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Towards the Design of Better Equity Benchmarks

Rehabilitating the Tangency Portfolio from Modern Portfolio Theory

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Abstract
Following recent research on the relevance of idiosyncratic risk in asset pricing models, I propose to use total volatility as a model-free estimate of a stock's excess expected return, and analyze the implications in terms of the design of improved equity benchmarks. I find that maximum Sharpe ratio portfolios consistent with such expected return proxies, and built upon improved estimates of the correlation parameters, significantly outperform market cap weighted schemes on a risk-adjusted basis. This analysis, which rehabilitates the role of the tangency portfolio from modern portfolio theory, suggests that better equity benchmarks can be designed, provided that a sophisticated portfolio optimization procedure is used that relies on robust estimates of moments and co-moments of stock return distributions. This paper has important potential implications for the ongoing debate on appropriate weighting schemes for equity indices.

Lionel Martellini would like to thank Daniel Mantilla for superb research assistance, as well as Noël Amenc and Devraj Basu for very useful discussions.

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1. Introduction

With an ever increasing volume of assets managed under passive indexing strategies, whether packaged under a mutual fund format or under an exchanged-traded fund format, the standard practice of constructing stock market indices based on a capitalisation-weighting scheme has recently been subject to heightened scrutiny and renewed criticism. More than fifteen years ago, a number of papers (including Haugen and Baker (1991) or Grinold (1992)) had already offered convincing empirical evidence that market-cap weighted indices provide an inefficient risk-return trade-off. From the theoretical standpoint, the poor risk-adjusted performance of such indices should not come as a surprise given that the efficiency of these weighting schemes comes only at the cost of heroic assumptions.

More recently, equity indices with different weighting schemes have emerged as alternatives. A particular type of alternative weighting scheme is currently implemented by indices that use accounting characteristics to weigh the component stocks (Arnott, *et al.* (2005)). In a different spirit, Fernholz, *et al.* (1998) propose to use stock market diversity, a measure of the distribution of capital in an equity market first introduced in Fernholz (1999), as a weighting factor. Under a set of simplifying assumptions, Fernholz (1999) shows that the return of a diversity weighting scheme relative to the market cap weighting scheme is, among other things, a function of the difference between the weighted-average variance of the individual stocks and the portfolio variance. Closely related is a recent paper by Choueifaty and Coignard (2006), who introduce in a portfolio optimization context a so-called "diversification index", given by the ratio of average volatility of the stocks in the portfolio divided by the portfolio volatility.[1] One last example is the emergence of equally weighted counterparts to well known indices such as the S&P500 index.

From the theoretical standpoint, however, it is not clear how any of these attempts can be grounded in modern portfolio theory, which states that a mean-variance utility-maximizing agent should hold the portfolio with the highest reward-to-risk ratio, also known as the tangency portfolio or the maximum Sharpe ratio (MSR) portfolio. In fact, the equally-weighted portfolio can be regarded as a MSR portfolio, albeit under the rather specific assumption that all stocks' expected returns, volatilities and pairwise correlations are identical. While discarding all possible information about the moment and co-moment structure of stock returns may appear as an extreme stance, most attempts to implement the prescriptions of scientific diversification on the basis of sample-based parameter estimates have generated rather disappointing results in thorough out-of-sample empirical tests.

In a recent paper, DeMiguel *et al.* (2007) argue that the presence of estimation error in input parameters almost entirely invalidates the performance of formal optimization models, even when improved estimators are used, unless for unreasonably large sample size. That paper, following most existing research on the domain, including Chan *et al.* (1999), Jagannathan and Ma (2003), has focused on testing the out-of-sample performance of global minimum variance (GMV) portfolios based on various improved estimators of the variance-covariance matrix, and has found they were dominated by the simple equally weighted rule in terms of risk-adjusted performance. The reason why these authors have chosen to focus on the out-of-sample performance of GMV portfolios, as opposed to MSR portfolios is related to the widespread consensus that expected returns are difficult to obtain with a reasonable estimation error. What makes the problem worse is that optimization techniques are known to be very sensitive to differences in expected returns, so portfolio optimizers typically allocate the largest fraction of capital to the asset class for which estimation error in the expected returns is the largest (*e.g.*, Britten-Jones (1999) or Michaud (1998)).

This paper is an attempt to put the focus back on the only truly optimal weighting scheme consistent with modern portfolio theory, *i.e.*, the one that is designed to achieve the highest Sharpe ratio. In a nutshell, I claim that using improved estimators for the variance-covariance matrix of stock returns, as well as for their expected returns, allows one to design proxies for the tangency portfolio

1 - Maximizing the diversification index is formally equivalent to maximizing the portfolio Sharpe ratio under the assumption that expected excess return parameters are proportional to volatility parameters.

that outperform their market-cap-weighted counterparts on a risk-adjusted basis in out-of-sample exercises. I also find evidence that such benchmarks may outperform equally weighted schemes, a rare occurrence in previous similar studies.

These results are obtained from an approach consisting of linking expected return estimation to volatility estimation, which builds on the recent literature on the explanatory power of idiosyncratic volatility for the cross section of expected returns. In particular, Malkiel and Xu (2006), extending an insight from Merton (1987), show that an inability to hold the market portfolio, whatever the cause, will force investors to care about total risk to some degree, in addition to market risk, so firms with larger firm-specific variances require higher average returns to compensate investors for holding imperfectly diversified portfolios (see also Barberis and Huang (2001) for a similar conclusion from a behavioral perspective). That stocks with high idiosyncratic volatility earn higher returns has also been confirmed in a number of recent empirical studies, including in particular Tinic and West (1986) as well as Malkiel and Xu (1997, 2002).

These findings, taken together with the fact that asset pricing theory implies a positive premium for systematic risk, suggest that expected returns should be positively related to total volatility, a model-free quantity given by the sum of systematic and idiosyncratic risk components as measured against the prevailing asset pricing model. In this context, I propose to use a stock's volatility as a robust proxy for its expected excess return, and investigate the implications in terms of portfolio construction and performance (see Basu and Martellini (2007) for the asset pricing implications of the relationship between total volatility and expected return).

2. On the Relationship between Total Volatility and Expected Return

In this section, I first report empirical evidence that supports the relationship between total volatility and expected returns. For this analysis, I use monthly excess stock return data from the Center for Research in Security Prices (CRSP), with a total of 682 stocks that survived over the entire 1975-2004 sample period.

I first consider the time-series evidence for whether stocks with high volatility have higher average returns than stocks with lower volatility. For this, I follow the procedure in Ang *et al.* (2006) and sort individual stocks into equally-weighted quintiles based on their total volatility calculated on the basis of the previous 120 months' (10 years) returns, hold these portfolios for 1 month and then repeat the process. I thus obtain a time series of returns for the quintiles starting in January 1985, and compute the difference, which I call the 1-5 premium, in average return between the top quintile (20% highest volatility stocks) and the bottom quintile (20% lowest volatility stocks). The results are reported in exhibit 1, and show that the 1-5 annualized premium over the 1985-2004 period reaches a spectacular annualized value of 8.47%, with a p-value of 2.38%, indicating that stocks with higher volatility earn significantly higher returns than stocks with lower volatility. While the premium is not significantly positive for quintiles 2, 3 and 4, a monotonic increase is found for annualized performance as a function of volatility. Exhibit 2 shows the cumulative returns on a strategy that goes long the top quintile and short the bottom quintile, which confirms the economic significance of the effect.

I further analyze whether total volatility has explanatory power for the cross-section of expected returns by following the procedure in Fama and Macbeth (1973), and at each point in time run a cross sectional regression of realized return on total estimated volatility:

$$R_{i,t} = \alpha_{i,t} + \gamma_t \sigma_{i,t} + \varepsilon_{i,t}$$

I then average the slope coefficient in this regression across time $(\bar{\gamma})$ to obtain an estimate for the unconditional slope coefficient. The t-statistic for the slope coefficient is given by

$$t_\gamma = \frac{\sqrt{T}\bar{\gamma}}{\sigma_\gamma} \text{ where } \sigma_\gamma = \frac{1}{T-1}\sum\left(\gamma_t - \bar{\gamma}\right)^2.$$

Total volatility is computed using the previous 120 months of data so that our regression begins in 1985. I find that the average slope coefficient when using total volatility as a regressor over the 1985-2004 period is 0.1095 with a p-value of .01%, showing that total volatility has explanatory power for the cross section of stock returns.

Given that total volatility is the sum of systematic volatility with respect to some asset pricing model and idiosyncratic volatility with respect to the same asset pricing model, one can wonder whether these results might be mostly driven by the systematic component of volatility. To test for this, I decompose total volatility into Fama-French systematic volatility and idiosyncratic volatility, and use these as separate regressors. I find that over the 1985-2004 period the slope coefficient for Fama-French systematic volatility is 0.0257 and is not significant, while that for idiosyncratic volatility is 0.1122 with a p-value of 0.03%, indicating that the explanatory power of total volatility is driven by idiosyncratic volatility with respect to the Fama-French model.

3. Implications for Portfolio Optimization and the Design of Efficient Equity Benchmarks

The focus of this paper is not to assess empirically the statistical significance and asset pricing implications of the relationship between total volatility and expected return at the individual stock level (a task performed in Basu and Martellini (2007)), but to try to assess the economic significance of such findings for portfolio selection and equity benchmark construction. To this end, I perform the following experiment. I randomly draw from our base universe of stocks 100 portfolios of 100 stocks each. For each of the 100 sub-universes, I estimate on a monthly basis the maximum Sharpe ratio (MSR) portfolio, as well as the global minimum variance (GMV) portfolio, based on the past 120 months of sample information, and record their performance on an out-of-sample basis.

In terms of the correlation matrix, I use the following 3 estimates: i) sample correlation matrix, ii) constant correlation matrix (see Elton and Gruber (1973)), and iiii) factor-based correlation matrix, where the Fama-French 3 factor model is used.[2] While the first estimator (sample correlation matrix) is likely to be plagued by estimation error due to the high number of parameters to estimate, the second estimate (constant correlation matrix) is likely to be impaired instead by the presence of significant specification risk and implies forgoing the extraction of *any* useful information about the co-movement structure of stock returns from the sample. On the other hand, the factor-based estimator is expected to allow a reasonable trade-off between sample risk and model risk. In terms of the volatility vector, I use in this section the sample estimate. These volatility estimates are used to obtain the covariance matrix needed for the GMV portfolios. In keeping with the discussion in the previous section, I also use them as proxies for excess expected return estimates in the case of the MSR portfolios. Hence, the MSR portfolio, whose composition is well known to be given by $\frac{\Sigma^{-1}\mu}{1_N \Sigma^{-1}\mu}$, can in this case be re-written as $\frac{\Sigma^{-1}\sigma}{1_N \Sigma^{-1}\sigma}$, where Σ is the N-dimensional stock return covariance-matrix, μ is the N-dimensional vector of excess expected returns, σ is the N-dimensional vector of volatilities, and 1_N is an N-dimensional vector of ones. It should be noted that if one assumes that volatility is identical for all stocks, then the MSR portfolio based on volatility as a proxy for excess expected return coincides with the GMV portfolio

$$\frac{\Sigma^{-1}1_N}{1_N \Sigma^{-1}1_N}.$$

2 - We have also tested a correlation matrix estimate based on an optimal shrinkage of the sample estimator towards the constant correlation estimator (Ledoit and Wolf (2003)), and have obtained results similar to the factor-based estimate, except when shrinkage intensity factors tend to reach values close to zero or one, leading to a trivial positioning with respect to the trade-off between sample and model risk. We do not report those results for the sake of brevity.

In exhibit 3, I report minimum, bottom 25th percentile, median, top 25th percentile and maximum out-of-sample values (over the 100 randomly chosen portfolios) for expected returns, volatilities and Sharpe ratios of the MSR and GMV portfolios, and I compare these values to risk and return indicators for the value-weighted and equally weighted counterparts. A number of conclusions can be drawn from this analysis. First, the value-weighted index is outperformed by its equally weighted counterpart in terms of Sharpe ratio, suggesting again that market cap weighting may not be the most appropriate weighting scheme as far as portfolio efficiency is concerned. Focusing then on GMV portfolios, it appears that they lead to lower out-of-sample volatility compared to the value-weighted and also equally weighted benchmarks when an improved estimator of the correlation matrix is used that offers a reasonable trade-off between sample and specifications risks. As expected, the sample and correlation estimators, on the other hand, do not yield very attractive out-of-sample results. If they typically dominate those of their value-weighted benchmarks, the Sharpe ratios for GMV portfolios however remain lower than those of the equally-weighted benchmark because of poorer average performance. Taken together, these results are consistent with those in DeMiguel *et al.* (2007) and suggest that focusing solely on risk minimization leads to opportunity costs with respect to a naïvely diversified benchmark as far as performance is concerned. Turning now to the MSR portfolios, which were not considered by DeMiguel *et al.* (2007), I find, on the other hand, that consistently higher out-of-sample Sharpe ratios are obtained on the basis of using volatility as a proxy for a stock expected return, provided that a reasonable (here, factor-based) estimator of the correlation structure of stock returns is used. In fact, when using a constant correlation estimator, equivalent to throwing away all information from the correlation matrix, Sharpe ratios for the MSR portfolios are already roughly equivalent to those of the equally weighted benchmarks.[3]

As a robustness check, I also looked at the maximum Sharpe ratio portfolios using expected return estimates implied by the Fama-French model, and based on the same competing covariance matrix estimates. The results, reported in exhibit 4, suggest that using a factor-model not only for the covariance estimates, but also for expected return forecasts, leads to out-of sample Sharpe ratios that are lower than when total volatility was used as a proxy for expected returns. Furthermore, the out-of-sample Sharpe ratio of the MSR portfolio is no longer greater than this of the corresponding GMV portfolio. This result, which stands in sharp contrast to the case when total volatility was used as an expected return estimate (see exhibit 3), is consistent with the aforementioned findings that the explanatory power of total volatility in the crosssection of expected returns is not driven by systematic volatility with respect to the Fama-French model.

4. Improving volatility estimates

The analysis conducted in the previous sections puts a strong emphasis on volatility inputs, which are used as proxies for both the risk and return parameters on individual stock returns. This suggests that particular attention should be devoted to the estimation of theses parameters of interest. In particular, a vast amount of empirical evidence has been accumulated documenting the fact that stock market volatility randomly evolves in time. To account for the presence of volatility risk, I extend the previous analysis to a setup where variances are forecasted with a simple GARCH (1,1) model, with the parameter estimates obtained by a maximum likelihood procedure for a model with normal innovations. Given the date t parameter estimates, estimates of the conditional variance for month $t+1$ are obtained and used as 1-step forecasts each month during the out-of-sample 1985-2004 period.

The results, which are reported in exhibit 5, show that using a simple dynamic volatility model allows for a further improvement in the risk-adjusted performance of the MSR portfolio (as well as the GMV portfolio) that strongly dominates its equally weighted and value-weighted counterparts.

3 - In case of a constant correlation across all pairs of stocks, it can be shown that when total volatility is taken as a proxy for a stock is excess expected return the MSR portfolio is obtained by using the reciprocal of each stock volatility as a characteristic-based weighting scheme.

5. Conclusions and Suggestions for Further Research

In this paper, I have reported empirical evidence of the relationship between a stock is total volatility estimate and its expected return, and have examined the implications in terms of portfolio selection. I have found that tangency portfolios with attractive out-of-sample risk-adjusted performance properties can be obtained when using volatility as a model-free proxy for a stock's excess return and when a reasonable amount of structure is imposed in the covariance matrix estimation process. These portfolios, which strongly dominate their simple value-weighted counterparts, can be used as convenient benchmarks for the normal performance on equity markets.

In the end, it should be acknowledged that total volatility is probably not the best possible proxy for excess expected returns. The interesting fact is that using even such a crude and simplistic measure allows for promising results, with a potential for outperforming market-cap-weighted indices, but also equally weighted benchmarks, in terms of out-of-sample Sharpe ratio.

This work can be extended in several directions. First, one can be tempted to improve upon the covariance matrix estimation process. While I have used the Fama-French model for simplicity, improved covariance forecasts can instead be generated from an implicit factor model in an attempt to alleviate the concern over factor specification risk. Besides, a conditional version of the factor model would allow one to capture time variations in correlation and/or volatility parameters. The analysis can/should also be extended beyond the mean-variance setting. In particular, recent research has shown that in the presence of non-normally distributed asset returns, investors' preferences over skewness and kurtosis have an impact on expected returns (see Harvey and Siddique (2000), Dittmar (2002), or Mitton and Vorkink (2007)). In this context, robust estimators for the coskewness and cokurtosis parameters can be introduced as in Martellini and Ziemann (2007) in an attempt to further enhance the risk-adjusted performance of equity benchmarks. These, as well as other related questions, are left for further research.

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Exhibit 1: Annualized performance of each of the 5 quintiles, as well as associated quintile premia (with respect to the high volatility quintile). In this table we provide the out-of-sample performance of volatility quintiles over the 1985-2004 period. Returns are given in percent per year. P-values indicate whether the respective annualized premium is statistically significant.

	annualized return	annualized premium	p-values (>0)
Quintile 1	22.99%	8.47%	2.38%
Quintile 2	15.94%	1.42%	30.39%
Quintile 3	15.31%	0.79%	35.62%
Quintile 4	15.24%	0.72%	32.68%
Quintile 5	14.52%	0.00%	NA

Exhibit 2: This figure shows the cumulative returns on a zero-cost portfolio strategy that is long the top equally weighted quintile of stocks sorted on total volatility and short the bottom equally weighted quintile sorted on total volatility over the 1985-2004 period.

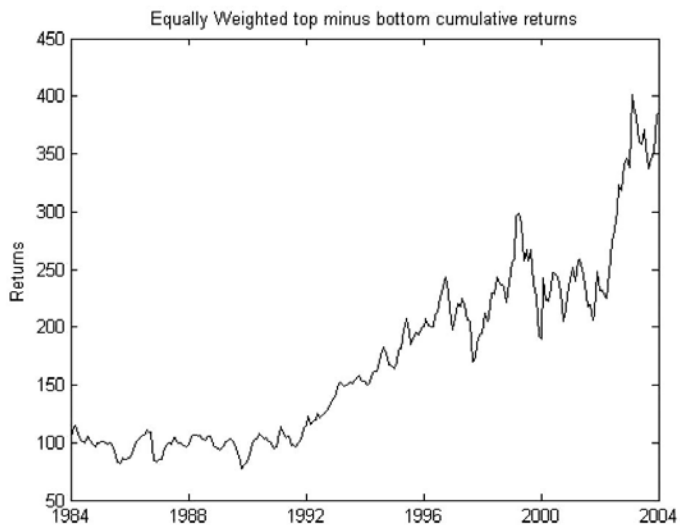

Exhibit 3: Risk and return performance indicators of various portfolios constructed from 100 random draws of 100 stocks, based on a sample estimate of stock volatilities and using total volatility as a proxy for a stock excess expected return. In this exhibit we provide the out-ofsample (geometric) annualized performance, annualized volatility and Sharpe ratio of various optimized portfolios over the 1985-2004 period. MSR stands for maximum Sharpe ratio portfolio, and GMV stands for global minimum variance portfolio. In all cases, the sample estimate for volatility has been used in this table. Expected return estimate is taken to be given by the sample volatility estimate for the MSR portfolio. Various estimates of the correlation matrix have been used, including the sample correlation estimate, the constant correlation estimate, and the factorbased (see details in the text).

Sample Volatility		Expected return					Volatility					Sharpe ratio				
		Min	25%	Median	75%	Max	Min	25%	Median	75%	Max	Min	25%	Median	75%	Max
Sample Correlation	MSR	-67.79%	4.31%	13.79%	25.23%	3233%	36.39%	55.05%	66.81%	111%	13082%	-0.233	-0.001	0.221	0.354	0.839
	GMV	-11.28%	0.40%	3.81%	7.23%	19.94%	22.12%	24.34%	25.28%	26.62%	49.08%	-0.665	-0.053	0.112	0.282	0.832
Constant Correlation	MSR	9.37%	10.97%	11.53%	12.07%	13.28%	13.75%	14.26%	14.40%	14.54%	14.90%	0.635	0.761	0.799	0.836	0.948
	GMV	2.71%	5.48%	6.54%	7.48%	11.25%	12.20%	12.85%	13.21%	13.50%	16.10%	0.177	0.397	0.493	0.567	0.852
Factor Correlation	MSR	5.76%	9.76%	11.27%	12.53%	15.85%	11.29%	12.65%	12.97%	13.43%	15.91%	0.449	0.747	0.874	0.955	1.192
	GMV	4.57%	6.98%	8.22%	9.14%	11.58%	9.84%	0.61%	10.85%	1.06%	11.97%	0.435	0.637	0.758	0.842	1.072
Equally Weighted	EW	9.71%	11.67%	12.59%	13.03%	14.81%	14.97%	15.71%	15.95%	16.26%	17.28%	0.601	0.727	0.783	0.812	0.917
Value Weighted	VW	7.22%	9.09%	9.83%	10.43%	12.12%	15.47%	15.83%	16.07%	16.27%	16.67%	0.455	0.574	0.613	0.651	0.770

Exhibit 4: Risk and return performance indicators of various portfolios constructed from 100 random draws of 100 stocks, based on a sample estimate of stock volatilities and using Fama-French model to forecast a stock excess expected return. In this exhibit we provide the out-ofsample (geometric) annualized performance, annualized volatility and Sharpe ratio of various optimized portfolios over the 1985-2004 period. MSR stands for maximum Sharpe ratio portfolio, and GMV stands for global minimum variance portfolio. In all cases, the sample estimate for volatility has been used in this table. Expected return estimate is taken to be given by the Fama-French factor model for the MSR portfolio. Various estimates of the correlation matrix have been used, including the sample correlation estimate, the constant correlation estimate, and the factor-based (see details in the text).

Sample		Expected return					Volatility					Sharpe ratio				
Volatility		Min	25%	Median	75%	Max	Min	25%	Median	75%	Max	Min	25%	Median	75%	Max
Sample	MSR	-430.7%	2.35%	11.77%	22.08%	166.9%	33.24%	44.62%	56.92%	98.74%	1982%	-0.2375	0.0349	0.1960	0.3785	0.8162
Correlation	GMV	-9.04%	3.60%	6.93%	9.89%	21.48%	21.70%	23.71%	24.80%	25.71%	34.24%	-0.3820	0.1399	0.2800	0.3820	0.8559
Constant	MSR	-94.02%	7.67%	11.58%	14.30%	62.18%	23.09%	26.69%	28.31%	32.56%	449%	-0.2093	0.2558	0.3802	0.4949	0.7395
Correlation	GMV	2.71%	5.48%	6.54%	7.48%	11.25%	12.20%	12.85%	13.21%	13.50%	16.10%	0.1768	0.3973	0.4931	0.5670	0.8517
Factor	MSR	8.05%	10.20%	11.33%	12.61%	16.23%	13.60%	14.17%	14.46%	14.81%	15.80%	0.5496	0.6951	0.7819	0.8652	1.0652
Correlation	GMV	4.50%	6.96%	8.20%	9.10%	11.56%	9.84%	10.61%	10.85%	11.06%	11.94%	0.4278	0.6335	0.7574	0.8401	1.0688
Equally Weighted	EW	9.71%	11.67%	12.59%	13.03%	14.81%	14.97%	15.71%	15.95%	16.26%	17.28%	0.601	0.727	0.783	0.812	0.917
Value Weighted	VW	7.22%	9.09%	9.83%	10.43%	12.12%	15.47%	15.83%	16.07%	16.27%	16.67%	0.455	0.574	0.613	0.651	0.770

Exhibit 5: Risk and return performance indicators of various portfolios constructed from 100 random draws of 100 stocks, based on GARCH forecasts for stock return volatilities. In this exhibit we provide the out-of-sample (geometric) annualized performance, annualized volatility and Sharpe ratio of various optimized portfolios over the 1985-2004 period. MSR stands for maximum Sharpe ratio portfolio, and GMV stands for global minimum variance portfolio. In all cases, volatility estimates have been taken to be the quintile estimates. Expected return estimate is taken to be given by the volatility estimate for the MSR portfolio. Various estimates of the correlation matrix have been used, including the sample correlation estimate, the constant correlation estimate, and the factor-based (see details in the text).

Sample		Expected return					Volatility					Sharpe ratio				
Volatility		Min	25%	Median	75%	Max	Min	25%	Median	75%	Max	Min	25%	Median	75%	Max
Sample	MSR	-2479%	-1.67%	11.88%	24.80%	3343%	39.56%	52.07%	61.13%	106%	14818%	-0.299	-0.032	0.212	0.354	0.663
Correlation	GMV	-6.52%	3.99%	6.95%	10.35%	21.66%	18.75%	20.72%	21.67%	22.54%	25.95%	-0.321	0.187	0.321	0.456	1.007
Constant	MSR	9.36%	11.02%	11.50%	11.98%	13.56%	13.66%	14.02%	14.16%	14.29%	14.73%	0.644	0.778	0.817	0.846	0.930
Correlation	GMV	5.16%	7.34%	8.12%	9.15%	12.32%	10.90%	11.68%	11.93%	12.15%	13.91%	0.408	0.601	0.692	0.755	1.023
Factor	MSR	6.46%	9.77%	11.26%	12.44%	15.61%	11.15%	12.23%	12.53%	12.90%	14.17%	0.502	0.782	0.902	0.988	1.267
Correlation	GMV	6.21%	8.56%	9.32%	10.52%	12.48%	9.60%	10.38%	10.60%	10.79%	11.93%	0.605	0.797	0.892	0.987	1.203
Equally Weighted	EW	9.71%	11.67%	12.59%	13.03%	14.81%	14.97%	15.71%	15.95%	16.26%	17.28%	0.601	0.727	0.783	0.812	0.917
Value Weighted	VW	7.22%	9.09%	9.83%	10.43%	12.12%	15.47%	15.83%	16.07%	16.27%	16.67%	0.455	0.574	0.613	0.651	0.770