important potential policy implications. In particular, it raises a series of questions related to current practices in liability cash-flow valuation (in which an arbitrary rate is used) and pension insurance pricing (in which the premium paid is independent of both the financial health of the sponsor and pension fund allocation decisions). Our model suggests that valuation principles for liability streams (and derivative assets written on them) should account for differences in financial health and capital structure decisions made by the sponsor, as well as differences in asset allocation policy chosen by the pension fund. Our analysis also suggests that any minimum funding requirements should be a function of these items as well. Although providing firm-specific regulatory constraints and accounting valuation principles might seem a formidable challenge, our paper presents an analytical model that could be used to achieve these objectives. Also somewhat related to ours is a recent paper by Lucas and Zeldes (2006), who likewise consider the problem of rational valuation of liability streams, but mostly by focusing on the pricing of market risks in liabilities (in particular, the pricing of uncertainty in earnings growth), whereas we also price the credit risk component, which turns out to account for a very substantial share of their value (Sundaresan and Zapatero 1997) also provide a valuation formula for liability obligations, but without taking into account default risk). Finally, related is a recent paper by Novy-Marx and Rauh (2009), who provide estimates for the present value of state pension liabilities using both a risk-free discount rate and a discount rate that reflects the state’s riskiness in some simplified manner (done by using the yield on the state’s general obligation debt).

The rest of the paper is organised as follows. In section 2, we develop the basic intuitions in the context of a simple one-period setting with default triggered only at terminal date, in the spirit of the early Merton (1974) model (referred to as M74 in the paper). Section 3 presents several extensions to the static model: the introduction of a pension guarantee fund, the indexation of pension payments on inflation, and the incorporation of pension assets and liabilities into the balance sheet of the
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sponsor company. In section 4, we extend the model to an intertemporal setting and analyse the impact of dynamic allocation strategies, a random date of default, short-term funding ratio constraints, and contingent contributions on the value of the various claims. Section 5 concludes and discusses possible directions for further research.
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2. A Stylised Model with Default at Terminal Date Only
In this section, we develop the main intuitions by studying a simple one-period setting with constant interest rates and default triggered only at terminal date. This model builds on the model of integrated ALM set up by Scherer (2005).

2.1 The Model
We first consider a situation in which the cost of default is fully borne by pensioners and employees of the sponsor. We subsequently introduce (see subsection 3.1 below) a pension benefit guarantee corporation (PBGC), which is intended to provide pension benefits for pensioners of companies that are in distress.

2.1.1 State Variables
We consider a firm issuing a single class of debt; we assume that it promises a fixed payment $D$ at time $T$. The firm has an after-tax unlevered asset value $V$, which we assume to evolve under the risk-neutral measure $\mathbb{Q}$ as:

$$dV_t = V_t[r \, dt + \sigma_V \, dz^V_t] \quad (2.1)$$

where $r$ is the constant short-term risk-free rate and $z^V_t$ is a standard Wiener process. This dynamics can be modified at minor cost to accommodate a payout rate representing all intermediate payments to the agents with claims on the firm: it would suffice to replace the drift $r$ with $r - \delta$. We do not introduce this payout rate for two reasons. First, we want our static model to be a straightforward extension of M74, where there are no dividend payments to equity-holders and no coupon payments to bondholders. Second, since default can be triggered only at time $T$, it is the terminal value $V_T$ that matters: subtracting $\delta$ from the risk-free rate amounts to multiplying this terminal value by the constant factor $e^{-\delta T}$.

In addition to the debt held by the market, the firm is indebted to former employees who are entitled to a payment $L$ at time $T$ as pension fund beneficiaries. The payment is made by the pension fund, which is assumed to be a separate entity. The pension plan is endowed with an initial asset $A_0$, which, as it happens, can be thought of as the initial contribution from the firm, and which is invested in a risky asset $S$ and in the bank account $B$. In this section, we assume away interest rate risk, so the cash is risk-free at all investment horizons. Therefore, it can be seen as a perfect hedge of promised liability value. Under the risk-neutral probability measure, $S$ and $B$ evolve as:

$$dS_t = S_t[r \, dt + \sigma_S \, dz^S_t], \quad \frac{dB_t}{B_t} = r \, dt \quad (2.2)$$

where $z^S_t$ is another Wiener process whose correlation with $z^V_t$ is denoted by $\rho$. We assume away intermediate contributions from the sponsor. Therefore, the financial portfolio of the pension fund is self-financed, the only contribution from the firm to the pension plan is the initial endowment $A_0$, and the value of the pension portfolio evolves as:

$$dA_t = A_t[r \, dt + \omega_t \sigma_S \, dz^S_t] \quad (2.3)$$

where $\omega_t$ is the weight allocated to the risky asset. In the numerical exercises (see subsection 2.2 below), we will consider fixed-mix strategies where $\omega_t$ is maintained constant (so that we can drop the index $t$).
With this assumption there is a clear link between the amount of risky assets that the pension fund holds in its portfolio and the volatility of $A_T$. Indeed, we have:

$$A_T = A_0 e^{\left(r - \frac{\sigma^2}{2}\right) T + \omega \sigma z^T}$$

$V_0$ is the initial unlevered asset value of the firm and $x$ the initial capital available to the firm. We also let $\theta$ denote the corporate tax rate. Throughout the paper, we will assume that any contribution to the pension fund is tax deductible, which imposes the following budget constraint:

$$V_0 = (1 - \theta) A_0 + x$$  \hspace{1cm} (2.4)$$

It says that the sum of the initial contribution to the pension fund, net of the tax deduction, and the capital allocated to the operating projects equals the total amount of capital brought by the initial owners of the firm.

### 2.1.2 Payoffs to Stakeholders

We consider three groups of agents with claims on the firm and perhaps on pension fund assets: pensioners are the retired workers of the sponsor, equity-holders (or shareholders) own shares in the firm, and bondholders hold bonds issued by the firm. The group of pensioners, equity-holders, and bondholders will be referred to as the claimholders (or stakeholders). Following M74, we assume away any payment to either agent between dates 0 and $T$, and default can be triggered only at time $T$.

The pension fund is committed to paying the amount $L$ at date $T$ to pensioners. Hence a pension contract is a collateralised form of debt held by the pensioners of the firm, where the pension fund assets serve as collateral. If the pension fund is insolvent (i.e., if $A_T < L$), the sponsor is called on to make a contribution $L - A_T$. If the operating assets $V_T$ are sufficient to cover the deficit, pensioners receive $L$ as promised; otherwise, default is triggered. In the opposite case, when the pension fund enjoys a surplus ($A_T > L$), this surplus is shared by pensioners and equity-holders: pensioners receive a contractual and constant fraction $1 - \gamma$ of the surplus, whereas the remainder, $\gamma$, net of taxes goes to equity-holders. In fact, any surplus returned to the sponsor plan is subject to a special tax regime known as "the reversion tax". New legislation was passed in 1986 regarding the tax treatment of excess pension assets: it levied a 10% excise tax on reversions from defined-benefit plans. This rate was raised to 50% in 1990, unless the sponsor gives at least 25% of the reversion to participants (in the form of contributions to some other plan), in which case the reversion tax is 20% (Ippolito 2001, 2002). The reversion (net of the excise tax) is also subject to the normal corporate tax. The amount remaining after debt payment, if strictly positive, goes to equity-holders. We let $\theta_{rev}$ be the reversion tax rate, which is equal to 50% if $\gamma \geq 75\%$ and 20% otherwise. In our base case we shall assume that surpluses reverted to the sponsor are not subject to taxation. To encompass both situations, we will let $\theta_{eff}$ be the effective tax rate. Hence we will have $\theta_{eff} = 0$ in the base case and $\theta_{eff} = 1 - (1 - \theta)(1 - \theta_{rev})$ in the presence of the reversion tax. If $\gamma = 0$ then the entire surplus goes to the beneficiaries of the pension plan in the form of enhanced pension benefits. On the other hand, when $\gamma = 1$ (which we shall treat as our base case in the numerical exercise), shareholders have full access to any after-tax pension fund surplus.
Bondholders are promised a fixed payment $D$ at time $T$. Equity-holders are responsible for redeeming debt, using the terminal unlevered value of the firm $V_T$, net of any bankruptcy costs, and a fraction of any pension plan surplus. As explained above, this fraction is equal to $(1 - \theta_{\text{eff}})\gamma$. In the end, bondholders receive the promised payment if any one of the following three conditions is met: both the sponsor and the pension fund are solvent ($A_T \geq L$ and $V_T \geq D$); the pension plan is insolvent ($A_T < L$) but the sponsor can make up for the deficit ($V_T \geq D + L - A_T$); the sponsor is insolvent ($V_T < D$) but the pension fund enjoys a surplus ($A_T \geq L$) and the current tax and surplus-sharing regimes make it possible to avoid bankruptcy ($V_T + \gamma(1 - \theta_{\text{eff}})(A_T - L) \geq D$).

In the other states of the world the firm is liquidated and bondholders receive a recovery payment. Since the recovery payment in the event of default may include a fraction of any pension fund surplus, it can be said that bondholders have a conditional and limited claim on pension surpluses.

As a general rule, equity-holders receive the aggregate assets of the firm and pension fund $A_T + V_T$, net of the payments to pensioners and bondholders if the firm is not in default at time $T$, and they receive zero as soon as the firm is bankrupt. They are also entitled to the tax savings at time $T$. We assume that the contributions to the pension plan as well as the interest payments on debt are tax deductible (when the firm is not in default). The amount of interest paid on debt is $D - D_p$, where $D_p = D e^{-rT}$ is the present value at time 0 of the promised repayment to debtholders. Hence the tax savings on interest payments are equal to $\theta(D - D_p)$. The tax savings on contributions are $\theta(L - A_T)^*$.

We now summarise the payoffs to each group of claim-holders at time $T$, in the various states of the world. For notational convenience, we let $C_T$ be the payoff of the call written on the pension fund’s assets with exercise price $L$; i.e., we set $C_T = (A_T - L)^*$, which is the surplus of the pension fund.

1. $AT + VT > L + D$: in this situation, equity-holders receive a positive payoff.
   1. $A_T \geq L$ and $V_T \geq D$: the firm pays $D$ to debt-holders, and pensioners receive $L + (1 - \gamma)C_T$, that is, the promised payoff $L$ plus a fraction $1 - \gamma$ of the pension fund surplus. Equity-holders receive the remaining assets of the firm plus the remaining part of the surplus net of the tax reversion, plus the tax shield. The payoff that they receive is thus $V_T - D + \gamma(1 - \theta_{\text{eff}})C_T + (D - D_p)$;
   2. $A_T \geq L$ and $V_T < D$: pensioners still receive $L + (1 - \gamma)C_T$. Default can be avoided if $V_T + \gamma(1 - \theta_{\text{eff}})C_T \geq D$, in which case bondholders receive $D$ and equity-holders $V_T - D + (1 - \theta_{\text{eff}})C_T + \theta(D - D_p)$. Otherwise, bankruptcy is triggered, entailing a loss to third parties in the form of bankruptcy costs. Bondholders receive the proceeds of the liquidation, $(1 - \alpha)V_T$, plus the fraction $\gamma(1 - \theta_{\text{eff}})$ of the surplus, and equity-holders receive nothing;
   3. $A_T < L$ and $V_T \geq D$: in this case, the sponsor makes a final additional contribution $L - A_T$ to the pension plan so that pensioners can receive the promised payment $L$. Bondholders also receive the promised payment $D$, and equity-holders receive the remaining assets of the firm plus the tax shield, $A_T + V_T - L - D + \theta((D - D_p) + (L - A_T))$. 

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In this case there is no pension fund surplus to be shared by pensioners and equityholders.

II. \( A_T + V_T < L + D \): in this case, equityholders receive nothing.

1. \( A_T < L \) and \( V_T < D \): both the firm and the pension fund default on their obligations. Pensioners receive the totality of the pension assets (collateralised nature of pension obligations) plus some fraction \( q \) of the proceeds \((1-\alpha)V_T\) of the liquidation of the firm’s assets, after bankruptcy costs. Assuming equal seniority of bondholders and pensioners, \( q \) must be equal to 0.10 In other words, both pensioners and debt-holders receive an amount proportional to the remaining amount due them before liquidation \((L-A_T)\) for pensioners, and \(D\) for bondholders. Pensioners thus receive \( A_T + q(1-\alpha)V_T \), and bondholders are paid \((1-q)(1-\alpha)V_T \).

2. \( A_T \geq L \) and \( V_T < D \): pensioners receive \(L + (1-\gamma)C_T\), The amount available for debt payment is \(V_T + \gamma(1-\theta_{eff})C_T\), which is strictly less than \(D\). Hence default is triggered: bondholders get \((1-q)(1-\alpha)V_T \), and equity-holders get nothing.

3. \( A_T < L \) and \( V_T \geq D \): this case is similar to the case where \( A_T < L \) and \( V_T < D \): bondholders receive \((1-q)(1-\alpha)V_T \); pensioners receive \( A_T + q(1-\alpha)V_T \).

Formally, the payoffs to each group of stakeholders can be written as:

\[
L_T = L \left[ \mathbb{1}_{\{A_T + V_T \geq L + D\}} + \mathbb{1}_{\{A_T \geq L, A_T + V_T < L + D\}} \right]
+ [A_T + q(1-\alpha)V_T] \mathbb{1}_{\{A_T < L, A_T + V_T < L + D\}}
+ (1-\gamma)C_T \tag{2.5}
\]

\[
D_T = D_T \left[ \mathbb{1}_{\{A_T + V_T \geq L + D\}} + (1-q)(1-\alpha)V_T \mathbb{1}_{\{A_T + V_T < L + D\}} \right]
+ [(1-\gamma)V_T + \gamma(1-\theta_{eff})C_T] \mathbb{1}_{\{A_T \geq L, A_T + V_T < L + D\}} \tag{2.6}
\]

Our model nests that of M74 when the pension fund is removed from the analysis: it suffices to set \( A_0 \) and \( L \) to zero, and the payoffs collapse to:

\[
E_T = (V_T - D)^+, \quad L_T = 0,
D_T = D_T \left[ \mathbb{1}_{\{V_T \geq D\}} + (1-\alpha)V_T \mathbb{1}_{\{V_T < D\}} \right]
\]

which is equivalent to equations (9.c) and (11) in M74.

2.1.3 Prices of the Claims

The fair values at time 0 of the pension fund liabilities and of corporate bonds are:

\[
L_0 = E^Q[e^{-rT}L_T] \tag{2.8}
\]

\[
D_0 = E^Q[e^{-rT}D_T] \tag{2.9}
\]

They can be interpreted as the cost of issuing these claims for the corporation. In fact, there is one important difference between the debt held by pensioners and that held by bondholders, which is that the latter is tradable while the former is not. The inability to trade pension claims implies that the value of receiving them for pensioners is, \textit{ex ante}, lower than the cost of issuing them for the corporation. We discuss this question in the next subsection. Similarly, the price at time 0 of equity is given by:

\[
E_0 = E^Q[e^{-rT}E_T] \tag{2.10}
\]
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The actual payment $L_T$ to pensioners can be decomposed as $L_T^1 + L_T^2$, where $L_T^1$ is the sum of the first two terms on the right-hand side of (2.5) and $L_T^2$ is the access to pension fund surpluses. With $L_T^1$ being the present value of $L_T^1$, the credit spread $s_L$ is the rate, in excess of the risk-free rate, at which the face value should be discounted to give the fair value:

$$L_T^1 = e^{-(r+s_L)T} L_T$$

$L_T^1$ rather than $L_0$ is used in this definition, to obtain a non-negative spread. In other words, we attempt to measure the impact of default risk on the valuation of promised liability payments, while excluding possible access to pension fund surpluses. We take this definition better to compare the fair value of liabilities and the “regulatory” liability value; this regulatory value is the present value of the promised liability, $L$, discounted at rate $r + s_{reg}$, where $s_{reg}$ is a spread given by the regulator:

$$L^{reg} = e^{-(r+s_{reg})T} L$$

Dividing the current assets of the pension fund by the regulatory liability value, one obtains the regulatory funding ratio:

$$F_t^{reg} = \frac{A_t}{L_t^{reg}}$$

As for pensioners, we can define a credit spread for bondholders by:

$$D_0 = e^{-(r+s_D)T} D$$

We now define the aggregate value of the firm and the pension fund. To do this, we first introduce the present values of bankruptcy costs, tax savings, and tax reversion:

$$BC_0 = E^Q\left[ e^{-rT} BC_T \right]$$
$$TS_0 = \theta A_0 + E^Q\left[ e^{-rT} TS_T \right]$$
$$RT_0 = E^Q\left[ e^{-rT} RT_T \right]$$

where:

$$BC_T = \alpha V_T \left[ \mathbb{P}_{\{A_T + V_T \geq L + D_T + \gamma (1-\theta)D_T \}} + \mathbb{P}_{\{A_T + V_T \leq L \}} \right]$$
$$TS_T = \theta (D - D_T) + (L - A_T) + \mathbb{P}_{\{A_T + V_T \geq L + D_T + \gamma (1-\theta)D_T \}}$$
$$RT_T = \gamma \beta C_T$$

(2.11)

It is straightforward to verify from the payoffs of subsection 2.1.2 that we have, in any state of the world:

$$L_T + E_T + D_T = V_T + A_T + TS_T - RT_T - BC_T$$

(2.12)

This simply states that the aggregate assets of the firm and the pension fund, net of taxes and liquidation costs, are shared by all stakeholders at time $T$. Taking the present value of (2.12) under the risk-adjusted probability measure $\mathbb{Q}$, we obtain that:

$$L_0 + E_0 + D_0 = V_0 + A_0 + TS_0 - \theta A_0 - BC_0$$

whence:

$$V_0 \equiv L_0 + E_0 + D_0 = V_0 + (1-\theta)A_0 + TS_0 - RT_0 - BC_0$$

(2.13)

We refer to this quantity as the total value of the firm and pension fund; it is denoted $V_0$. If the firm is not subject to taxation ($\theta = 0$), no taxes are levied on surpluses ($\theta_{rev} = 0$), and there are no liquidation costs ($\alpha = 0$), the total value is just $x$ and is thus independent of any choice of face value of debt, $D$, of initial pension funding, $A_0$ ($Le^{-rT}$), and of portfolio allocation decision $(\omega_t)_t$, although the components $L_0$, $E_0$, and $D_0$ may still depend on these parameters.

2.1.4 A Comment on Valuation of Liability Claims

We have argued that pension liabilities can be regarded as defaultable bonds held by
employees. It should be recognised, however, that the private valuation of these claims for pensioners is, in general, lower than the cost for the firm of issuing these claims, and this for at least two related reasons. 14 On the one hand, the pension claims are not tradable, and the valuation of these claims should therefore incorporate an illiquidity premium. On the other hand, employees of the firm are severely under-diversified with respect to sponsor risk, to which their human capital is already highly exposed. The non-tradable nature of the claim prevents employees from hedging away this risk. Similar arguments have been made for stock and stock option compensation packages, for the valuation of which preference-based pricing rules (e.g., a certainty equivalent principle) have been proposed (Detemple and Sundaresan 1999). It remains to be seen why a company puts workers at risk when it could sell this exposure at a lower cost on the bond market. One explanation may be that pensions are not publicly traded contracts and therefore decreases in their value might not be as obviously recognisable by pensioners as decreases in bond values from deterioration in credit ratings. 15 One of the contributions of this paper, as it happens, is to provide pensioners and other stakeholders with a means of pricing pension claims consistently.

In the absence of information regarding preferences, endowments, and trading and consumption strategies for employees, we may recognise only that \( L_0 \), the cost of issuing these claims for the corporation, provides an upper bound for the private value of the pension claims. One should note, however, that the difference between the value and the cost of the pension claims disappears in a complete market situation, in which the value of the claim can be obtained by arbitrage arguments only, or where the payoff generated by the claim can be replicated by pensioners through suitably designed dynamic trading strategies involving the available assets. In fact, our model is cast in a complete market setting, so any exposure to pension liability claims can be hedged away, if needed, through suitably designed dynamic replication strategies involving two risky assets, the stock index, as well as the sponsor company stock, in addition to the risk-free asset.

An additional source of complexity here is that the values of the stock of the sponsor and of the pension fund assets are endogenous and depend on the decisions made by stakeholders (contribution and leverage decisions made by the sponsor, and allocation decisions made by the pension fund). In fact, if these decisions are constant, or readjusted in a predictable manner, the pension payoffs remain attainable/replicable by trading in existing securities. In what follows, we present an extremely simplified example to explain how the replication strategy can be accommodated to account for predictable changes to allocation policy. More general situations can be handled similarly. Let us assume, for instance, that the entire initial capital \( x \) is invested in the pension fund and that there is no corporate debt; that is, we take \( V_0 = 0 \) and \( D = 0 \). Assume, moreover, that \( \gamma = 1 \). Hence the payoff to pensioners is: \( L_T = \min(L, A_T) = L - (L - A_T)^+ \). In this extremely simplified setting, pensioners hold a short position in a put written on pension fund assets, with a strike price given by the promised liability payment. If \( A \) follows a geometric Brownian motion, then it is well known that the put can be dynamically replicated using a delta-neutral strategy. Hence \( L_T \) can be replicated as well. More
generally, it is sufficient to assume that \( A \) has a deterministic volatility over the lifetime of the option. To see this, let us assume that the weight allocated to the risky asset \( S \), \( \omega_t \), is a deterministic function of time and that the remainder is invested in the cash (as in equation (2.3)). Then it can easily be shown that the price of the put is:

\[
P(t, A_t) = Le^{-r(T-t)} \mathcal{N}(-d_{2,t}) - A_t \mathcal{N}(-d_{1,t})
\]

where:

\[
d_{1,t} = \frac{1}{\sqrt{\int_t^T \sigma_s^2 \omega_s^2 ds}} \left[ \ln \frac{A_t}{L} + r(T - t) - \frac{1}{2} \int_t^T \sigma_s^2 \omega_s^2 ds \right]
\]

\[
d_{2,t} = d_{1,t} - \sqrt{\int_t^T \sigma_s^2 \omega_s^2 ds}
\]

The delta of the put is \(-\mathcal{N}(-d_{1,t})\), so buying \(-\mathcal{N}(-d_{1,t})\) shares of \( A \) starting from the initial wealth \( Le^{-rT} - p(0, A_0) \) yields the payoff \( L_T \). Examples of situations in which \( \omega_t \) is deterministic include the case in which it is constant (fixed-mix strategies) and in which it is piecewise constant over time, with the jump instants being deterministic:

\[
\omega_t = \sum_{i=0}^{n} \omega^i \mathbb{I}_{\{t_i \leq t < t_{i+1}\}}
\]

where \( 0 = t_0 < t_1 < ... < t_n = T \) is a subdivision of the interval \([0, T]\) and the \( \omega^i, i = 1, ..., n \), are constant. In the more general case in which \( V \) and \( D \) are not zero and \( \omega_t \) is a deterministic function of time, the pair \((S, V)\) follows a Markov process under \( \mathbb{Q} \), so there is a function \( \ell \) such that \( L_T = \ell(t, S_t, V_t) \).

In this case, the payoff \( L_T \) can be replicated by a dynamic trading strategy in the stock index \( S \) and the stock issued by the firm \( E \), if it is publicly traded (to hedge against unexpected changes in the firm asset value process \( V \)).

2.2 Numerical Results

In this section, we present numerical results for the pricing of the claims written on the pension fund and firm assets. These prices have been obtained by simulating the joint distribution of the final asset value and final payoff, a joint log-normal distribution given the assumption of a constant weight \( \omega \) in (2.3). We assume in the base case that \( \gamma = 1 \), i.e., that the entire pension surplus goes (after tax) to the shareholders. The initial capital \( x \) is set to 100, so all prices can be directly interpreted as percentages of this quantity. The other base-case parameters are displayed in table 1.

2.2.1 Leverage Decisions

We first analyse the impact of leverage decisions on total value of the firm and fair liability value, for different values of the promised payment \( L \). \( L \) is taken as a measure of the size of commitments to pensioners. The leverage ratio is defined as the market value of debt over the sum of debt and equity values. Since we focus on leverage choices, we set in this subsection the initial endowment of the pension fund equal to the regulatory value of liability, so the pension fund is fully funded at the starting date, at least from a regulatory standpoint. The initial contribution net of tax deduction cannot exceed the initial capital available to the firm before the pension fund is in place. This requirement implies that the set of possible promised payments is bounded from above. Precisely, the following must hold:

\[
L < x \frac{e^{(r+s_{reg})T}}{1 - \theta}
\]  

(2.14)

Given our base-case parameters, the upper bound is equal to 253.6.
In the presence of frictions (taxes and bankruptcy costs), the trade-off theory of capital structure suggests that an optimal leverage decision can be achieved to maximise total firm value. Intuitively, one expects that a firm without a pension fund will optimally take on more debt than an otherwise identical firm sponsoring a pension plan, since the latter firm has already issued a form of debt by committing to a payment to retired employees. The results of figure 1 confirm that the total value-maximising leverage ratio is indeed a decreasing function of $L$. In the absence of the pension fund (for $L = 0$), the optimal leverage ratio is 34.92%, a value that is of the same order of magnitude as what is found in related papers that use a dynamic capital structure model with a random default date (Leland 1994, 1998; Ju and Ou-Yang 2006) and find leverage ratios ranging from 30% to 50% depending on parameter values. For sufficiently high promised payments $L$ (e.g., $L = 100$ or $L = 150$), the optimal leverage ratio is in fact zero, suggesting that the amount of debt already issued in the form of pension claims makes it sub-optimal for the firm to issue any more debt to market participants. The collateralised nature of the pension obligations and the existence of potentially complex surplus-sharing rules imply, however, that debt held by pensioners and debt held by bondholders are not perfect substitutes. On the whole, the main insight that we obtain from figure 1 is that the existence of a pension plan should have an effect on capital structure decisions.

Figure 2 takes the reciprocal point of view and plots the credit spread $s_L$ implied by the fair liability value $L_0$ as a function of the leverage ratio. It shows that capital structure decisions will have an impact on the fair value of pension claims. Other things being equal, pensioners will prefer a sponsor with a small amount of outstanding debt to a heavily indebted one, since a more financially constrained firm is less likely to be able to afford to make additional contributions if and when needed. The impact is very substantial: for an initially fully funded pension plan, increasing the leverage ratio from 15% to 45% also leads to multiplying by three the defaultable liability spread $s_L$ (from less than fifty basis points to more than 150). We find that for a given level of funding the credit spread is larger if promised payments to pensioners are a large fraction of the firm’s total commitments ($L = 150$) than if they are of the same order of magnitude as debt ($L = 50$). One important finding is that the regulatory valuation (based on an arbitrary spread, taken to be equal to 100 basis points in the base case) leads to overestimating the fair value of liabilities for highly leveraged firms, whereas it leads to underestimating the liability value for firms with little debt outstanding. For firms with large commitments to pensioners ($L = 150$) and a pension plan with initial funding of 70% or 100% only, liabilities are found to be undervalued, whatever the leverage ratio. Although these effects are straightforward from a qualitative standpoint, one of the contributions of the model is to show that the magnitude of the over- or underestimate can be quantitatively substantial for reasonable parameter values.

2.2.2 Allocation Decisions

We now turn to the impact of the allocation decision $\omega$ made by the pension fund on the liability.
2. A Stylised Model with Default at Terminal Date Only

The value of the claims. Figures 3 to 6 display the market values of claims held by pensioners, bondholders, and equity-holders as functions of the pension fund allocation $\omega$ to stocks, for different values of the regulatory funding ratio and of the promised payment to pensioners $L$. We consider five values for the initial regulatory funding ratio. Since the initial capital made available to the pension fund net of tax deduction cannot exceed $x$, this ratio is bounded from above:

$$\frac{A_0}{L_0^{\text{reg}}} < \frac{x}{(1-\theta)\Delta^{(r+s_{\text{reg}})T}}$$

In the base case, the right-hand side is equal to five. When the ratio gets closer to this upper bound, the initial amount of cash available for investment in firm activities, $V_0$, shrinks to zero. Finally, we consider three values of the correlation between the process $V$ of unlevered asset value and the value of the risky asset $S$: $\rho = 0.5$ (the base case), $\rho = 0$, and $\rho = -0.5$.

Figure 3 shows that the fair value of payment to pensioners, $L_0$, is generally a decreasing function of $\omega$. As a first level of explanation, this can be explained by the fact that pensioners in the base case have no access to the surplus ($\gamma = 0$), which corresponds to panel (a) in figure 3). In essence, if we assume away for a moment the presence of the sponsor, pensioners hold a short option written on the assets of the pension fund with payoff min $(L, AT)$, so a riskier strategy unambiguously leads to a decrease in $L_0$. When the presence of the sponsor is accounted for, the situation is subtler, since the payoff to pensioners is contingent on the realisation of both $A_T$ and $A_T + V_T$ (see equation (2.5)). In this context, greater allocation to the risky asset leads to increasing the volatility of the pension fund assets, but it may also introduce a diversification effect at the total financial plus real asset value $A_T + V_T$ level, at least when the stock index is negatively correlated to the unlevered value of the firm. This competition between the two effects is expected ex ante to lead to an interior optimal solution. As a matter of fact, we do obtain that, for negative correlation, there is an optimal interior allocation, at least when the initial funding does not exceed 100%. When the pension fund is sufficiently funded, pensioners have no interest in the pension fund taking risks given the collateralised nature of their claims. On the other hand, the diversification effect disappears when the correlation is non-negative, which explains why $L_0$ then becomes a monotonically decreasing function of $\omega$. Giving pensioners partial access to surplus (i.e., setting $\gamma$ to a value less than one) introduces an additional dimension to the analysis–namely, pensioner access to the upside performance of the pension fund investments. Indeed, panel (b) in figure 3 shows that, when pensioners have access to the full surpluses of the fund ($\gamma = 0$), $L_0$ generally becomes an increasing function of the allocation to the risky asset. This effect is more pronounced for a negative correlation because holding the risky asset diversifies away firm risk.

In the base case ($\gamma = 1$), equity-holders are entitled to the full surplus of the pension fund and are thus expected to benefit from riskier investment strategies, especially when the pension plan is highly funded, an expectation borne out by panel (a) of figure 4. This increase in shareholder value...
as a function of increasing volatility of the pension fund assets through an increase in the allocation to stocks, as opposed to cash, comes at the cost of a related decrease in bondholder wealth (see figure 5), which is a clear case of asset substitution (Jensen and Meckling 1976). A look at figures 3 and 5 shows obvious similarities between the fair value of the promised payment to bondholders and the fair value of pension payments, with a key difference again related to the collateralised nature of pension liabilities. That pension fund assets serve as collateral for pension liabilities explains why the fair value of pension claims \( L_0 \) is higher, other things equal, than the fair value of debt \( D_0 \). To put it differently, credit spreads for corporate debt are larger than credit spreads for pension liabilities. When funding falls, on the other hand, the diversification benefits start to become effective, as long as the correlation \( \rho \) is negative, so the volatility of the aggregate assets of firm and pension fund, \( A + V \), exhibits a minimum as a function of \( \omega \). This results in an interior optimal value for the allocation parameter. Finally, we consider in figure 6 the overall impact of investment decisions on firm and pension fund value \( L_0 + E_0 + D_0 \). It appears that, in general, total firm value \( v_0 \) is maximised for zero investment in the risky asset, hence providing some support to the proponents of fully investing pension assets in safe instruments. It is only when the correlation \( \rho \) is negative that a riskier strategy may lead to an increase in the total value and that there may be an interior optimal weight. For initial funding of 70% or 100%, the optimal weight is around 50%, and it grows to 70% for funding of 150%.

### 2.2.3 Funding Decisions

Funding decisions are made by the initial owners of the firm at time 0, who decide how to allocate the initial capital \( x \) to the firm \( (V_0) \) and the pension fund \( (A_0) \). Figures 7 to 9 show the impact on claim values of changes in the initial contribution to the pension fund \( A_0 \). On the whole, figure 7 confirms that pensioners always benefit from increases in funding to the pension plan because the plan assets are used as a collateral for the pension claims. This increase in the value of liabilities takes place at a decreasing rate: when the pension plan is already very well funded, marginal increases in asset value generate only marginal improvements in pensioners’ welfare. The impact on equity-holders is also straightforward: increases in funding lead to increases in benefits from the tax advantage and thus to higher shareholder value. This effect is obvious in panel (a) of figure 8, which represents the base case in which shareholders have full access to the pension fund surplus, at least for low face values of debt to pensioners. In this context, the tax-shield effect dominates, and investing more in the pension fund is not detrimental to the sponsor, since it can enjoy negative contributions in the end. On the other hand, for large pension claims (e.g., \( L = 100 \)), an increase in funding starting from low funding levels can lower the value of equity: when it is highly unlikely that pension assets will ever be in excess of pension liabilities, an additional dollar invested in the pension fund is certain to be lost for shareholders, who would rather have it invested in the firm. Panel (b) of figure 8 shows that shareholder value is a decreasing function of the allocation to the pension fund when \( \gamma = 0 \), a situation in which equity-holders...
have no access to the upside of pension asset values. Figure 9 focuses on the impact of funding decisions on bondholders’ welfare. When surpluses go to pensioners alone, substantial increases in funding ratios lead to lower bond values because higher funding to the pension fund might make it harder for the firm to redeem its debt. For reasonably low funding levels, bondholders benefit from higher funding because it is in their interest that the pension fund be able to fulfill its obligations without needing additional contributions, but beyond a certain funding, the value of corporate bonds decreases with the funding level. When equity-holders have access to surpluses, on the other hand, increases in the funding ratio do not hurt bondholders because part of any pension surpluses can be transferred to them if the sponsor is not wealthy enough to pay back the debt. On the whole, figure 10 suggests that higher funding leads unambiguously to higher total firm value. When equity-holders have full access to surpluses, this comes as no surprise, because making a high initial contribution makes it possible to derive greater benefits from the tax regime, and exceedingly large surpluses can be returned to the firm so that it does not lead to increases in the likelihood of bankruptcy. When pensioners have access to surpluses, large contributions still lead to high tax benefits, but they also lead to increases in bankruptcy probability. However, panel (b) of figure 10 shows that the total value is still an increasing function of the funding level, which shows that the present value of the tax savings increases faster than bankruptcy costs. In figure 11, finally, we analyse bondholder value as a function of the pension fund’s initial funding ratio. As expected, we find that a firm with a pension fund in surplus has a better rating than an otherwise comparable firm with a pension fund in deficit, and the impact of pension funding decisions on credit ratings is substantial.\[22\]

2.2.4 Impact of Surplus-Sharing Rules

In our base-case model, equity-holders have access to full surpluses; i.e., the parameter $\gamma$ is set to 1. Figures 12 to 15 show the impact of this parameter on the prices. $\gamma$ is the fraction of any pension fund surplus that goes to equity-holders. Obviously, this parameter has a great impact on the present values of the actual payments to pensioners and equity-holders, as can be directly seen from the payoffs (2.5) and (2.7): $L_0$ is a linearly decreasing function of $\gamma$, whereas $E_0$ is quasi-linearly increasing in $\gamma$. A higher $\gamma$ value is beneficial to equity-holders because they are entitled to a larger share of pension fund surpluses, as a result of which they are more likely to receive a positive payment at date $T$. Indeed, a higher $\gamma$ makes it possible for the firm to receive negative contributions from its pension fund at terminal date, and these contributions make default on the bond payment less likely. Of course, the impact of $\gamma$ is greater when the pension fund is highly funded, because a poorly funded pension plan is unlikely ever to enjoy any surplus. Since bondholders have an indirect access to surpluses when $\gamma$ is positive, they also benefit from a higher $\gamma$, which can be verified in figure 14. However, this upside potential is capped because they can never receive a payment higher than the face value of debt. Hence, the impact of $\gamma$ on the value of corporate bonds is less pronounced than on the value of equity. Finally, figure 15 measures the impact of the surplus-sharing rule on the
total value of the firm. The overall effect of increasing $\gamma$ is shown to be slightly positive. Again, the intuition is straightforward: increasing $\gamma$ allows for larger negative contributions, thanks to which default can be avoided in some states of the world, which decreases bankruptcy costs without adversely impacting the tax shield. However, the effect on the total value is not very great because the increase in $E_0$ induced by a larger fraction $\gamma$ offsets the related decrease in $L_0$ almost entirely.
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3. Extensions to the Static Model

This section presents possible extensions to the model developed in the previous section. Although they are presented independently, they could well be combined.

3.1 Introducing a Pension Benefit Guarantee Scheme

In the base-case model, it is assumed that the costs of corporate default are borne by the pensioners, who may not get their promised payment if the pension fund is insolvent and the sponsor is unable to cover the deficit. We now introduce a pension benefit guarantee scheme meant to provide pension benefits for pensioners of companies in distress. In the United States, the PBGC provides protection, but the protection is limited to a statutory maximum ($47,659 in 2006 for employees retiring at age sixty-five). One could price this so-called pension put and obtain its rational value as a function of the pension funding status, the firm’s capital structure, and pension fund allocation policy (Marcus 1987; Hsieh, Chen, and Ferris 1994; and Boyce and Ippolito 2002). If the guarantee scheme charges sponsors a premium proportional to the actual credit riskiness of liabilities, the costs are transferred to shareholders. In practice, pension insurance is not fairly priced (the premium paid is independent of both capital structure and pension fund allocation) and mostly underpriced for the most vulnerable companies, so incentives for asset substitution (extracting value from the scheme, that is, from other, healthier companies) remain intact (see Sharpe [1976] or more recently Bodie [1996] for a discussion of the pension put, and Bodie et al. [1985] for empirical evidence that pension insurance creates incentives for distressed companies to underfund their pension plans and invest in risky assets).23

3.1.1 Payoffs to Claimholders

In the United States, the actual provisions of the PBGC as defined by the Employee Retirement Income Security Act (ERISA) are as follows. The sponsoring firms with defined-benefit plans must enroll in the PBGC’s pension benefit insurance programme, which insures pension benefits up to a fraction \( \lambda \) of the promised payment \( L \). The fixed dollar amount of the insurance premium charged by the PBGC is currently $2.60 per employee per year. Under ERISA, the PBGC can preempt assets of the sponsor company, to the limit of 30% of their value. This preemptive right is senior to all unsecured liabilities of the company except wages.

We now provide a stylised model that will define the payoffs to each group of claimholders in the presence of a guarantee scheme. When \( A_T \geq L \) or \( A_T + V_f \geq L + D \), there is no need for the PBGC to intervene. Indeed, the assets of the firm are sufficient to cover the pension deficit. Hence, the payoffs to each group are identical to the payoffs in the absence of the PBGC (see item I in section 2.1.2). The situation is different if \( A_T + V_f < L + D \) and \( A_T < L \). Here, the firm does not have the assets to make up for the deficit, so liquidation takes place. The PBGC can then withdraw a fraction \( p \) of the assets net of bankruptcy costs, where \( p = \min (0.3, q) \), and:

\[
q = \frac{L - A_T}{D + L - A_T} \tag{3.15}
\]

Putting together pension fund assets and a fraction \( p \) of the firm assets, the amount available to compensate pensioners is...
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If the PBGC sells the put at fair value, i.e., if $P_0^{eq} = P_0$, then the value of the firm with a guarantee is equal to the value without a guarantee. But the share of total firm value that goes to each of the three groups of claimholders is still affected by the presence of the PBGC. In practice, the premium that the PBGC charges the sponsor is not equal to the fair value of the put and is taken equal to a fraction $\eta$ of the regulatory value $L_0^{reg}$. The total value of the firm will be increased if the pension put is underpriced ($P_0 > P_0^{eq}$) and decreased if it is overpriced ($P_0 < P_0^{eq}$). In our model, however, maximising the total value in the presence of the PBGC with respect to corporate and pension fund policies and maximising the difference between the fair and the regulatory values of the pension put are not equivalent objectives. Indeed, the first term $V_0$ is usually affected by these policies as well. Were the frictions equal to zero, $V_0$ would be equal to $V_0^{eq}$, and the equivalence would be recovered. This is a major difference between our model and Sharpe (1976), who derives optimal policies by maximising the share of the fair value of the insurance in excess of the premium paid by the sponsor.

3.1.2 Numerical Results

In this subsection, we study the impact of the PBGC on the values of the various claims. The base-case parameters are those of table 2. We take $\lambda = 0.85$, as in Bicksler and Chen (1985), and $\eta = 0.1$. This choice of parameter value implies that the PBGC charges the sponsor an amount equal to 10% of the regulatory value $L_0^{reg}$, and that its contribution is capped at 85% of promised pension payments. In the base-case situation, in which equity-holders have full access to pension surpluses, figure 16...
shows that the presence of the PBGC makes pensioners become almost indifferent to the asset allocation policy of the pension fund. It is only for very low funding ratios and very aggressive strategies that they can suffer a loss, so the credit spreads $s_j$ which are virtually zero in all other cases, then become slightly positive. Equity-holders and bondholders are also affected by the introduction of the pension guarantee, even though the impact is less substantial, as can be seen by comparing figures 4 and 17 with figures 5 and 18. Sharpe (1976) and Bicksler and Chen (1985) argue that it is optimal for shareholders to maximise the benefits obtained from the mispricing of the pension put, which is at its greatest when the allocation to the risky asset is also at its greatest, as confirmed in figure 19. For a negative value of the correlation parameter $\rho$, there is an interior value of the allocation that minimises the value of the pension put, as the volatility of the aggregate assets of firm and pension fund is minimal for some non-trivial allocation to the risky asset. Figure 19 also suggests that the difference between the fair value of the pension put $P_0$ and the regulatory value of the pension put $P_0^{\text{reg}}$, which, for reasonable parameter values, can be substantial, can be positive or negative depending on parameter values. Looking at the overall effect on total firm and pension value (figure 20), we see that only extreme allocations are optimal; an optimal allocation can be $\omega = 0$ or $\omega = 1$, depending on parameter values.

The next series of figures (from 21 to 24) focuses on the impact of funding decisions in the presence of the PBGC. As expected, figure 21 shows that, as long as surpluses go to equity-holders, these decisions have a very slight impact on the fair value of the payment to pensioners. It is only for declining funding ratios that credit spreads become strictly positive, although they are still much smaller than in figure 7. Comparing figures 8 to 22 reveals that the impact of funding decisions on equity-holders is not substantially different in the presence and the absence of the guarantee corporation. However, bondholders are more heavily affected when the promised payment to pensioners is large (more than 100% of the initial capital), as can be seen from figures 9 and 23. These are the situations in which the premium charged by the PBGC is particularly high, which reduces the initial unlevered value of the firm (see the budget constraint (3.16)). Moreover, with a large debt to pensioners, the pension fund is unlikely to enjoy surpluses that could eventually help the sponsor repay its debt. The result is a non-monotonic pattern, with the price of corporate bonds a decreasing function of funding for low or medium funding ratios. The same effects are at work for the total value (see figure 24). For large promised payments to pensioners, this aggregate quantity exhibits a U-shape. Similar results have been obtained by Bicksler and Chen (1985), in the context of a model with pension termination costs and a progressive tax on corporate income. For our choice of base-case parameter values, we find that funding to the maximum, as opposed to the minimum, is the optimal solution when the aim is to maximise firm value.
3. Extensions to the Static Model

3.2 Indexation to Inflation

The base-case model assumes constant promised pension benefits, but, in practice, pension payments are often indexed to inflation. In a DB pension plan, this indexation would be unconditional, but conditional indexation is more common. We present both cases in what follows.

3.2.1 An Economy with Inflation Risk

We introduce a stochastic price index $\Phi$, which is assumed to have the following dynamics under $\Omega$:

$$\frac{d\phi_t}{\phi_t} = \pi dt + \sigma_\phi dz^\phi_t$$

The drift $\pi$ is equal to the expected inflation rate minus the inflation risk premium. If this risk premium is zero, the drift coincides with the anticipated inflation rate. For simplicity, we assume that the coefficients $\pi$ and $\sigma_\phi$ are constant.\(^{26}\) In the presence of such stochastic inflation, the assumption of a constant interest rate would be irrelevant. Hence, we relax it and we consider a stochastic short-term interest rate:

$$dr_t = a(b - r_t) dt + \sigma r dz^r_t$$

$a$ is the speed of mean reversion towards zero, $\sigma r$ is the short-term volatility, and $b$ is the long-term mean under the risk-adjusted probability measure $\lambda$, the unit market price of interest rate risk, then we have that $b = b - \sigma^2 a$.\(^{27}\) The dynamics of the stock index $S$ and of the unlevered value of the firm thus become:

$$dS_t = S_t \left[ (r_t + \sigma S \lambda_s) dt + \sigma S dz^S_t \right]$$

$$dV_t = V_t [r_t dt + \sigma_V dz_t]$$

Since there are now four independent sources of risk, the model can be rewritten in vector form using a four-dimensional Brownian motion $z$ and four volatility vectors:

$$d\Phi_t = \Phi_t \left[ \pi dt + \sigma_\phi dz^\phi_t \right]$$

$$dr_t = a(b - r_t) dt + \sigma r dz^r_t$$

$$dS_t = S_t \left[ (r_t + \sigma S \lambda_s) dt + \sigma S dz^S_t \right]$$

$$dV_t = V_t [r_t dt + \sigma_V dz_t]$$

These equations make the model identical to the model in Martellini and Milhau (2010), with an additional state variable, which is the process $V$. We let $\rho_\Phi$, $\rho_\Phi S$, $\rho_\Phi V$, $\rho_\Phi V$, and $\rho_V$ be the pairwise correlations of the Brownian motions driving the dynamics of the state variables.

To stay in a complete market, we must assume that inflation risk is traded. A perfect hedge against inflation risk can be achieved by investing in a zero-coupon bond that delivers the nominal payoff $\Phi_T$ at time $T$. Given the above dynamics, Martellini and Milhau (2010) show that the price of this bond is:

$$l(t, T) = \Phi_T e^{\alpha (T-t) r_t + \beta (T-t)}$$

with:

$$\alpha(T-t) = -\frac{1 - e^{-\alpha(T-t)}}{a}$$

$$\beta(T-t) = \left( \frac{\pi}{2} - \frac{\sigma_\phi^2}{2} - b \right) (T-t) + \frac{1 - e^{-\alpha(T-t)}}{a}$$

$$- \frac{\sigma r \sigma_\phi \rho_\phi}{\sigma} \left[ T-t - \frac{1 - e^{-\alpha(T-t)}}{a} \right]$$

$$+ \frac{\sigma^2}{2a^2} \left[ T-t - \frac{1 - e^{-2\alpha(T-t)}}{a} + \frac{\sigma_\phi^2}{2} \right]$$

\(^{26}\) Brennan and Xia (2002) enrich the model by assuming a stochastic, mean-reverting inflation rate.

\(^{27}\) If $\lambda$ is the long-term mean under the real-world probability measure and $\lambda$, the unit market price of interest rate risk, then we have that $b = b - \sigma^2 a$.\(^{27}\) The dynamics of the stock index $S$ and of the unlevered value of the firm thus become:
3. Extensions to the Static Model

with a volatility vector given by:

$$\sigma_j(t, T) = \sigma_{\Phi} + \alpha(T - t)\sigma_r$$

This indexed bond is the liability-hedging portfolio (LHP) here, since it delivers a payoff that is perfectly correlated with promised pension benefits. It is the true safe asset in this framework, so it will replace the cash in the fixed-mix strategy used by the pension fund. The dynamic of the wealth is now given by:

$$\frac{dA_t}{A_t} = r_t \, dt + [\omega \sigma_S + (1 - \omega)\sigma_j(t, T)]' \, dz_t$$

Since the volatility vector is deterministic and $r_t$ is Gaussian, $A$ is still a log-normal process, as it was in the base-case model of section 2. It is shown therein that the impact of a given allocation policy on the sponsor and its pension plan depends on the value of the instantaneous correlation of $A$ and $V$. Now, this correlation is driven by two parameters. The first, denoted $\rho_{S\Phi}$, is the correlation of $S$ and $V$, and the second, denoted $\rho_{V\Phi}$, the correlation of $V$ and $\Phi$.

### 3.2.2 Unconditional Indexation

We first consider the case of pension benefits that are fully indexed to inflation. That is, the pension fund promises the payment $L_{\Phi T}$. All actual payoffs to bondholders remain the same as in the base model (see subsection 2.1.2), with the face value $L$ replaced by $L_{\Phi T}$ wherever needed. Having decomposed $L_T$ as $L_1^T + (1 - \gamma)C_T$, one can write down the new definition of the spread $s_L$:

$$\mathbb{E}^Q \left[ e^{-\int_0^T (r_t + s_t) \, dt} L_{\Phi T} \right] = L_0^1$$

or equivalently:

$$s_L = -\frac{1}{T} \ln \frac{L_0^1}{L(0, T)}$$

### Numerical Results

The base-case parameter values for the numerical analysis are given in table 3. They have been obtained from quarterly US data on the S&P 500, US three-month T-bills and the US consumer price index over the period from 1953 to 2009, except for the speed of mean reversion $a$ and the volatility $\sigma_r$, which have been set at a respective 20% and 1.5% to reduce the likelihood of simulating negative values for the nominal rate.

In each figure, we show the static analysis in the case of unconditionally indexed pension benefits (panels (a)) as well as in the case of a promised pension benefit equal to $L$. We do so for comparison purposes (because of the many differences in the underlying model and in parameter values, the prices that we obtain here are not directly comparable to those reported for the base model in subsection 2.2). A comparison of panels (a) and (b) in figures 25 and 27 reveals that two effects are at work. First, the promised payoff is larger as a result of indexation to inflation. As a consequence, the prices of the claims held by pensioners are higher than in the case of a nominal contract. Second, a larger promised payoff also means that it is harder for the pension fund to be solvent. The outcome is a credit spread for pension liabilities larger than when benefits are fixed in nominal terms. But because the promised payment to pensioners is larger and the pension fund must rely more often on an exceptional contribution from the sponsor to make it, there is in the end less money left for equity-holders, and even a greater probability that they receive nothing at all. As a result, equity prices are adversely affected, as can be seen by comparing panels (a) and (b) in figures 26 and 28.
3. Extensions to the Static Model

Impact of Correlations with Inflation
Intuition suggests that an increase in the correlation parameters $\rho_{V\Phi}$ and $\rho_{S\Phi}$ will have a positive impact on fair liability value. Indeed, if the correlation of the unlevered asset value of the firm and the price index is high, then those states of the world in which realised inflation is high are also, in general, those in which the sponsor is wealthy. If the pension fund needs an exceptional contribution, it is then more likely that the sponsor can make it. This greater likelihood should result in an increase in fair liability value. The impact on equity value is less straightforward. In the base case, in which they have full access to surpluses, equityholders receive the remaining assets of the firm and pension fund, net of payments to pensioners and debtholders, plus the tax savings. But high correlation of firm value and inflation does not mean that greater surpluses can be expected. In fact, a closer examination of equity payoff (2.7) shows that equities contain an embedded exchange option between the payoffs $A_T + V_T$ and $L\Phi_T + D$. If the correlation of $V$ and $\Phi$ increases, the volatility of the difference between these two payoffs is, all else being equal, expected to decrease. Since an exchange option has positive vega, an increase in the correlation parameter $\rho_{V\Phi}$ should have a negative impact on equity. A rise in the correlation parameter $\rho_{S\Phi}$ can be expected to have a similar effect. It will increase the correlation of the terminal wealth of the pension fund and the promised payment to pensioners, so the pension fund is more likely to deliver this payment without requiring any contribution from its sponsor. On the other hand, the volatility of the difference $A_T - L\Phi_T$ will be reduced, which has a negative impact on equity.

Figures 29 and 30 show the impact on fair liability value and equity of raising the correlation parameters $\rho_{V\Phi}$ and $\rho_{S\Phi}$ from $-70\%$ to $70\%$. As expected, we find that a positive shock to these correlations has a positive impact on liability and a negative impact on equity—although in neither case does this impact exceed an absolute value of $3\%$. That the impact is only slight may be explained by the fact that, in our simulations, the maximum realised inflation over the ten-year period is $85.2\%$. In unreported results, we have set the volatility of inflation to $5\%$, a setting that magnifies the effects of the correlation parameters.

3.2.3 Conditional Indexation
In most real situations, indexation is granted only if the pension fund is sufficiently funded to make the nominal promised payment plus inflation. The minimum amount promised to pensioners is still equal to $L$, and indexation is granted only if the final real funding ratio is higher than some threshold $f$, which we shall take equal to $100\%$ in the base case.

Payoffs to Stakeholders
The situation at time $T$ when equity-holders have full access to surpluses ($\gamma = 1$) and no tax is applied to negative contributions ($\theta_{eff} = 0$) can be described as follows:

I. If $A_T + V_T \geq L + D$:
1. If $A_T \geq L$ and $V_T \geq D$, then pensioners receive $L$ and perhaps inflation. Debt is paid back to bondholders, and equity-holders receive the remaining assets of the firm and the pension fund, plus the tax savings from interest payments, that is, $E_T = A_T + V_T - L - D + \theta(D - D_p);$
3. Extensions to the Static Model

2. If \( A_T \geq L \) and \( V_T < D \), then pensioners again receive \( L \) plus inflation if the real funding ratio permits. The total asset left to equity-holders is thus \( A_T + V_T - L_T \). If it is larger than \( D \), then default can be avoided, bondholders receive \( D \), and equity-holders receive the remaining asset net of \( L_T \) and \( D \), plus the tax shield. If \( A_T + V_T - L_T > D \), then the firm's equityholders trigger bankruptcy. They receive nothing, whereas bondholders get a recovery payment equal to \( (1 - \alpha)V_T + A_T - L_T \).

3. If \( A_T < L \) and \( V_T \geq D \), the sponsor makes a final contribution \((L - A_T)\) to the pension fund, so pensioners receive \( L \). Corporate bonds are redeemed, and equity-holders receive \( A_T + V_T - L_T - D + (D - DP) + (L - A_T) \), where the last term corresponds to the tax saving on the terminal contribution.

II. If \( A_T + V_T < L + D \):

1. If \( A_T < L \), then the pension fund needs a contribution from the sponsor, but the sponsor cannot make up for the deficit. Then the firm defaults, which entails a loss \( V_T \) to third parties in the bankruptcy procedure. The proceeds of the liquidation are shared by pensioners, who receive \( q = \frac{L - A_T}{D + L - A_T} \) of the asset net of bankruptcy costs, and bondholders, who receive \( 1 - q \). Equity-holders receive nothing;

2. If \( A_T \geq L \), then pensioners receive \( L \) plus possible indexation to inflation. The remaining aggregate asset of the firm and pension fund, \( A_T + V_T - L_T \), is still less than \( D \), so the firm is unable to pay back its debt. It is then liquidated, and pensioners receive \((1 - \alpha)V_T + A_T - L_T \).

Therefore, conditional indexation to inflation is a form of surplus sharing: pensioners are promised a nominal amount \( L \), but if the portfolio held by the pension fund performs sufficiently well, they receive in fact \( L \) plus the realised inflation. If the indexation threshold \( b \) rises to infinity, then the payoffs are the same as when pensioners are promised a fixed nominal amount (see section 2). The credit spread \( s_L \) is defined as:

\[
s_L = -\frac{1}{T} \ln \frac{L_0^1}{LB(0, T)}
\]

where \( L_0^1 \) is the present value of the payoff \( \min(L_T, L) \). Since the minimum capital promised to pensioners is the nominal amount \( L \), the risk-free asset is the nominal zero-coupon bond delivering \( L \) at time \( T \). We consider fixed-mix strategies in which the weight \( \omega \) is allocated to the stock index, with the remaining fraction of wealth allocated to the nominal bond.

Numerical Analysis

Figure 31 shows the probability, as a function of the allocation to the stock index, of pensioners' receiving the promised nominal payment \( L \) plus inflation. For high initial funding, this probability appears to be a decreasing function of the weight allocated to the stock index: indeed, if the initial funding ratio is high, investing more in the stock index leads only to an increase in the volatility of the terminal funding ratio, hence to a lower probability of being able to deliver the indexed payout. On the other hand, for low funding levels, a riskier strategy can lead to a greater probability of enjoying high funding ratios. Indeed, if the allocation is tilted towards nominal bonds, the funding ratio computed with respect to the nominal payment \( L \) stays close to its initial low value,
and it is unlikely that it will end up high enough to permit indexation. Increasing the allocation to the stock index leads to better performance, which in turn increases the probability of indexing the payout.

In figures 32 and 33, we study the impact of allocation decisions on fair liability value and equity value in the presence of conditional indexation. The first observation is that pensioners’ claims have a higher value than they do when pensioners are promised only a nominal payment, but the increase in value is not as great as it would have been had indexation been unconditional (see panels (a) in figures 25 and 32). Figure 32 also shows that the allocation to the stock index that maximises the value of the claim held by pensioners is greater when indexation is conditional than when it is unconditional. This optimal allocation can be compared to the allocation that maximises equity value (see figure 33), and which is always equal to 100%. If promised pension benefits were fixed in nominal terms, maximising equity would call for investing only in the stock index, but the allocation that maximises liability value would be closer to zero than it is with conditional indexation. Hence, introducing conditional indexation makes it possible to reduce the conflicts of interest (over the riskiness of the pension fund’s investments) between pensioners and equity-holders. This effect is similar to that obtained by granting partial access to a pension fund’s surpluses by the means of a lower coefficient γ (see subsection 2.2).

3.3 Internal vs. External Funding

In this section we modify the actual payoffs to model a situation in which the pension fund is held on balance sheet. In this case, the pension fund assets and liabilities are fully incorporated into the balance sheet of the sponsor. As a result, there is a single asset, \( A_T + V_T \), and a single liability, \( L + D \). In this framework, there is no notion of surplus sharing. All pension fund surpluses, if any, are returned to equity-holders (this is also the case when we assume \( \gamma = 1 \) in the off-balance-sheet model), who are then responsible for paying back corporate debt.

3.3.1 Payoffs to Stakeholders

In case the entity is wealthy enough to pay back all liabilities \( A_T + V_T \geq L + D \), the actual payoffs to pensioners and debtholders equal the promised payoffs, and equity-holders receive the share of assets in excess of these promised values. In “bad” states of the world, the available assets are insufficient to make the promised payoffs. The firm is liquidated, and the global asset net of liquidation costs is shared by pensioners and bondholders, proportionally to the payoffs that they were promised. Hence a strict equal seniority rule is enforced here. Formally, the payoffs can be written as:

\[
L_T = L \mathbb{1}_{\{A_T + V_T \geq L + D\}} + [A_T + (1 - \alpha) V_T] \frac{L}{D + L} \mathbb{1}_{\{A_T + V_T < L + D\}}
\]

\[
D_T = D \mathbb{1}_{\{A_T + V_T \geq L + D\}} + [A_T + (1 - \alpha) V_T] \frac{D}{D + L} \mathbb{1}_{\{A_T + V_T < L + D\}}
\]

\[
E_T = [A_T + V_T - L - D + \theta (D - D_p)] \mathbb{1}_{\{A_T + V_T \geq L + D\}}
\]

(3.17)

As in the off-balance-sheet case, we have the following accounting equality:

\[
L_T + D_T + E_T = A_T + V_T + TS_T - BC_T
\]
where:
\[ BC_T = \alpha V_T I_{\{A_T + V_T < L + D\}} \]
\[ TS_T = \theta (D - D_p) I_{\{A_T + V_T \geq L + D\}} \]

As a consequence, the total value of the firm and the pension fund can still be computed as \( v_0 = L_0 + D_0 + E_0 \) or as \( A_0 + V_0 + TS_0 - BC_0 \).

The payoffs (3.17) are the same as in the model of internal funding of Scherer (2005), up to the introduction of frictions (non-zero bankruptcy costs associated with the default of the firm and the tax shield). In "good" states of the world, where \( A_T + V_T \geq L + D \), the payoffs are exactly the same as in the base-case model described in section 2, with a coefficient \( \gamma \) equal to 1 (surpluses go to equity-holders only). However, when "things go wrong" \( A_T + V_T < L + D \), pensioners have the same seniority as bondholders, and they receive less than they would if the pension fund were held off of the balance sheet. Indeed, in the off-balance-sheet case they receive an amount at least equal to \( A_T + q(1 - \alpha)VT \) where \( q = \frac{L - A_T}{D + L - A_T} \).

With internal funding, they receive only
\[ [A_T + (1 - \alpha)VT] \frac{L}{D + L} \]

Comparing these payoffs, we obtain that:
\[ A_T + q(1 - \alpha)VT - [A_T + (1 - \alpha)VT] \frac{L}{D + L} = \frac{A_T}{L + D} \frac{D + L - A_T - (1 - \alpha)VT}{D + L - A_T} \]

which is positive when \( D + L > A_T + V_T \). This result formally shows that internal funding rules are less favourable to pensioners than external funding rules when the aggregate asset of the firm and pension fund is less than the aggregate debt to bondholders and pensioners. The numerical analysis below will confirm that pension claims are worth less under internal funding.

The impact of internal funding on corporate debt depends on two factors. The first is the surplus-sharing rule enforced under external funding. In good states of the world (where \( A_T + V_T \geq L + D \)), corporate debt will be fully paid back to bondholders under internal funding, whereas it may not be under external funding, if equity-holders do not have full access to surpluses. If the surplus-sharing rule allows them to receive all the surpluses, bondholders are treated equally under internal and external funding, as long as \( A_T + V_T \geq L + D \). The second parameter is the seniority rule applied in bad states of the world (where \( A_T + V_T < L + D \)). Under internal funding, bondholders always receive
\[ \frac{D}{D + L} [A_T + (1 - \alpha)VT] \]
while they receive either
\[ \frac{D}{D + L - A_T} (1 - \alpha)VT \]
or
\[ (1 - \alpha)VT + A_T - L \]
under external funding if they have indirect access to the pension fund’s surpluses. Comparing these payoffs, it can be seen that the recovery payment to bondholders is greater under internal funding than under external funding. Therefore, one can expect

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3. Extensions to the Static Model

An unambiguous positive effect of internal funding on corporate bond prices. This effect will be even greater if the surplus-sharing rule enforced under the external regime prohibits negative contributions from the pension fund to the firm, because, in that case, internal funding will make default much less likely.

As far as equity-holders are concerned, the impact of holding the pension fund on the balance sheet is straightforward if they have access to surpluses in the external funding regime of reference. Since there are no longer contributions from the sponsor to the pension plan, they will lose the benefit of the tax shield from terminal contributions. So a negative impact of internal funding on equity prices should be expected. When surpluses go to pensioners in the external regime, the situation is more complex. On the one hand, the loss of the tax shield still tends to have an adverse impact on equity value. On the other, having an internal pension fund prevents bankruptcy when \( A_T + V_T \geq L + D \), although default may still occur under external funding when \( A_T + V_T \geq L + D \) but the firm is insolvent \( (V_T < D) \) and the surplus-sharing rule prevents equity-holders from accessing the totality of surpluses. With a surplus-sharing rule favourable to equity-holders, the default probability when \( A_T + V_T \geq L + D \) is already zero, so internal funding never improves the payout to equity-holders. This predicted impact is the same as in Scherer (2005), and the numerical analysis that we present below provides quantitative estimates of its magnitude.

3.3.2 Numerical Analysis

Figures 34 to 37 show the relative change in each claim value when the firm chooses internal rather than external funding. This change is measured for different initial funding ratios and different values of the real promised payment to pensioners, \( L \). We have also considered two reference situations. The first is an external funding regime in which surpluses go to equity-holders \( (\gamma = 1) \); the second is an external funding regime in which surpluses go to pensioners \( (\gamma = 0) \). As expected, other things being equal, pension claims are lower in the case of internal funding than in the case of external funding. When the reference situation is \( \gamma = 1 \), the decrease in value is greater for large promised payments. Indeed, a larger \( L \) increases the probability that the final aggregate asset \( A_T + V_T \) will be less than \( L + D \); that is, it increases the probability of being in a situation in which pensioners will get less than in the case of external funding. We also verify that switching from external to internal funding increases the value of corporate bonds. This is especially true if the reference situation is \( \gamma = 0 \), since bankruptcy is much less frequent under internal than under external funding. The increase in value is less spectacular if internal funding is compared to an external funding situation in which equity-holders have access to surpluses, but it is still significant. It is greater when promised pension benefits are high, because, in these situations, it is likely that the pension fund will be insolvent and the sponsor will be unable to make up for the deficit. These are the cases in which bondholders will enjoy the benefits of a larger recovery payment. Figure 35 shows that, as expected, internal funding has a negative impact on the value of equity when equity-holders have access to surpluses in the external regime of reference. The magnitude of this impact is far from negligible for low funding levels: it is in these very situations...
3. Extensions to the Static Model

that equity-holders would have enjoyed the tax shield on exceptional contributions from the sponsor to the pension fund if the pension plan were being held off of the balance sheet. For higher funding levels or for smaller promised pension benefits (or for both), such an exceptional contribution is very unlikely, so the present value of the associated tax savings is close to zero, and there is almost no difference between the two situations. When the reference situation is $\gamma=0$, the picture is completely different. In general, equity-holders will benefit from internal funding because they were highly penalised by the surplus-sharing rule under the external regime, especially for high funding levels or for large promised pension benefits. For low funding, the negative effect induced by the loss of the tax shield on contributions dominates, and equity value is lower under internal funding.

When it comes to the impact of internal funding on the total value of the firm and the pension fund, the same two factors that drive the impact on equity value must be taken into account. The first is the loss of the tax shield on exceptional contributions made by the sponsor, and the second is the possible decrease in bankruptcy costs that follows from the decrease in the probability of default. When equity-holders have access to surpluses in the external regime, only the first impact is present, and it results in a decrease of the total value. Like equity value, the total value is the same under external and internal funding when the pension fund is highly funded or promised pension benefits are low, because, in these situations, the tax shield and the bankruptcy costs are the same under both regimes. When pensioners have access to surpluses under external funding, the two factors are in competition. The first dominates only for low funding levels, as a result of which exceptional contributions are most likely to be needed. For higher funding levels, internal funding has a positive global impact.
4. Extending the Model to an Intertemporal Setting
This section develops intertemporal extensions to the base-case model, which was purely static. We first relax the assumption of fixed-mix strategies to analyse dynamic risk-controlled strategies. We then extend the model to account for the presence of a random default date of the sponsor, as well as minimum funding requirements.

4.1 Dynamic Risk-Controlled Strategies

We have so far considered fixed-mix asset allocation strategies. It has been shown, however, that some particular forms of risk-controlled state-dependent strategies would be utility-maximising in the presence of funding ratio constraints (Martellini and Milhau 2010). In a related effort, Bodie (1990a) discussed the benefit of contingent immunisation for an overfunded pension plan. These papers, however, have addressed the problem in a simplified setting in which the pension fund’s situation is analysed somewhat in isolation from the sponsor. In this section, we test for the impact of introducing dynamic risk-controlled strategies on stakeholder welfare. We focus on basic constant proportion portfolio insurance (CPPI) strategies, which of course have no particular reason to be optimal from a perspective of maximising total firm value (see section 5 for a discussion of more general strategies that can prove to generate more substantial welfare improvement).

4.1.1 Description of CPPI Strategies

The literature has considered two main families of risk-controlled strategies: constant proportion portfolio insurance (CPPI) strategies and option-based portfolio insurance (OBPI) strategies. CPPI strategies were introduced by Black and Jones (1987) and Black and Perold (1992), and their advantage over OBPI strategies is that they lead to a very simple and intuitive expression for the weight allocated to the risky asset: this weight is a multiple, denoted by $m$, of the outstanding risk budget, defined as the difference between current wealth and some minimum wealth known as the floor. The commitments of the pension fund suggest a simple form for the floor: since pensioners in this simplified model are promised a fixed payment $L$ at time $T$, we define the floor as the present value of the promised payment, $Le^{-r(T-t)}$. More generally, one can consider a floor equal to a fraction $k$ of the previous present value. The risk budget will then be a decreasing function of $k$, and the weight allocated to the risky asset will be:

$$\omega_t = m \frac{A_t - kL_t}{A_t} \quad (4.19)$$

where $m$ denotes the multiplier, and $k$ can be interpreted as some minimum funding ratio requirement. As shown in appendix A, such strategies can be rationalised in a reduced-form expected-utility framework, which also makes it possible to relate the multiplier to the risk aversion of the investor and to the Sharpe ratio of the risky asset. Further details, including an extension to the case of multiple asset classes, can be found in Basak (2002) and in Martellini and Milhau (2010). Having defined a strategy by (4.19), one can write the state-dependent terminal wealth achieved by an investor following this strategy, as shown in the proposition below.
4. Extending the Model to an Intertemporal Setting

Proposition 4.1 Assume that the interest rate \( r \) and volatility \( S \) are both constant, and that the pension fund follows the strategy (4.19). Terminal wealth is then given by:

\[
A_T = kL + (A_0 - kLe^{-rT}) \exp \left[ (1 - m) \left( r + \frac{m \sigma^2}{2} \right) T \right] \left( \frac{S_T}{S_0} \right)^m
\]

Proof. See appendix B.

In particular, if the initial condition

\[
A_0 \geq kLe^{-rT}
\]

holds, then the terminal funding ratio, \( A_T/L \), is greater than \( k \) almost surely, at least in the context of a continuous-time implementation. If \( k \) is taken equal to or greater than 1, the pension fund will be able to deliver the promised payment \( L \) in any state of the world, without requiring any unexpected contribution from the sponsor. If \( k \) is less than 1, implementing the CPPI strategy does not ensure that the funding ratio will end up greater than 1, but at least the deficit \( L - A_T \) will be bounded from above by \( (1 - k)L \). This assigns a limit to the size of the contribution that the sponsor could have to make in the event of a deficit.

In practice, the possible choice for \( k \) will be constrained by the initial contribution \( A_0 \). Condition (4.20) indeed implies that \( k \) cannot exceed \( A_0/(Le^{-rT}) \), or, put in terms of regulatory quantities, \( F_0^{req} e^{-sreqT} \). If \( A_0 \) is greater than the discounted promised payment to pensioners, \( k \) can be set at a level greater than 1. Of course, starting from such a funding level, the pension fund might also deliver the payment \( L \) with full certainty by investing in cash alone, but it would be unable to take advantage of the performance of the risky asset. The above risk-controlled strategy implemented with a minimum funding level \( k \) ensures that there will never be a deficit exceeding \( 1 - k \) but also opens access to some upside performance. If the initial endowment \( A_0 \) is less than the discounted promised payment, setting \( k \) to its maximum value cannot ensure that the final funding ratio will be greater than 1, but the size of the worst deficit will be limited to \( (1 - k)L \), whereas the upper bound would be \( L \) in the absence of risk management. The choice of the multiplier allows one to define the riskiness of the strategy: the larger the multiplier, the greater the volatility of the payoff. The case in which \( m \) is zero corresponds to an investment in the risk-free asset only, whereas the case in which \( m = 1 \) corresponds to a buy-and-hold strategy: starting from the initial weight \( A_0 \), one buys \( kLe^{-rT} \) shares of the risk-free asset and invests the remaining wealth \( A_0 - kLe^{-rT} \) in the stock index. Beyond these fixed-mix and buy-and-hold strategies recovered as special cases, any value for \( m \) strictly greater than 1 defines a truly dynamic risk-controlled strategy.

When this risk-controlled strategy is implemented with a minimum terminal funding ratio \( k \) at least as large as 1 and an initial funding compatible with (4.20), the pension fund ends up more than fully funded in all states of the world. Many simplifications in the payoffs to stakeholders (see equations (2.5), (2.6) and (2.7)) follow from this property. In particular, with our base-case parameter values (see table 1), equity-holders have access to surpluses (\( \gamma = 1 \)) and the sponsor pays no taxes on these surpluses (\( \theta_{eff} = 0 \)). Therefore, pensioners
4. Extending the Model to an Intertemporal Setting

receive the promised payoff:

\[ L_T = L \quad (4.21) \]

On the other hand, equity-holders hold a long position in a call written on \( A + V \), plus a long position in a digital option paying the tax savings on interest payments if the firm does not default on its debt:

\[ E_T = [A_T + V_T - L - D + \theta(D - D_p)] \mathbb{1}_{\{A_T + V_T \geq L + D\}} \]

Finally, the payoff to bondholders becomes:

\[ D_T = D_L \mathbb{1}_{\{A_T + V_T \geq L + D\}} + [(1 - \alpha)V_T + A_T - L] \mathbb{1}_{\{A_T + V_T < L + D\}} \]

4.1.2 Numerical Exercise

In this subsection, we conduct an empirical investigation of the impact of introducing a CPPI strategy on the prices of the claims. We take the minimum funding ratio \( k \) to be equal to 1. Given condition (4.20), this implies that the initial regulatory funding ratio must satisfy \( F_0^{\text{reg}} \geq e^{p_0 T} \). With our base-case values, this lower bound is equal to 1.11, so the pension fund must be more than fully funded in the regulatory sense. For each possible initial weight \( \omega_0 \), the multiplier \( m \) is chosen according to the following rule, which ensures that the initial allocation to the risky asset is between 0 and 1:

\[ m = \frac{\omega_0}{1 - \frac{k}{F_0^{\text{reg}}} e^{p_0 T}} \]

where \( F_0^{\text{reg}} \) is the initial regulatory funding ratio. We also limit the multiplier to values less than five, which is considered a maximum value by most practitioners. Thus, the set of possible multipliers depends on the initial funding level. For example, the highest multiplier is 3.8 for \( F_0^{\text{reg}} = 200\% \); it is approximately equal to 2.24 for \( F_0^{\text{reg}} = 150\% \); and is five for \( F_0^{\text{reg}} = 130\% \).

Unsurprisingly, figure 38 shows that pensioners are completely insensitive to the choice of multiplier, which is consistent with (4.21). Indeed, the pension fund delivers \( L \) in all states of the world, and since they have no access to surpluses, pensioners have no appetite for performance. On the other hand, figure 39 shows that, for high funding ratios of 150% and 200%, equity-holders are willing to increase the riskiness of the investment strategy by increasing the multiplier \( m \). This behaviour jibes with the insights obtained with the fixed-mix strategies (see figure 4) and is explained mainly by our base-case assumption that shareholders have (full) access to surpluses and that it is thus in their interest to engage in riskier investment strategies. The main insight from this analysis is that implementing risk-controlled strategies that attempt to ensure a minimum funding ratio allows shareholders some (limited) access to the upside performance of risky assets and, at the same time, ensures that pensioners will not be greatly hurt by the resulting increase in risk. Indeed, increasing the multiplier \( m \) has no impact on pensioners but, as a result of the access to the performance, it does lead to an increase in equity value. In the fixed-mix case, increasing the weight allocated to the risky asset also benefited equity-holders, but was in general detrimental to pensioners, except when there was a sufficiently negative correlation of the firm value process and the stock index value process. The effect of diversifying firm risk and market risk, which exists only for a negative correlation \( p \), is still

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29 - Even if the multiplier of the CPPI is set between these bounds, nothing guarantees that the weight allocated to stocks will remain between 0 and 1 over the investment period.

30 - The reason is that if the multiplier is set too high, a downwards jump in the price of the risky asset will lead to a sharp decrease in the value of the portfolio. If \( S \) suddenly falls by \( 1/m \), \( A \) falls to \( A_0 \), and the risk budget in (4.19) becomes zero. Then the entire wealth is invested in the cash, and the final performance of the strategy is just that of the cash. A multiplier of five thus corresponds to a 20% jump in the stock. This consumption of the entire risk budget can also occur if there is no jump risk, but continuous trading is impossible, and the risky asset loses \( 1/m \) in relative terms between two trading dates.
present, and it may account for the local minimum of $E_0$ that we observe around $m = 1$ for a funding ratio of 130%. As for the fixed-mix strategies, a riskier strategy leads to a decrease in bondholder welfare, but this side effect is less pronounced. The decrease can be seen in figure 5, which shows the relative change in $D_0$ defined as:

$$\frac{D_0(\text{CPP}) - D_0(\text{fixed-mix})}{D_0(\text{fixed-mix})}$$

When the correlation parameter $\rho$ is non-negative, bondholders’ claims are worth more than they are when the pension fund follows a fixed-mix strategy. From the aggregate perspective (see figure 41), an increase in the multiplier has ambiguous effects: taking more risk has a positive impact on the total value for a funding ratio of 150%, an impact that turns out to be negative for a funding ratio of 200%. Here again, the intuition is that the implicit opportunity cost of downside risk protection may not be worth paying when insurance against downside risk is highly likely not to be needed in the first place. However, if the multiple is set to its highest value (subject to the constraint of an initial allocation to the risky asset less than 100%), which maximises the access to surpluses for equity-holders, the total value of the firm is greater than if the pension fund were investing only in the risky asset.

### 4.2 A Fully Dynamic Capital Structure Model

As noted in the introduction, the literature has, for the most part, considered the valuation of pension liabilities in a simplified setting in which default is either assumed away or happens only at terminal date, as in Merton (1974). Although they provide useful insight, these papers cannot address such features as minimum funding ratio constraints. In this section we extend the previous model to account for the presence of intermediate payments, random default date of the sponsor, and minimum funding ratio constraints.

#### 4.2.1 Introducing Intermediate Payments and a Random Default Date

So far we have considered only zero-coupon debt contracts, promising fixed payments at the terminal date $T$ to pensioners and bondholders. In practice, pensioners receive periodic pension payments and bondholders receive coupon payments. It is straightforward to introduce these payments in the model. We assume that bondholders are promised a coupon payment $c$ to time $T$, plus the face value of debt $D$ at this date, whereas pensioners are promised a payment $l$ to time $T$, plus a final lump-sum payment $L$. The coupon and pension payments are assumed to be continuous for notational convenience, but accounting for discrete payments would involve no additional difficulties. The present value of the promised payments to pensioners is denoted by $L^P_t$, and is given by:

$$L^P_t = \int_t^T le^{-r(s-t)} \, ds + Le^{-r(T-t)}$$

It is to be distinguished from the fair value, denoted by $L$, of actual pension payments after incorporation of default risk as well as surplus sharing, the estimation of which is one of the focuses of the paper. An ad hoc way to account for the presence of
default risk in the valuation of the liability stream is to use an exogenously specified spread, as recommended by the regulator. This leads to the regulatory value of pension liabilities:

\[
L_t^p = \int_t^T e^{-r(s-t)} \, ds + Le^{-r(T-t)}
\]

Similarly, the present value of the promised payments to bondholders is:

\[
D_t^p = c \int_t^T e^{-r(s-t)} \, ds + De^{-r(T-t)}
\] (4.22)

Because of the presence of a stream of promised payments to pensioners, we also introduce a continuous stream of contractual contributions \( \kappa \), paid by the sponsor to the pension fund. For simplicity, \( \kappa \) is assumed constant, but the actual contribution may differ, given the presence of regulatory funding requirements, as will be explained below (see subsection 4.2.2).

The coupon payments to bondholders and the contributions to the pension fund will impact the firm’s unlevered value. This impact can be taken into account by introducing a payout rate \( \delta \) in the drift of \( V \). In the absence of funding ratio constraints, we would thus have:

\[
dV_t = V_t[(r - \delta) \, dt + \sigma \, dz_t^V]
\] (4.23)

where \( \delta \) is the dividend rate paid to equity-holders. The dynamics (4.23), however, will be slightly modified below (see subsection 4.2.2) to account for the existence of minimum funding constraints and contribution holidays.

We now assume that default is triggered when the unlevered value of the firm falls below a fraction \( \beta \in [0, 1] \) of \( D_t^p \):

\[
\tau = \inf \{ t \geq 0 ; \ V_t \leq \beta D_t^p \}
\] (4.24)

This notion of default is consistent with that of Briys and de Varenne (1997), who define the default date as the first time unlevered value falls below a fraction of the present value of face value of debt.

Given that the pension fund assets are not held on the sponsor company balance sheet, they have no direct impact on the definition of default. Conversely, the presence of default risk will obviously have an impact on the liability value. Indeed, early default by the sponsor will force liquidation of the pension fund before terminal date \( T \).

4.2.2 Introducing Minimum Funding Ratio Constraints

In the United States, the Employment Retirement Income Security Act (ERISA), passed in 1974 and subsequently complemented by several rounds of related legislation, establishes a system of mandatory pension contributions when the deficit of the pension plan exceeds given threshold values. The presence of minimum funding ratio constraints at intermediate dates, which exist in most developed countries (Pugh 2003), was
not accounted for in the previous model, which allowed only for the presence of contributions at the initial and terminal dates. We now take into account such required contributions, which increase the probability of default on the non-pension debt and depress capital investment by the sponsor (Rauh 2006), as a potentially important ingredient of the model. Our objective is not to derive the optimal dynamic contribution strategy, a question that is arguably difficult to address given the complexity of the underlying model, but to analyse a strategy consisting of contributing only if, as required by regulation, minimum funding ratios are breached.

We now describe a series of contributions inspired by the regulatory practice of checking the funding status of the pension fund at predefined dates $0 < t_1 < \ldots < t_N$. We assume that there is a regulatory minimum floor whose value at time $t$ is denoted by $F_t$. If the assets of the pension fund just before time $t_i$, $A_{t_i-}$, are found to be lower than the minimum funding $F_{t_i}$, the sponsor is called on to contribute. This contribution is equal to a fraction $a_1$ of the current deficit: $a_1(F_{t_i} - A_{t_i-})^+$. Taking $a_1 = 1$ implies that the sponsor should make up for the entire deficit immediately, while taking $a_1 < 1$ allows partial recovery.33

On the other hand, the contractual contribution can be decreased when the pension fund enjoys sufficiently large surpluses. By large surpluses, we mean that $A_{t_i-} > G_{t_i-}$, where $G$ is some stochastic adapted process such that $G > F$. In this case, the sponsor benefits from what is known as a contribution holiday: the normal contribution stream, denoted by $\kappa dt$, is decreased by $a_2(A_{t_i-} - G_{t_i-})^+$. There is a limit to the amount of over-funding in DB plans, which was formalised in 1988 to be such that no additional contribution can be made to the pension fund if the assets are in excess of 150% of termination benefits (Ippolito 2001, 2002). However, the overall contribution from the sponsor plan to its pension fund must be non-negative, because contributions are essentially irreversible. As a consequence, the contribution holiday lasts only as long as the overall instantaneous contribution $[\kappa - a_2(A_{t_i-} - G_{t_i-})]_+$ is non-negative. Finally, we allow a fraction $a_3$ of the assets in excess of $F_{t_i-}$ to go to pensioners if $A_{t_i-}$ exceeds some level $H_{t_i}$, where $H$ is an adapted stochastic process such that $H < F$. This covenant implies that pensioners have partial access to large pension fund surpluses. In what follows, we define $F$, $G$, and $H$ as proportions of the regulatory liability value $L^{\text{reg}}$.

\[
F_t = kL^{\text{reg}}_t, \quad G_t = k' L^{\text{reg}}_t, \quad H_t = k'' L^{\text{reg}}_t
\]

with $k \leq k' \leq k''$. In particular, these processes are continuous, so that:

\[
F_{t_i-} = F_{t_i}, \quad G_{t_i-} = G_{t_i}, \quad H_{t_i-} = H_{t_i}.
\]

The previous covenants imply that the pension fund asset value $A$ evolves as:

\[
dA_t = rA_t \, dt + A_t \sigma_\Delta \sigma_\Delta \, dz_t^\Delta - I \, dt
\]

\[
+ \sum_{i=1}^N a_1(kL^{\text{reg}}_t - A_{t_i-})^+ \, dM_i^t
\]

\[
- \sum_{i=1}^N a_2(A_{t_i-} - kL^{\text{reg}}_t)^+ \mathbb{1}_{\{A_{t_i-} > kL^{\text{reg}}_t\}} \, dM_i^t
\]

\[
+ \left[ \sum_{i=1}^N (\kappa - a_2(A_{t_i-} - kL^{\text{reg}}_t)^+) \mathbb{1}_{\{t \leq t_i \leq t_{i+1}\}} \right] \, dt
\]

(4.25)
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where \( dM^t \) is the Dirac measure at time \( t \):

\[
dM^t_s = \begin{cases} 
1 & \text{if } s = t \\
0 & \text{otherwise} 
\end{cases}
\]

For further use, we let \( \kappa_t dt \) be the overall continuous contribution rate:

\[
\kappa_t = \sum_{i=1}^{N} \left( \kappa - \sigma_2(A_{t_{i-1}} - k^* L^V_{t_{i-1}})^+ \right) \mathbb{1}_{\{t_i < t \leq t_{i+1}\}}
\]

and let \( dK_t \) be the total contribution at time \( t \):

\[
dK_t = \kappa_t dt + \sum_{i=1}^{N} \sigma_1(kL^V_{t_{i-1}} - A_{t_{i-1}})^+dM^t_i
\]

which may differ from \( \kappa dt \) if the sponsor enjoys a contribution holiday or if some exceptional contribution takes place.

The dynamics of \( A \) in our model will, in general, differ from the dynamics of financial wealth in papers that consider asset allocation with labour income (Bodie, Merton, and Samuelson 1992; Munk and Sørensen 2010) because of the obligation for the sponsor to contribute \( (a_1 \neq 0) \) and the existence of contribution holidays \( (a_2 \neq 0) \). Moreover, the asset \( A_t \) may fall below the regulatory floor \( F_t \) between two consecutive checking dates \( t_i \) and \( t_{i+1} \).

On the whole, the dynamics of the unlevered value of the firm are given by a modified version of (4.23) in which we have accounted for exceptional contributions:

\[
dV_t = V_t[(r - \delta) dt + \sigma_V dz^V_t] - \sum_{i=1}^{N} \sigma_1(kL^V_{t_{i-1}} - A_{t_{i-1}})^+dM^t_i
\]

with \( \delta V_t = c + \kappa_t + d_t \). Like \( A, V \) is a right-continuous process and can exhibit discontinuities only at points \( t_i \).

4.2.3 Payoffs to Stakeholders

We now provide a detailed analysis of the actual payoff accruing to each stakeholder. The last payments to each group of stakeholders (pensioners, equity-holders, and bondholders) take place at time \( \tau_1 = \tau \wedge T \), and they are contingent on the values of \( A \) and \( V \) just before \( \tau_1 \). The pension fund is committed to paying the amount \( L \) at date \( T \) to pensioners. Hence a pension contract is a collateralised form of debt held by the employees and pensioners of the firm, with the pension fund assets serving as collateral. If the pension fund is insolvent at terminal date \( (i.e., if A_T < L) \) or at default date \( (i.e., if A_T < L^V_T) \), the assets of the firm are used to compensate pensioners. Of course, these assets may not be sufficiently high to cover the deficit of the pension plan, in which case the firm is technically in default (default triggered by the pensioners).

As in section 2, we assume the presence of a rule by which pensioners and equity-holders share the surplus if the firm has not defaulted before time \( T \); if the pension fund enjoys a surplus at time \( T \), pensioners receive a contractual fraction \( 1 - \gamma \) of this surplus, whereas the remainder, \( \gamma \), goes to equity-holders. In the first case the entire surplus goes to the beneficiaries of the pension plan in the form of enhanced pension benefits. On the other hand, when \( \gamma = 1 \) (which we treat as our base case), shareholders have full access to any (after-tax) pension fund surplus (see below for the tax treatment of pension...
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reversion). More generally, equity-holders are responsible for debt payment, using the terminal unlevered value of the firm $V_f$ plus a fraction of any pension plan surplus. In practice, however, surpluses that reverted to the sponsor are subject to a special tax regime, as described in subsection 2.1.2. As in section 2, we let $\theta$ be the normal corporate tax rate, $\theta_{\text{rev}}$ the reversion tax rate, and $\theta_{\text{eff}}$ the effective tax rate on surpluses. In our base case we will assume that reverted surpluses are not subject to taxation, so $\theta_{\text{eff}}$ will be set to zero.

The amount remaining after debt payment, if strictly positive, goes to equity-holders. Default is triggered by bondholders when the actual payment made to them is less than the promised payment. Since the recovery payment in the event of default may include a fraction of the pension fund surplus (assuming $\gamma > 0$), it can be said that bondholders have a conditional and limited claim on pension surpluses.

We thus have the following covenants:

1. If $\tau > T$: then we have that $\tau_1 = T$. We assume that, in this case, any pension fund surplus, is shared by equity-holders and pensioners before debt repayment. The shareholders receive the fraction $\gamma$ of the surplus and the the pensioners the fraction $1 - \gamma$. If the sum of firm assets and of the fraction $\gamma$ of the surplus is insufficient to repay corporate debt, equity-holders must cede their access to the surplus to bondholders. That is, in the event of bankruptcy, overfunded pension plans can be terminated, with the excess assets distributed to the creditors (Ippolito 2001, footnote 10). The payoffs to the claimholders in this situation are identical to the payoffs in the base model (see subsection 2.1.2);

2. If $\tau \leq T$: we then have that $\tau_1 = \tau$. The firm is liquidated and equity-holders receive nothing. By definition of $\tau$ and by right-continuity of $V$ and $D^p$, we have that $V_f \leq \beta D^p \leq D^p$. In this case, we assume that equity-holders receive nothing, whatever the value of $\gamma$ and that any pension fund surplus is shared by bondholders and pensioners.

(a) If $A_{\tau-} \geq L^p_f$: bondholders receive $D_{\tau_1} = \min(D^p_{\tau_1}, A_{\tau-} - L^p_f + (1 - \alpha)V_{\tau-})$ and pensioners receive the remaining assets of the pension fund $L_{\tau_1} = A_{\tau-} - \min(D^p_{\tau_1}, (1 - \alpha)V_{\tau-}, A_{\tau-} - L^p_f)$

(b) If $A_{\tau-} < L^p_f$: pensioners receive $(1 - q)(1 - \alpha)V_{\tau-}$ and bondholders receive $A_{\tau-} + q(1 - \alpha)V_{\tau-}$, where $q = \frac{L^p_f - A_{\tau-}}{D_T + L^p_f - A_{\tau-}}$.

4.2.4 Fair Values of the Claims and Total Value of the Firm

Having written the payoffs received by each group of claimholders, we can write their present values and define the total value of the firm and the pension fund. Bondholders receive the coupon payment to time $\tau_1$, plus a terminal payment $D_{\tau_1}$, so the present value of the claim that they hold is:

$$D_0 = \mathbb{E}^Q \left[ \int_0^{\tau_1} e^{-rt} c \, dt \right] + \mathbb{E}^Q \left[ e^{-r\tau_1} D_{\tau_1} \right]$$

(4.27)

and the corresponding credit spread $s_D$ is implicitly defined by the relation:

$$D_0 = c \int_0^T e^{-(r+s_D)T} \, ds + De^{-(r+s_D)T}$$

It is straightforward to show that $D_0$ is smaller than $D^p$ as defined in equation (4.22), so the previous definition ensures that $s_D$ is indeed non-negative.
On the pension fund side, the present value of actual pension payments is:

\[
L_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} dL_t \right] + \mathbb{E}^Q \left[ \sum_{i=1}^N a_i e^{-r(t-i)} (A_{t-i} - F_k) \mathbb{1}_{\{A_{t-i} > F_k \}} \mathbb{1}_{\{t-i < t\}} \right] + \mathbb{E}^Q \left[ L_0 e^{-rt} \right]
\]

To define a credit spread for the claim held by pensioners, we decompose the present value of this claim, as given in (4.28), into two terms: the first, \( L_0^1 \), is the present value of the actual payments, excluding access to pension fund surpluses, whereas the second, \( L_0^2 \), is the present value of the access to these surpluses. We have that:

\[
L_0^1 = \mathbb{E}^Q \left[ \int_0^{T_1} e^{-rt} dL_t \right] + \mathbb{E}^Q \left[ e^{-rT_1} \min(L_{T_1}, L_{R_1}) \right]
\]

\[
L_0^2 = \mathbb{E}^Q \left[ \sum_{i=1}^N a_i e^{-r(t-i)} (A_{t-i} - F_k) \mathbb{1}_{\{A_{t-i} > F_k \}} \mathbb{1}_{\{t-i < t\}} \right] + \mathbb{E}^Q \left[ e^{-rT_1} (L_{T_1} - L_{R_1})^{+} \right]
\]

The credit spread \( s_L \) is then the value of the regulatory spread that would be needed to make the regulatory value equal to the fair value:

\[
L_0^1 = l \int_0^T e^{-(r+s_L)t} dt + Le^{-(r+s_L)T} + \mathbb{E}^Q \left[ e^{-rT_1} L_{T_1}^{+} \right]
\]

Again, it can be verified that \( s_L \) is a non-negative quantity. Equity-holders, for their part, receive the dividend payment \( \delta V_t - \kappa_t - c \) dt to time 1 and the payoff \( E_{T_1} \) at time \( T_1 \). Hence the fair value of equity is:

\[
E_0 = \mathbb{E}^Q \left[ \int_0^{T_1} e^{-r(t-u)} (\delta V_t - (1 - \theta)\kappa_t + c) du \right]
\]

\[
+ \mathbb{E}^Q \left[ \sum_{i=1}^N a_i (k_{t+i} e^{r(i+1)} - A_{t+i})^+ \right] + \mathbb{E}^Q \left[ E_{T_1} e^{-rT_1} \right] (4.29)
\]

The total value of the firm and the pension fund is:

\[
v_0 = L_0 + D_0 + E_0 \quad (4.30)
\]

This is the aggregate fair value of the pensioners’, bondholders’, and equity-holders’ claims to firm and pension fund assets. Intuitively, this should also be the aggregate value of operating assets and pension fund assets, augmented by the tax shield and decreased by the present value of bankruptcy costs and other tax payments. This property indeed holds here, as will be shown in the next proposition. Before formulating this result, we introduce a few notations. The present value of bankruptcy costs is defined by:

\[
BC_0 = \mathbb{E}^Q \left[ e^{-rT_1} BC_{T_1} \right]
\]

where \( BC_{T_1} \) is the amount lost to bankruptcy procedure at time 1, namely:

\[
BC_{T_1} = \alpha V_t \mathbb{1}_{\{t \leq T_1\}} + \alpha V_T \mathbb{1}_{\{T_1 > T\}} \left( \mathbb{1}_{\{T_1 + V_T \leq L + D, V_T + \gamma (1 - \theta d)}} + \mathbb{1}_{\{T_1 + V_T > L + D\}} \right)
\]

The tax shield is defined as the present value of future tax savings from contributions and coupon payments, plus the tax savings from the initial contribution:

\[
TS_0 = \theta A_0 + \theta \mathbb{E}^Q \left[ \int_0^{T_1} e^{-r(t-u)} (c + \kappa_t) du \right] + \mathbb{E}^Q \left[ e^{-rT_1} TS_{T_1} \right]
\]

where \( TS_{T_1} \) is the tax savings corresponding to the last contribution, if any, made at time \( T_1 \):

\[
TS_{T_1} = \theta (L - A_T)^+ \mathbb{1}_{\{T_1 \geq T\}} \mathbb{1}_{\{A_T + V_T \leq L + D, V_T + \gamma \}}
\]

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Finally, the present value of the tax reversion is given by:

$$RT_0 = E^Q [e^{-r^Q} R_{T_1}]$$

where:

$$R_{T_1} = \gamma \theta_{eff} 1_{\{r > T\}} (A_T - L)^+$$

**Proposition 4.2** The total value of the firm satisfies:

$$v_0 = (1 - \theta) A_0 + V_0 + TS_0 - BC_0 - RT_0$$

**Proof.** See appendix C.

Because of the presence of minimum funding ratio constraints and the intermediate payments, there is no closed-form expression for the prices. In the numerical exercise of the next subsection, we have thus run Monte-Carlo simulations to estimate these prices.

### 4.2.5 Numerical Analysis

In the numerical application, we choose to focus on the impact of short-term funding constraints and consider a simplified version of the model, in which there is no intermediate payment to claimholders, no contractual contribution stream $\kappa$, and no surplus sharing at intermediate dates. We assume that the minimum funding ratio is $k_1 = 100\%$, the funding ratio is checked every year, and the sponsor has to cover a fraction $a_1 = 1/3$ of the difference between the actual value of pension assets and the regulatory floor. These assumptions are those that prevail in the Netherlands (Pugh 2003), where the sponsor is given three years to make up for a deficit. We impose no protective covenant on debt: that is, the parameter $\beta$ in (4.24) is set to 0. As a consequence, default can be triggered before time $T$ only at a checking date at which the pension fund posts a deficit and the sponsor cannot afford to make the regulatory contribution. The other parameter values are set as in table 1. As a result, the only difference between the results we obtain here and those obtained in the base case are the result of minimum funding ratio constraints, a difference that allows us to analyse their marginal impact.

First, figure 42 shows that in the presence of short-term constraints, pensioners are less sensitive to the choice of the investment strategy than in the absence of such constraints (see figure 3). For a negative correlation $\rho$ between the firm value risk and stock index risk, one still obtains an interior optimal allocation for initial funding ratios of 70% and 100%, but liability value is a flatter function of the allocation to the risky asset compared to figure 3, where no minimum funding was imposed. The intuition behind this result is that short-term constraints protect pensioners from the insolvency risk of the pension fund, regardless of the funding status or the investment policy of the pension fund. Unlike risk-controlled strategies, they do not guarantee that the promised payment will be delivered to pensioners in all states of the world, for any riskiness of the strategy, so the weight $\omega$ still has an impact, but this effect is largely dampened. As far as equity-holders and bondholders are concerned, $\omega$ also still has an impact, which is qualitatively similar to what was obtained in figures 4 and 5. Finally, figure 45 shows that, in the presence of short-term constraints, a 100% allocation to the risky asset is more often optimal from the aggregate perspective than in the absence of these constraints (see figure 6). The intuition is that the introduction of minimum
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funding requirements allows more risk taking, which is beneficial to shareholders but does not make a great impact on pensioner welfare.

In figures 46 to 49, we measure the impact of introducing short-term constraints for several rules for the sharing of surpluses by pensioners and equity-holders. The rule is determined by the parameter $\gamma$, giving the fraction of surpluses that goes to equity-holders after liquidation of the pension plan at terminal date. The impact is measured in terms of the relative change, defined as:

$$\frac{L_0(\text{with constraints}) - L_0(\text{without constraints})}{L_0(\text{without constraints})}$$

for the value pension liabilities and similarly for equity value, debt value, and total value. Unsurprisingly, figure 46 shows that, for any value of $\gamma$, pensioners benefit from the enforcement of minimum funding requirements. This benefit could be expected from the comparison of the credit spreads in figures 3 and 42, which shows that spreads are greatly reduced when short-term constraints are introduced. The greatest benefits are to be expected when the initial funding ratio is low, because it is then that the pension fund is unlikely to deliver the promised payment without needing additional contributions. The situation is more ambiguous for equity-holders. On the one hand, it can be argued that early contributions make insolvency of the pension fund less likely, thus reducing bankruptcy costs and making a positive impact on the tax shield. On the other hand, early additional contributions may turn out, after the fact, to have been unnecessary in those states of world in which the fund can recover a better funding situation simply because of strong stock market performance. As a matter of fact, figure 47 shows that equity-holders will benefit from the introduction of short-term constraints only if $\gamma$ is sufficiently close to 1. Indeed, if they are entitled to a large fraction of the surpluses, they can expect unnecessary contributions to be returned to them at the terminal date. If $\gamma$ is lower, however, any such contribution is lost for them. Thus, the main message from this analysis is that equity-holders are hurt by the introduction of short-term funding constraints if there is no surplus-sharing rule (or at least a contribution holiday covenant) allowing some form of negative contributions at terminal date. Again, the introduction of surplus-sharing schemes makes it possible to align the interests of stakeholders more closely. The impact of short-term constraints on bondholders depends on the value of correlation $\rho$ of stock market performance and firm asset value. For a negative $\rho$, they benefit from such constraints. Indeed, intermediate contributions lead equity-holders to invest more in the risky asset $S$ through the pension fund, which diversifies away firm risk if the firm and stock market are negatively correlated. Indeed, negative correlation results in lower volatility for the aggregate asset $A + V$, which increases the value of corporate bonds. Such a diversification effect is not present for a positive correlation. Here, the presence of short-term constraints will, in general, hurt bondholders because they worsen the financial health of the sponsor. The opposite effect will take place if the pension fund can return surpluses to the firm. Turning to the aggregate impact of
short-term constraints, figure 49 shows that, in most situations, introducing such constraints will increase the total value of the firm. These results shed new light on the fierce debate between advocates of tighter pension fund regulation, not only in the United States but also in Europe, and those arguing that it would result only in severe welfare loss. The imposition of funding ratio constraints has come in for particularly pointed criticism by experts who believe that imposing such short-term constraints on long-term investors could be counter-productive (Pugh 2003). Our results suggest that, as long as some form of explicit surplus-sharing rule is put in place, short-term funding constraints could in fact be a welfare-improving mechanism for both pensioners and shareholders.
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5. Conclusions and Extensions
5. Conclusions and Extensions

Correctly assessing the value of a pension plan requires a comprehensive model for the joint quantitative analysis of capital structure choices, pension fund allocation decisions, and their impact on rational pricing of liability streams. This paper is an attempt to analyse the valuation of pension liabilities regarded as defaultable claims issued by the sponsor to workers and pensioners in the context of an intertemporal capital structure model with contingent contributions. Our model has important policy implications in that it takes a first step towards a much-needed methodological framework for the design of firm-specific regulatory constraints and accounting valuation principles. It also has a number of implications for investment decisions made by the pension fund and funding decisions made by the sponsor.

Our work can be extended in a number of directions. First, the model could be extended to account for widespread indexation of promised pension payments to wage inflation rather than to price inflation. Such an indexation rule would, however, introduce market incompleteness, since bonds perfectly indexed to wages are not traded. In terms of allocation strategies, we have tested fixed-mix strategies as well as basic CPPI strategies and found that the benefits of moving away from static allocation strategies to consider even the simplest form of dynamic risk-controlled strategies were substantial, in particular for liability-holders. It would be of interest to try to test more sophisticated forms of welfare-improving strategies in a more general dynamic context, including strategies with a floor given as a function of the liability portfolio value (with a key distinction to be made between the regulatory floor and the fair value floor), strategies with a performance cap in addition to floors, which can enable a decrease in the cost of downside risk protection, as well as strategies involving corporate bonds in the liability-hedging portfolio. It would likewise be of interest to test strategies that attempt to control sponsor risk by incorporating state variables that measure the financial health of the sponsor. The intuition is that one could further decrease the cost of downside protection by relaxing risk constraints when the sponsor is financially healthy and could therefore make up for eventual deficits through increased contributions, while focusing on hedging away the states of the world characterised by joint occurrences of poorly performing pension assets and ill financial health on the part of the sponsor. We leave these questions for further research.
6. Appendices
A. Optimality of Portfolio Insurance

We assume that the Sharpe ratio $\lambda_S$ of the risky asset $S$ is constant. The dynamics of $S$ under the actual probability measure $\mathbb{P}$ is:

$$\frac{dS_t}{S_t} = [r + \sigma_S \lambda_S] dt + \sigma_S d\tilde{z}_t^S$$

where $\tilde{z}_t^S = z_t^S - \lambda_S t$ defines a $\mathbb{P}$-Brownian motion. Let us now consider an agent (e.g., a pension fund manager) who invests the weight $\omega_t$ at time $t$ in $S$ and the remainder in cash, and maximises expected utility from the random variable $A_T/L - k$, which is the excess of the funding ratio at time $T$ over the minimum funding requirement $k$. We assume that her preferences are represented by a constant relative risk aversion utility function, with relative risk aversion $\nu$. The maximisation programme at time $t$ can be written as:

$$\max_{(\omega_t,z_t^S)} \mathbb{E}_t^P \left[ \frac{1}{1-\nu} \left( \frac{A_T}{L_T} - k \right)^{1-\nu} \right]$$

This problem can be solved via the duality technique of Cox and Huang (1989). First, the optimal terminal wealth is:

$$A_T^* = kL + \left( uM_T \right)^{-\frac{1}{\nu}}$$

where $u$ is some Lagrange multiplier, given by $\mathbb{E}_T^P [M_T A_T] = M_T A_T$, and $M_t$ is the pricing kernel at time $t$:

$$M_t = e^{-\left( r + \frac{\sigma^2}{2} \right)(T-t) - \lambda_S \tilde{z}_t^S}, \quad t \leq T$$

The optimal wealth at time $t$ is thus:

$$A_t^* = kLe^{-(T-t)} + u^{-\frac{1}{\nu}} M_t^{-\frac{1}{\nu}} \mathbb{E}_t \left[ \left( \frac{M_T}{M_t} \right)^{1-\frac{1}{\nu}} \right]$$

Applying Ito’s lemma and matching the diffusion term in $dA_t^*$ with $A_t^* \omega_t \sigma_S d\tilde{z}_t^S$, we obtain the optimal instantaneous allocation to the risky asset at time $t$:

$$\omega_t^* = \frac{1}{A_t^*} \left[ A_t^* - kLe^{-(T-t)} \right] \frac{\lambda_S}{\nu \sigma_S}$$

which is of the form (4.19) with $m = \frac{\lambda_S}{\nu \sigma_S}$.

Introducing the optimal terminal wealth when no constraint is imposed (i.e., when $k = 0$), $A_T^u = (u_0 M_T)^{-\frac{1}{\nu}}$, we obtain that:

$$A_T^* = kL + \left( 1 - \frac{kLe^{-(T-t)}}{A_0} \right) A_T^u$$

In particular, in those states of the world where $A_T^u$ is greater than $A_0 e^{rT}$, $A_T^*$ will be lower than $A_T^u$. Therefore, there is an opportunity cost associated with downside risk protection if it proves ex post to have been unnecessary.

It is also interesting to compare in a mean-variance setting the properties of the insurance strategy and those of a strategy taking no risk at all. It can easily be checked that the expected value of the payoff $A_T^*$ under the physical measure is:

$$\mathbb{E}^P [A_T^*] = kL + \left( A_0 e^{rT} - kL \right) e^{m \sigma^2 \lambda_S T}$$

As soon as the Sharpe ratio $\lambda_S$ is positive and the initial risk budget satisfies (4.20), this expected value exceeds the expected terminal wealth from a strategy invested in the cash only (for which the expected payoff would be $A_0 e^{rT}$). Of course, the variance of $A_T^*$ will also exceed the zero variance that would be achieved by investing in the cash alone, but investors with non-zero risk aversion may prefer to take some risk for more average return.
6. Appendices

**B. Proof of Proposition 4.1**

We start from the dynamics of $A$:

$$dA_t = m(A_t - kL^{e^{-r(T-t)}}) dS_t$$

and we then introduce

$$Y_t = (A_t - kL^{e^{-r(T-t)}})/S_t^m.$$

An application of Ito’s lemma shows that:

$$\frac{dY_t}{Y_t} = (1 - m) \left[ r + \frac{m \sigma_S^2}{2} \right] dt$$

A straightforward integration then shows that, for any $t \leq T$:

$$v_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[ L_t + D_t + E_t \right] dt \right]$$

The expectations of the integrals in $dz^S$, $dz^r$ and $dz^V$ are zero, and the covenants listed in subsection 4.2.3 imply that $L_t + D_t + E_t = A_t + V_t + TS_t - BC_t - RT_t$. Hence:

$$v_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[ e^{-rT} A_t + V_t \right] dt \right] + \mathbb{E}^Q \left[ \int_0^T \theta(\kappa_t + c) e^{-rt} dt \right]$$

which implies that $v_0 = (1 - \theta)A_0 + V_0 + TS_0 - BC_0 - RT_0$.

**C. Proof of Proposition 4.2**

From (4.28), (4.27), (4.29) and (4.30), the total value of the firm satisfies:

$$v_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[ l + \delta V_t + \theta(\kappa_t + c) - \kappa_t \right] dt \right]$$

$$+ \mathbb{E}^Q \left[ \int_0^T \sum_{i=1}^N e^{-rT_i} \theta a_1 (kL_t - A_{t_i})^+ \right.$$  

$$\left. + a_3 \left( A_{t_i} - kL_t \right) \mathbb{I}_{\left\{ A_{t_i} > kL_t \right\}} dM^q_t \right]$$

$$+ \mathbb{E}^Q \left[ e^{-rT_i} \left( L_{t_i} + E_{t_i} + D_{t_i} \right) \right]$$

Using the dynamics of $A$ and $V$, as given in (4.25) and (4.26), we can rewrite the integrals in the first two terms, and arrive at:

$$v_0 = \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[ e^{-rT} (A_t + V_t) \right] dt \right]$$

$$+ \mathbb{E}^Q \left[ \int_0^T e^{-rt} \left[ A_t (\sigma_S dz_t^S + (1 - \omega)\sigma_Y dz_t^Y) + V_t \sigma_Y dz_t^Y \right] dt \right]$$

$$+ \mathbb{E}^Q \left[ \int_0^T \theta(\kappa_t + c) e^{-rt} dt \right]$$

$$+ \mathbb{E}^Q \left[ \sum_{i=1}^N e^{-rT_i} \theta a_1 (kL_{t_i}^o - A_{t_i})^+ dM^q_t \right]$$

$$+ \mathbb{E}^Q \left[ e^{-rT_i} \left( L_{t_i} + D_{t_i} + E_{t_i} \right) \right]$$
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C. Figures & Tables

Figure 1: Leverage decisions with fixed-mix strategies – Total value of the firm as a function of the leverage ratio

This figure plots the leverage ratio against the total value for different values of the promised payment to pensioners, $L$. The curves are parametrised by the face value of debt, $D$. The initial endowment of the pension fund is set to the regulatory liability value, i.e., we set $A_0 = L_0^{(m)}$, and the initial unlevered value of the firm is set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Other parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the leverage ratio in the base case, where $L = 50$.

Figure 2: Leverage decisions with fixed-mix strategies – Fair liability value as a function of the leverage ratio

This figure plots the leverage ratio against the credit spread $s_L$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and two values of the promised payment to pensioners (50 and 150). The curves are parametrised by the face value of debt, $D$. The initial endowment of the pension fund is set to the regulatory liability value, i.e., we set $A_0 = L_0^{(m)}$, and the initial unlevered value of the firm is set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. The case where $L = 150$ and the initial funding ratio is 200% has not been represented because it is not compatible with the condition of a positive $V_0$. Other parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the leverage ratio in the base case, where $L = 50$, and the horizontal dashed-dot line represents the regulatory liability spread, which is taken to be 100 basis points.
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Figure 3: Allocation decisions with fixed-mix strategies – Impact on pensioners

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for $\omega$. 
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Figure 4: Allocation decisions with fixed-mix strategies – Impact on equity-holders

(a) Equity-holders have access to surpluses (γ = 1)

(b) Pensioners have access to surpluses (γ = 0)

These figures perform comparative static analysis with respect to the allocation to the risky asset, ω, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation ρ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, A₀, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V₀ = x - (1 - \theta)A₀ \), where x is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for ω.
These figures perform comparative static analysis with respect to the allocation to the risky asset, \( \omega \), for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta)A_0 \), where \( x \) is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for \( \omega \).

These figures perform comparative static analysis with respect to the allocation to the risky asset, \( \omega \), for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta)A_0 \), where \( x \) is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for \( \omega \).
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Figure 7: Funding decisions with fixed-mix strategies – Impact on pensioners

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures plot the regulatory funding ratio of the pension fund against the fair liability value for different values of the promised payment to pensioners, $L$ (25, 50, 100, and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.
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Figure 8: Funding decisions with fixed-mix strategies – Impact on equity-holders

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures plot the regulatory funding ratio of the pension fund against the market capitalization of the firm for different values of the promised payment to pensioners, $L$ (25, 50, 100, and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.
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Figure 9: Funding decisions with fixed-mix strategies – Impact on bondholders

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures plot the regulatory funding ratio of the pension fund against the market value of debt for different values of the promised payment to pensioners, $L$ (25, 50, 100, and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.
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Figure 10: Funding decisions with fixed-mix strategies – Impact on aggregate firm and pension fund

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures plot the regulatory funding ratio of the pension fund against the total value of the firm for different values of the promised payment to pensioners, $\text{L}$ (25, 50, 100, and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 1). The vertical dashed line represents the regulatory funding ratio in the base case.

Figure 11: Funding decisions with fixed-mix strategies – Debt value as a function of the regulatory funding ratio

This figure plots the regulatory funding ratio against the market value of debt and the credit spread $s_D$. The curves are parametrised by the initial endowment to the pension fund, $A_0$. Other parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the regulatory funding ratio in the base case.
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Figure 12: Surplus sharing rule – Impact on pensioners

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity-holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for $\gamma$.

Figure 13: Surplus sharing rule – Impact on equity-holders

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity-holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for $\gamma$.

Figure 14: Surplus sharing rule – Impact on bondholders

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity-holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for $\gamma$.
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Figure 15: Surplus sharing rule – Impact on aggregate firm and pension fund

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. $\gamma = 1$ describes the situation where equity-holders are entitled to the full surpluses, while $\gamma = 0$ corresponds to the case where surpluses go to pensioners. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1). The vertical dashed line identifies the base case for $\gamma$. 
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Figure 16: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on pensioners

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to

$$V_0 = x - (1 - \theta)A_0$$

where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values [see table 2]. The vertical dashed line identifies the base case for $\omega$. 
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Figure 17: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on equity-holders

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case for $\omega$. 
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Figure 18: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on bondholders

These figures perform comparative static analysis with respect to the allocation to the risky asset, \( \omega \), for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta A_0) - P_0^{\pi^f} \), where \( x \) is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case for \( \omega \).

Figure 19: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on the pension put

These figures perform comparative static analysis with respect to the allocation to the risky asset, \( \omega \), for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation \( \rho \) between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, \( A_0 \), is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V_0 = x - (1 - \theta A_0) - P_0^{\pi^f} \), where \( x \) is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case for \( \omega \) and the dash-dot line represents the regulatory premium \( P_0^{\pi^f} \).
These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0 - \rho \rho^{\text{un}}$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case for $\omega$. 

Figure 20: Allocation decisions with fixed-mix strategies in the presence of the PBGC – Impact on aggregate firm and pension fund
Figure 21: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on pensioners

(a) Equity-holders have access to surpluses ($\gamma = 1$)

(b) Pensioners have access to surpluses ($\gamma = 0$)

These figures plot the regulatory funding ratio of the pension fund against the fair liability value for different values of the promised payment to pensioners, $L$ (25, 50, 100, and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0 - \frac{E_{L}}{\theta}$, where $x$ is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.
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Figure 22: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on equity-holders

(a) Equity-holders have access to surpluses \( \gamma = 1 \)

(b) Pensioners have access to surpluses \( \gamma = 0 \)

These figures plot the regulatory funding ratio of the pension fund against the market capitalization of the firm for different values of the promised payment to pensioners, \( L \) (25, 50, 100, and 150). The curves are parametrised by \( A_0 \), the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to \( V_0 = x - (1 - \theta)A_0 - P_0^{\text{mm}} \), where \( x \) is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.
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Figure 23: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on bondholders

These figures plot the regulatory funding ratio of the pension fund against the market value of debt for different values of the promised payment to pensioners, $L$ (25, 50, 100, and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0 - P^0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.

Figure 24: Funding decisions with fixed-mix strategies in the presence of the PBGC – Impact on aggregate firm and pension fund

These figures plot the regulatory funding ratio of the pension fund against the total value of the firm for different values of the promised payment to pensioners, $L$ (25, 50, 100, and 150). The curves are parametrised by $A_0$, the initial capital made available to the pension fund. The initial unlevered asset value of the firm is set to $V_0 = x - (1 - \theta)A_0 - P^0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, other parameters are fixed at their base-case values (see table 2). The vertical dashed line identifies the base case regulatory funding ratio.
Figure 25: Allocation decisions with unconditional indexation – Impact on pensioners

(a) Indexed pension payments

(b) Nominal pension payments

This figure shows the impact of the weight allocated to the stock index on the fair liability value, as well as on the creditworthiness of liabilities. This impact is assessed for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150%, and 200%), and for different values of the correlation $\rho_{SV}$ between the stock index and the firm’s unlevered value ($-50\%$, $0\%$, and $50\%$). Promised pension payments are assumed to be unconditionally indexed to inflation in panel (a), and fixed in nominal terms in panel (b). Unless otherwise stated, parameters are set at their base-case values (see table 3).
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Figure 26: Allocation decisions with unconditional indexation – Impact on equity-holders

(a) Indexed pension payments

(b) Nominal pension payments

This figure shows the impact of the weight allocated to the stock index on equity value, for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and different values of the correlation $\rho_{SV}$ between the stock index and the firm’s unlevered value (−50%, 0, and 50%). Promised pension payments are assumed to be unconditionally indexed to inflation in panel (a), and fixed in nominal terms in panel (b). Unless otherwise stated, parameters are set at their base-case values (see table 3).
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Figure 27: Funding decisions with unconditional indexation – Impact on pensioners

(a) Indexed pension payments

(b) Nominal pension payments.

This figure shows the impact of the initial regulatory funding level on the fair liability value as well as on the credit worthiness of pension liabilities. This impact is assessed for different values of the real promised payment to pensioners ($L = 25, 50, 100$ and $150$), and for different values of the correlation $\rho_{SV}$ between the stock index and the firm’s unlevered value ($-50\%, 0\%$, and $50\%$). Promised pension payments are assumed to be unconditionally indexed to inflation in panel (a), and fixed in nominal terms in panel (b). Unless otherwise stated, parameters are set at their base-case values (see table 3).
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Figure 28: Funding decisions with unconditional indexation – Impact on equity-holders

(a) Indexed pension payments

(b) Nominal pension payments

This figure shows the impact of the initial regulatory funding level on equity value. This impact is assessed for different values of the real promised payment to pensioners (L = 25, 50, 100, and 150), and for different values of the correlation \( \rho_{SV} \) between the stock index and the firm’s unlevered value (−50%, 0, and 50%). Promised pension payments are assumed to be unconditionally indexed to inflation in panel (a), and fixed in nominal terms in panel (b). Unless otherwise stated, parameters are set at their base-case values (see table 3).
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Figure 29: Increase in the correlation parameters $\rho_{V\varphi}$ and $\rho_{S\varphi}$ – Impact on pensioners

(a) Increase in $\rho_{V\varphi}$

(b) Increase in $\rho_{S\varphi}$

These figures show the separate impacts of positive shocks on the correlation parameters $\rho_{V\varphi}$ and $\rho_{S\varphi}$ on pensioners. Panel (a) shows the effect of increasing $\rho_{V\varphi}$, the correlation between firm value and inflation, from $-70\%$ to $70\%$; panel (b) shows the effect of increasing $\rho_{S\varphi}$, the correlation between stock index and inflation, from $-70\%$ to $70\%$. This impact is assessed for different values of the initial funding ratio, different values of the promised payment to pensioners, and different values of the correlation $\rho_{SV}$ between the stock index and firm value. Pension payments are unconditionally indexed to inflation. Unless otherwise stated, parameters are set at their base-case values (see table 3).
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Figure 30: Increase in the correlation parameters $\rho_{V}\phi$ and $\rho_{S}\phi$ – Impact on equity-holders

(a) Increase in $\rho_{V}\phi$

(b) Increase in $\rho_{S}\phi$

These figures show the separate impacts of positive shocks on the correlation parameters $\rho_{V}\phi$ and $\rho_{S}\phi$ on equity-holders. Panel (a) shows the effect of increasing $\rho_{V}\phi$, the correlation between firm value and inflation, from $-70\%$ to $70\%$; panel (b) shows the effect of increasing $\rho_{S}\phi$, the correlation between stock index and inflation, from $-70\%$ to $70\%$. This impact is assessed for different values of the initial funding ratio, different values of the promised payment to pensioners, and different values of the correlation $\rho_{SV}$ between the stock index and firm value. Pension payments are unconditionally indexed to inflation. Unless otherwise stated, parameters are set at their base-case values (see table 3).

Figure 31: Allocation decisions with conditional indexation and fixed-mix strategies – Impact on the indexation probability

This figure shows the impact of the weight allocated to the stock index on the probability that pensioners receive a terminal promised payment $L$ plus inflation. This impact is assessed for different initial regulatory funding levels ($70\%, 100\%, 150\%, and 200\%$), and for different values of the correlation $\rho_{SV}$ between the stock index and the firm’s unlevered value ($-50\%, 0$, and $50\%$). Promised pension payments are assumed to be unconditionally indexed to inflation. Unless otherwise stated, parameters are set at their base-case values (see table 3).
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Figure 32: Allocation decisions with conditional indexation – Impact on pensioners

(a) Conditionally indexed pension payments

(b) Unconditionally indexed payments

This figure shows the impact of the weight allocated to the stock index on the fair liability value. This impact is assessed for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150%, and 200%), and for different values of the correlation $\rho_{SV}$ between the stock index and the firm’s unlevered value (−50%, 0, and 50%). Promised pension payments are assumed to be conditionally indexed to inflation in panel (a), unconditionally indexed in panel (b) and fixed in nominal terms in panel (c). Unless otherwise stated, parameters are set at their base-case values (see table 3).
Figure 33: Allocation decisions with conditional indexation – Impact on equity-holders

(a) Conditionally indexed pension payments

(b) Unconditionally indexed payments

(c) Nominal pension payments

This figure shows the impact of the weight allocated to the stock index on equity value, for different values of the initial regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and different values of the correlation $\rho_{SV}$ between the stock index and the firm’s unlevered value (−50%, 0 and 50%). Promised pension payments are assumed to be conditionally indexed to inflation in panel (a), unconditionally indexed in panel (b) and fixed in nominal terms in panel (c). Unless otherwise stated, parameters are set at their base-case values (see table 3).
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Figure 34: Impact of internal funding on pensioners

(a) Equity-holders had access to surpluses in the external funding regime ($\gamma = 1$)

(b) Pensioners had access to surpluses in the external funding regime ($\gamma = 0$)

This figure shows the impact on liability value of integrating pension assets and liabilities in the corporate balance sheet. This impact is assessed for different levels of the initial regulatory funding ratio, different values of the promised payment to pensioners ($L = 25, 50, 100, \text{ and } 150$), and different values of the correlation $\rho$ between the stock index and the firm’s unlevered value ($-50\%, 0, \text{ and } 50\%$). In panel (a), the reference situation is an external funding regime where equity-holders have access to surpluses; while in panel (b) the benchmark situation is an external regime where surpluses are paid to pensioners. Unless otherwise stated, parameters are set at their base-case values (see table 1).
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Figure 35: Impact of internal funding on equity-holders

(a) Equity-holders had access to surpluses in the external funding regime ($\gamma = 1$)

(b) Pensioners had access to surpluses in the external funding regime ($\gamma = 0$)

This figure shows the impact on equity value of integrating pension assets and liabilities in the corporate balance sheet. This impact is assessed for different levels of the initial regulatory funding ratio, different values of the promised payment to pensioners ($L = 25, 50, 100, \text{ and } 150$) and different values of the correlation $\rho$ between the stock index and the firm's unlevered value ($-50\%, 0$, and $50\%)$. In panel (a), the reference situation is an external funding regime where equity-holders have access to surpluses; while in panel (b) the benchmark situation is an external regime where surpluses are paid to pensioners. Unless otherwise stated, parameters are set at their base-case values (see table 1).
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Figure 36: Impact of internal funding on bondholders

(a) Equity-holders had access to surpluses in the external funding regime ($\gamma = 1$)

(b) Pensioners had access to surpluses in the external funding regime ($\gamma = 0$)

This figure shows the impact on market debt value, as well as on the credit spread of the sponsor firm, of integrating pension assets and liabilities in the corporate balance sheet. This impact is assessed for different levels of the initial regulatory funding ratio, different values of the promised payment to pensioners ($L = 25, 50, 100, \text{ and } 150$) and different values of the correlation $\rho$ between the stock index and the firm’s unlevered value ($-50\%, 0, \text{ and } 50\%$). In panel (a), the reference situation is an external funding regime where equity-holders have access to surpluses, while in panel (b) the benchmark situation is an external regime where surpluses are paid to pensioners. Unless otherwise stated, parameters are set at their base-case values (see table 1).
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Figure 37: Impact of internal funding on aggregate firm and pension fund

(a) Equity-holders had access to surpluses in the external funding regime \((\gamma = 1)\)

(b) Pensioners had access to surpluses in the external funding regime \((\gamma = 0)\)

This figure shows the impact on the total value of aggregate firm and pension fund, of integrating pension assets and liabilities in the corporate balance sheet. This impact is assessed for different levels of the initial regulatory funding ratio, different values of the promised payment to pensioners \((L = 25, 50, 100, \text{ and } 150)\) and for different values of the correlation \(\rho\) between the stock index and the firm’s unlevered value \((-50\%, 0, \text{ and } 50\%)\). In panel (a), the reference situation is an external funding regime where equity-holders have access to surpluses; while in panel (b) the benchmark situation is an external regime where surpluses are paid to pensioners. Unless otherwise stated, parameters are set at their base-case values (see table 1).
6. Appendices

Figure 38: Allocation decisions with risk-controlled strategies – Impact on pensioners

These figures perform comparative static analysis with respect to the multiplier, $m$, for different values of the initial regulatory funding ratio (130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), and the minimum terminal funding ratio $k$ is set to 100%. The vertical dashed line identifies the base case for $\alpha$.

Figure 39: Allocation decisions with risk-controlled strategies – Impact on equity-holders

These figures perform comparative static analysis with respect to the multiplier, $m$, for different values of the initial regulatory funding ratio (130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), and the minimum terminal funding ratio $k$ is set to 100%. The vertical dashed line identifies the base case for $\alpha$. 
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Figure 40: Allocation decisions with risk-controlled strategies – Impact on bondholders

(a) Prices

(b) Comparison with fixed-mix strategies

These figures perform comparative static analysis with respect to the multiplier, $m$, for different values of the initial regulatory funding ratio (130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), and the minimum terminal funding ratio $k$ is set to 100%. The vertical dashed line identifies the base case for $\omega$. 
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Figure 41: Allocation decisions with risk-controlled strategies – Impact on aggregate firm and pension fund

(a) Prices

(b) Comparison with fixed-mix strategies

These figures perform comparative static analysis with respect to the multiplier of the CPPI strategy, m, for different values of the initial regulatory funding ratio (130%, 150%, and 200%) and the correlation ρ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, A₀, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to V₀ = x - (1-θ)A₀, where x is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), and the minimum terminal funding ratio k is set to 100%. The vertical dashed line identifies the base case for ω.
These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x(1-\theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$. 

Figure 42: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on pensioners
Figure 43: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on equity-holders

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$.

Figure 44: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on bondholders

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$.

Figure 45: Allocation decisions with fixed-mix strategies and short-term constraints – Impact on aggregate firm and pension fund

These figures perform comparative static analysis with respect to the allocation to the risky asset, $\omega$, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\omega$. 

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6. Appendices

Figure 46: Surplus-sharing rule in the presence of short-term constraints – Impact on pensioners

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\gamma$.

Figure 47: Surplus-sharing rule in the presence of short-term constraints – Impact on equity-holders

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\gamma$.

Figure 48: Surplus-sharing rule in the presence of short-term constraints – Impact on bondholders

These figures perform comparative static analysis with respect to the parameter $\gamma$ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation $\rho$ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, $A_0$, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to $V_0 = x - (1 - \theta)A_0$, where $x$ is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for $\gamma$. 
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Figure 49: Surplus-sharing rule in the presence of short-term constraints – Impact on aggregate firm and pension fund

These figures perform comparative static analysis with respect to the parameter γ driving access to surpluses by equity-holders, for various levels of the regulatory funding ratio (70%, 100%, 130%, 150%, and 200%) and the correlation ρ between the unlevered value of operating assets of the firm and the risky asset in which the pension fund invests. The initial endowment of the pension fund, A₀, is adjusted to make the regulatory funding ratio equal to the target. The initial unlevered asset value of the firm is then set to \( V₀ = x - (1 - \theta)A₀ \), where x is a constant normalised to 100. Unless otherwise indicated, parameters are fixed at their base-case values (see table 1), the funding ratio is checked every year and the sponsor is given three years to make up for a deficit. The vertical dashed line identifies the base case for γ.

Table 1: Base case parameters in the absence of the PBGC

<table>
<thead>
<tr>
<th></th>
<th>Firm</th>
<th>Pension Fund</th>
<th>Interest Rate</th>
<th>Correlation</th>
<th>Default</th>
<th>Regulatory Environment</th>
<th>Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_V )</td>
<td>0.20</td>
<td>0.30</td>
<td>r</td>
<td>0.50</td>
<td>0.50</td>
<td>( \theta )</td>
<td>0.35</td>
</tr>
<tr>
<td>( V₀ )</td>
<td>80.29</td>
<td>( \omega )</td>
<td>( \rho )</td>
<td></td>
<td></td>
<td>( s_{\text{reg}} ) (bp)</td>
<td>100.00</td>
</tr>
<tr>
<td>( \sigma_S )</td>
<td></td>
<td>( \nu )</td>
<td>( \rho )</td>
<td></td>
<td></td>
<td>( \gamma )</td>
<td>1.00</td>
</tr>
<tr>
<td>( D )</td>
<td>50.00</td>
<td>( \Delta )</td>
<td>( \alpha )</td>
<td></td>
<td></td>
<td>( T ) (years)</td>
<td>10.00</td>
</tr>
</tbody>
</table>

This table lists the parameter values in the base model. The unlevered asset value of the firm, \( V \), starts at \( V₀ \) and has volatility \( \sigma_V \). The risky security in which the pension fund invests has volatility \( \sigma_S \) and its returns have instantaneous correlation \( \rho \) with changes in \( V \). The short-term interest rate is equal to \( r \) and the promised payments to pensioners and bondholders at time \( T \) are respectively \( D \) and \( \Delta \). The pension fund is fully funded in the regulatory sense, that is the initial endowment \( A₀ \) is equal to the regulatory liability value, \( V₀ \) is then computed as \( V₀ = 100 - (1 - \theta)A₀ \), where \( \theta \) denotes the corporate tax rate. \( \theta_{\text{eff}} \) denotes the effective tax reversion rate, and \( \alpha \) is the proportional rate of bankruptcy costs. At date \( T \), the fraction \( \gamma \) of pension fund surpluses goes to equity-holders, and pensioners receive the remaining part \( 1 - \gamma \).
Table 2: Base-case parameters in the presence of the PBGC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm</td>
<td></td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>0.20</td>
</tr>
<tr>
<td>$V_0$</td>
<td>80.29</td>
</tr>
<tr>
<td>$D$</td>
<td>50.00</td>
</tr>
<tr>
<td>Pension Fund</td>
<td></td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.50</td>
</tr>
<tr>
<td>$A_0$</td>
<td>30.33</td>
</tr>
<tr>
<td>$L$</td>
<td>50.00</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.04</td>
</tr>
<tr>
<td>Correlation $\rho$</td>
<td>0.50</td>
</tr>
<tr>
<td>Default $\sigma$</td>
<td>0.50</td>
</tr>
<tr>
<td>Regulatory Environment</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\theta_{eff}$</td>
<td>0.00</td>
</tr>
<tr>
<td>$s_{reg}(bp)$</td>
<td>100.00</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.00</td>
</tr>
<tr>
<td>Horizon $T$(years)</td>
<td>10.00</td>
</tr>
<tr>
<td>PBGC</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.10</td>
</tr>
</tbody>
</table>

This table lists the parameter values in the base model. The unlevered asset value of the firm, $V$, starts at $V_0$ and has volatility $\sigma_V$. The risky security in which the pension fund invests has volatility $\sigma_S$ and its returns have instantaneous correlation $\rho$ with changes in $V$. The short-term interest rate is equal to $r$, and the promised payments to pensioners and bondholders at time $T$ are respectively $L$ and $D$. The pension fund is fully funded in the regulatory sense, i.e., the initial contribution $A_0$ equals the regulatory liability value. $V_0$ is then computed as $V_0 = 100 - (1 - \theta)A_0 - \beta$, where denotes the corporate tax rate and $P_l^{(L)} = P_l^{(D)}$ is the premium charged by the PBGC. $\theta_{eff}$ denotes the effective tax reversion rate, and $\alpha$ is the proportional rate of bankruptcy costs. At date $T$, the fraction $\gamma$ of pension fund’s surpluses goes to equity-holders, and pensioners receive the remaining part $1 - \gamma$. $s_{reg}$ denotes the regulatory spread used to valuate pension payments. $\lambda$ is the fraction of the promised payment to pensioners at which the PBGC caps its contribution.
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Table 3: Base case parameters with inflation risk.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm</td>
<td></td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>0.20</td>
</tr>
<tr>
<td>$\delta$</td>
<td>50</td>
</tr>
<tr>
<td>Pension fund</td>
<td></td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>50</td>
</tr>
<tr>
<td>Interest rate</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>0.2</td>
</tr>
<tr>
<td>$b$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>0.015</td>
</tr>
<tr>
<td>Price index</td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.0773</td>
</tr>
<tr>
<td>$\sigma_\pi_0$</td>
<td>0.0183</td>
</tr>
<tr>
<td>Correlations</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\pi\pi}$</td>
<td>-0.2113</td>
</tr>
<tr>
<td>$\rho_{\pi S}$</td>
<td>-0.6823</td>
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<tr>
<td>$\rho_{\pi V}$</td>
<td>0.1415</td>
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<tr>
<td>$\rho_{S V}$</td>
<td>0.80</td>
</tr>
<tr>
<td>$\rho_{V V}$</td>
<td>0</td>
</tr>
<tr>
<td>Default</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.50</td>
</tr>
<tr>
<td>Regulatory environment</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\pi}$</td>
<td>0.35</td>
</tr>
<tr>
<td>$s_{reg}[\pi\pi]$</td>
<td>100</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>$f$</td>
<td>1</td>
</tr>
<tr>
<td>Horizon</td>
<td></td>
</tr>
<tr>
<td>$T$ (years)</td>
<td>10</td>
</tr>
</tbody>
</table>

This table lists the parameter values in the model with inflation risk. The unlevered asset value of the firm, $V$, starts at $V_0$ and has volatility $\sigma_V$. The risky security in which the pension fund invests has volatility $\sigma_S$ and its returns have instantaneous correlation $\rho$ with changes in $V$. $a$, $b$, and $\sigma_\pi$ are respectively the speed of mean reversion, the long-term mean (under the risk-adjusted probability measure), and the volatility of the nominal short-term interest rate. $\pi$ is the anticipated inflation rate and $\sigma_\pi_0$ is the volatility of unanticipated inflation. The correlation parameters denote correlations between the Brownian motions driving the dynamics of the state variables, and are indexed by the names of the variables. The promised payments to pensioners and bondholders at time $T$ are respectively $L$ and $D$. $\theta$ denotes the corporate tax rate, $s_{reg}$ is the regulatory spread used for the discount of liabilities, and $\alpha$ is the proportional rate of bankruptcy costs. At date $T$, the fraction $\gamma$ of pension fund surpluses goes to equity-holders, and pensioners receive the remaining part $1-\gamma$. $f$ is the minimum real terminal funding ratio required in order to deliver the indexed payment.
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7. References


7. References


7. References


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About EDHEC-Risk Institute
The Choice of Asset Allocation and Risk Management
EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market.

On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation.

On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context.

This strategic choice is applied to all of the Institute’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds.

An Applied Research Approach
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute's advisory board. This board is made up of internationally recognised researchers, the Institute's business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:
• Asset allocation and alternative diversification
• Style and performance analysis
• Indices and benchmarking
• Operational risks and performance
• Asset allocation and derivative instruments
• ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in London, Nice, and Singapore.

In addition, EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:
• Regulation and Institutional Investment, in partnership with AXA Investment Managers
• Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners
• Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS
About EDHEC-Risk Institute

- Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)
- Dynamic Allocation Models and New Forms of Target-Date Funds, in partnership with UFG-LFP
- Advanced Modelling for Alternative Investments, in partnership with Newedge Prime Brokerage
- Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank
- Core-Satellite and ETF Investment, in partnership with Amundi ETF
- The Case for Inflation-Linked Corporate Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie
- Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan
- Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, in partnership with CME Group
- Structured Equity Investment Strategies for Long-Term Asian Investors, in partnership with Société Générale Corporate & Investment Banking
- The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with Eurex
- Solvency II Benchmarks, in partnership with Russell Investments

Each year, EDHEC-Risk organises a major international conference for institutional investors and investment management professionals with a view to presenting the results of its research: EDHEC-Risk Institutional Days.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 42,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 700,000 readers.

**EDHEC-Risk Institute: Key Figures, 2009–2010**

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nbr of permanent staff</td>
<td>66</td>
</tr>
<tr>
<td>Nbr of research associates</td>
<td>18</td>
</tr>
<tr>
<td>Nbr of affiliate professors</td>
<td>6</td>
</tr>
<tr>
<td>Overall budget</td>
<td>€9,600,000</td>
</tr>
<tr>
<td>External financing</td>
<td>€6,345,000</td>
</tr>
<tr>
<td>Nbr of conference delegates</td>
<td>2,300</td>
</tr>
<tr>
<td>Nbr of participants at EDHEC-Risk Indices &amp; Benchmarks seminars</td>
<td>582</td>
</tr>
<tr>
<td>Nbr of participants at EDHEC-Risk Institute Risk Management seminars</td>
<td>512</td>
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<tr>
<td>Nbr of participants at EDHEC-Risk Institute Executive Education seminars</td>
<td>247</td>
</tr>
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</table>

The philosophy of the Institute is to validate its work by publication in international journals, as well as to make it available to the sector through its position papers, published studies, and conferences.
About EDHEC-Risk Institute

Research for Business
The Institute's activities have also given rise to executive education and research service offshoots. EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.

The EDHEC-Risk Institute PhD in Finance
www.edhec-risk.com/Aeducation/PhD_Finance
The EDHEC-Risk Institute PhD in Finance is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities and enjoying the support of the research centre with the greatest impact on the financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

FTSE EDHEC-Risk Efficient Indices
www.edhec-risk.com/indexes/efficient
FTSE Group, the award winning global index provider, and EDHEC-Risk Institute launched the first set of FTSE EDHEC-Risk Efficient Indices at the beginning of 2010. Offered for a full global range, including All World, All World ex US, All World ex UK, Developed, Emerging, USA, UK, Eurobloc, Developed Europe, Developed Europe ex UK, Japan, Developed Asia Pacific ex Japan, Asia Pacific, Asia Pacific ex Japan, and Japan, the index series aims to capture equity market returns with an improved risk/reward efficiency compared to cap-weighted indices. The weighting of the portfolio of constituents achieves the highest possible return-to-risk efficiency by maximising the Sharpe ratio (the reward of an investment per unit of risk). These indices provide investors with an enhanced risk-adjusted strategy in comparison to cap-weighted indices, which have been the subject of numerous critiques, both theoretical and practical, over the last few years. The index series is based on all constituent securities in the FTSE All-World Index Series. Constituents are weighted in accordance with EDHEC-Risk's portfolio optimisation, reflecting their ability to maximise the reward-to-risk ratio for a broad market index. The index series is rebalanced quarterly at the same time as the review of the underlying FTSE All-World Index Series. The performances of the EDHEC-Risk Efficient Indices are published monthly on www.edhec-risk.com.

EDHEC-Risk Alternative Indexes
www.edhec-risk.com/indexes/pure_style
The different hedge fund indexes available on the market are computed from different data, according to diverse fund selection criteria and index construction methods; they unsurprisingly tell very different stories. Challenged by this heterogeneity, investors cannot rely on competing hedge fund indexes to obtain a “true and fair” view of performance and are at a loss when selecting benchmarks. To address this issue, EDHEC Risk was the first to launch composite hedge fund strategy indexes as early as 2003. The thirteen EDHEC-Risk Alternative Indexes are published monthly on www.edhec-risk.com and are freely available to managers and investors.
About BNP Paribas Investment Partners
BNP Paribas Investment Partners is the dedicated autonomous asset management business line of the BNP Paribas Group.

BNP Paribas Investment Partners offers a full range of investment management services to institutional and retail clients around the world. Central to the way we work is the concept of partnership—both in terms of how we behave as a family of companies and our relationships with our clients. Around 1,000 investment professionals work across our network of some 60 investment centres, each of which is a specialist in a particular asset class or type of product. With total assets under management of EUR 539 billion (USD 736 billion) as of 30 September 2010, BNP Paribas Investment Partners is the third-largest asset manager in Europe and the ninth-largest in the world.1

BNP Paribas Investment Partners combines the financial strength, distribution network and focus on compliance of its parent company with the reactivity, specialisation and entrepreneurial spirit of investment boutiques.

BNP Paribas Investment Partners provides a broad range of expertise and local solutions from its various Partners across the world:
- Fundamental management of the major assets classes (BNP Paribas Asset Management)
- Quantitative global equity (Alfred Berg)
- Global and emerging fixed income (FFTW)
- Funds of hedge funds (Fauchier Partners)
- Currency management (Overlay Asset Management)
- Private equity (BNP Paribas Private Equity)
- Infrastructure management (Antin Infrastructure Partners)
- Multi-management (FundQuest)
- Environmental markets (Impax Asset Management, BNP Paribas Clean Energy Partners)
- Non-listed real estate (BNP Paribas Asset Management)
- Wealth management (CamGestion, BNP Paribas Discretionary Portfolio Management)
- Employee retirement and saving schemes (BNP Paribas Epargne & Retraite Entreprises)
- Australia (Arnhem Investment Management)
- Brazil (BNP Paribas Asset Management Brasil)
- Chile (BancoEstado Administradora General de Fondos)
- China (HFT Investment Management Co. Ltd.)
- India (BNP Paribas Mutual Fund)
- Indonesia (PT. BNP Paribas Investments Partners Indonesia)
- Morocco (BMC Gestion)
- Russia (TKB BNP Paribas Investments Partners)
- Saudi Arabia (SAIB BNP Paribas Asset Management)
- South Korea (Shinhan BNP Paribas AMC)
- Turkey (TEB Asset Management, Fortis Investments Turkey)2

On April 1, 2010, the operations of Fortis Investments were merged with those of BNP Paribas Investment Partners. Fortis Investments’ investment experts and international locations are a natural and complementary fit with BNP Paribas Investments Partners, whose flexible partnership model has proven successful in integrating new expertise in the past. Together, our combined company provides clients with an even broader range of investment solutions and even better client service than before.

BNP Paribas Investment Partners has offices in the world’s major financial centres, including Hong Kong, London, New York, Paris and Tokyo.

1 - Source: Based on the IPE Top 400, June 2010
2 - The final set-up in Turkey is to be determined after shareholder discussions.

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