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Overlay Hedging in a Fund of Funds

September 2008

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Abstract

Hedge funds are often referred to as absolute return strategies, yet investors are aware that most hedge funds do in fact take on a variety of systematic and quasi-systematic exposures. If a manager of a fund of hedge funds (FoF) finds that the exposure of the FoF to a certain systematic exposure or the risk level broadly has become excessive, then the FoF manager may want to hedge. The purpose of this article is to outline the major issues involved with overlay hedging in a fund of funds portfolio and to provide relevant solutions to these issues. These include the determination of whether to hedge, exposure estimation, hedging single exposures with futures, options, and other instruments, and hedging exposures broadly using a multi-factor approach.

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Introduction

Hedge funds are often referred to as absolute return strategies, yet investors are aware that most hedge funds do in fact take on a variety of systematic and quasi-systematic exposures. These exposures typically include a combination of standard "betas" (exposures to equities, rates, credit, currencies, and commodities) as well as to "alternative betas" (exposures to various spreads, volatility, the currency carry trade, merger arbitrage spreads, etc.). If a manager of a fund of hedge funds (FoF) invests too much in a particular type of manager, then the exposure of the FoF to a certain systematic exposure, or the risk level broadly, may become excessive. This may lead to a portfolio risk profile that is not consistent with the needs and desires of the FoF's clients. Similarly, the FoF manager may have a need to quickly reduce risk more broadly across all known exposures.

The FoF manager's first concern with respect to systematic exposures is to obtain a reasonably accurate estimate of these exposures, which may be difficult given the limited transparency offered by many hedge funds. Once a good estimate has been established, the FoF manager must consider the extent to which these exposures are appropriate for any given portfolio of hedge funds. If the exposures broadly or any individual exposures are excessive, the FoF manager has two choices—reduce risk by redeeming from managers, or hedge using derivatives contracts.[1] If the FoF manager has a deep belief in the alpha generating abilities of these managers despite the systematic exposures, or believes that these excessive exposures are temporary, then it may make sense to hedge. Without the ability to hedge, the FoF manager may be artificially constrained to redeem from high quality managers, thus potentially inducing lower returns for clients. This artificial constraint runs contrary to an important notion of hedge fund investing—that there are wider alpha opportunities available to managers once artificial constraints (e.g., too much beta) are lifted.[2] While articles have been written concerning FoF risk management[3] and portable alpha programs tied to FoFs,[4] little has been written on the topic of overlay hedging within a FoF portfolio.

The purpose of this article is to outline the major issues involved with overlay hedging in a fund of funds portfolio and to provide relevant solutions.[5] It is organised as follows. First, we provide a deeper analysis of the motivation for hedging. Second, we consider the choice of whether to hedge or to redeem from managers in order to reduce systematic (or quasi-systematic) exposures. Third, we consider the problem of estimating systematic exposures. Fourth, we consider single-instrument hedging with respect to the use of futures, options, and other instruments. Finally, we consider multiple-instrument hedging using a multi-factor approach (hedging both standard and alternative betas).

1. Motivation—Why Hedge?

In exhibit 1, we show the exposures of the HFRI Fund of Funds Composite index to the five broad asset classes.[6] It is clear that the systematic exposures of hedge funds vary widely through time. As markets trend, hedge fund managers may, over time, become more comfortable in taking particular types of risk. In particular, we note that equity exposure increased to approximately 50% during 2007 and the first half of 2008. If a fund of funds has broad exposure to the industry, then its exposure to equities is likely to match the relatively high levels shown here. Additionally, underlying manager beta may temporarily increase due to manager constraints such as uneven redemption patterns combined with illiquid positions.

1 - For a good overview of the use of derivatives in order to hedge and otherwise modify portfolio exposures, see Collins and Fabozzi (1999).

2 - See, for example, Grinold and Kahn (2000).

3 - See, for example, Christie (2007), Christory, Daul, and Giraud (2006), Pochon and Teiletche (2006), Krokhmal, et al. (2002), Lo (2001), and Jaeger (2000).

4 - See, for example, Amenc, Malaise, and Martellini (2006) and Schneeweis, Kazemi, and Gupta (2006).

5 - Note that we do not broach the issue of foreign currency denominated share classes in this paper, but are focused on the overlay hedging of standard and alternative betas at the master fund level.

6 - These are derived by taking the beta exposures from a rolling 24-month regression of the returns of the HFRI Fund of Funds Composite Index against the returns of five broad asset classes. The r-squared statistics of these regressions range from 51% to 89% with an average of 77%. Other researchers have used a similar methodology in this context. See for example Kuenzi and Shi (2007), Hasanhodzic and Lo (2006), and Fung and Hsieh (1997, 2004). The factors here are chosen so as to represent directional exposure to the five main global asset classes. All hedge fund data in this article comes from Pertrac/HFR. All other data (prices, rates, volatilities, etc.) comes from Bloomberg. Exhibits 1, 2, and 4 show 60 months of results 7/2003 to 6/2008, which means we use 83 months of data—8/2001 to 6/2008.

Exhibit 2 shows the percentage contribution to portfolio risk of these exposures.[7] This graph is even more dramatic, as it shows a significant increase in equity risk as a percentage of total systematic risk from 2004 to 2007. Given this level of exposure to a single systematic factor, the ability to hedge rather than redeem from potentially high-alpha managers may be critical to maintaining edge in a FoF portfolio. Hedging a FoF portfolio may also be desirable from the perspective of managing structured products based on FoF portfolios. If the delta-hedging of a structured product or uneven maturity and launch dates requires a firm to bring FoF positions into inventory, then it may make sense to hedge all of the exposures shown in exhibit 1 (and perhaps others as well).

2. The Choice: Hedge or Redeem from Managers

The first question to consider is the general purpose of the hedge—to reduce the overall risk of the portfolio (pro-rata across all risk types) or to reduce one key risk, such as equity exposure. If it is the former, then the choice is whether to raise cash or hedge broadly using a multi-factor approach. If it is the second (there is a factor exposure that is too high, such as equity risk), the choice is either to hedge or to redeem from high beta managers (or from those managers most associated with the undesirable systematic exposure). Redeeming has two very clear drawbacks: manager liquidity provisions and manager relationships.

Hedge funds offer different levels of liquidity to their investors. Liquidity provisions are usually quoted as the redemption frequency and the required notice period. Quarterly redemption/45-day notice means that the FoF can redeem every calendar quarter so long as the manager is notified of the redemption at least 45 days before quarter-end. While these are probably the most typical redemption provisions, other managers might be quarterly/60, quarterly/90, semi-annual/90, annual/90, and so forth. Additionally, many managers (and especially activist and related high beta managers) may also have lock-ups such that investors are unable to redeem for some long initial period (say, two years). Given the variety of possible liquidity provision, it may not be possible for a FoF manager to reduce standard and/or alternative beta in a timely fashion through manager liquidations.

Investor relationships with hedge fund managers are also important, especially with regard to highly sought-after managers. A large or full redemption from a manager may preclude the ability to increase the FoF's allocation to that manager in the future. As such, if the FoF manager believes that the hedge fund manager has alpha and wants to preserve a strong relationship, he or she may not want to signal otherwise with a large redemption.

Overlay hedging can be implemented and rebalanced quickly and in response to changes in manager behaviour, changes in the FoF portfolio, and changes in the risk inherent in the overall environment. The most serious potential drawbacks of taking this approach in order to reduce exposures are basis risk and negative fee arbitrage. We discuss basis risk with respect to hedging individual exposures and in the multi-factor context in the pertinent sections below. The negative fee arbitrage is based on the fact that most systematic exposures have a positive expected return over time, from which the manager is collecting an enormous incentive fee, while at the same time the overlay hedging serves to cancel that exposure.

If one is considering embarking on an overlay hedging program, then it is critical to quantitatively consider this negative fee arbitrage. We provide a methodology and example using US equities. This analysis involves the determination of the level of manager alpha needed in order to overcome the drag of paying fees on large amounts of beta. At a particular expected return for US equities and a given estimated manager beta, how much alpha does the manager need to generate in order to

7 - Following Grinold and Kahn (2002), we define portfolio standard deviation as: $\sigma_p = \sqrt{h_p^T V h_p}$, where h_p is a vector of portfolio weights, V is the factor return covariance matrix, and T indicates the vector transpose. To get the marginal contribution to total portfolio risk (MCTR), we take the partial derivative of this standard deviation with respect to the portfolio weight vector:
$\frac{\partial \sigma_p}{\partial h_p} = \frac{1}{2} \cdot \frac{1}{\sqrt{h_p^T V h_p}} \cdot 2Vh_p = \frac{Vh_p}{\sigma_p} = MCTR$.
Since MCTR is a vector with each element being a portfolio allocation's MCTR, we can calculate the total percentage contribution to portfolio standard deviation (CPSD) of each portfolio factor exposure as:
$CPSD = \frac{MCTR \otimes h_p}{\sigma_p}$, *where* $\otimes$ *denotes element by element multiplication. We use a rolling 24-month data set of factor returns in order to calculate V.*

overcome the fact that we are paying 2/20 (typical hedge fund terms—a 2% management fee and a 20% performance fee) to get an exposure that we are promptly cancelling through a hedging program? If the FoF portfolio manager believes that certain hedge funds in a portfolio are not able to clear this alpha hurdle, then perhaps manager redemptions would be preferred to hedging.

The framework for determining how much alpha a manager must provide to offset the fees paid on low-cost beta is as follows. First, define the percentage incentive fee (paid to the manager) as f (e.g., f = 20%), and then define $\bar{f} = 1 - f$, or the portion of after-management fee returns retained by the FoF. Using the capital asset pricing model we can write the expected manager return as:

$$R_i - R_F = \alpha + \beta(R_M - R_F) \Rightarrow$$
$$R_i = R_F + \alpha + \beta(R_M - R_F) \quad (1)$$

where R_i denotes the manager's gross returns, β is the manager's stock market exposure, R_M is the return of the market index (for the example, we assume 9%), R_F is the risk-free rate (we assume 4%), and α is the manager's alpha. Now define the net returns to the FoF as $N = \bar{f}(R_i - m)$, and the manager's compensation as $f(R_i - m) + m$, where m is the percentage annual management fee (e.g., 2%). (Management fees are paid first from gross returns; incentive fees are then calculated on the reduced returns.)

Define an alternative (naïve) strategy of investing in an S&P 500 ETF in order to get the desired portfolio beta. Leverage can then be defined as β - 1. The cost of this additional leverage is $(\beta - 1)R_F$, and the net return to the alternative strategy (denoted by A) is as follows:

$$A = \beta R_M - (\beta - 1)R_F = R_F + \beta(R_M - R_F) = R_{0\alpha} \quad (2)$$

These are the returns of a zero-alpha strategy ($R_{0\alpha}$) that charges very low fees.[8] Now define the advantage to the FoF of the hedge fund manager who provides a net return N compared to the alternative A as D, where

$$D = N - A = \bar{f}(R_i - m) - R_{0\alpha} \quad (3)$$

We now define α^* as the break-even value of α that makes the FoF indifferent between the two alternatives (D = 0). For any manager whose alpha is expected to be greater than α^*, the FoF will prefer the manager investment; however, if the alpha is lower, the FoF will prefer the mechanical ETF strategy. Using the definition of R_i and the fact that $\bar{f} - 1 = -f$ we can simplify the equation for D = 0 to solve for α^*:

$$\alpha^* = m + \frac{fR_{0\alpha}}{\bar{f}} \quad (4)$$

In exhibit 3 we calculate the breakeven alpha minus the management fee component for a variety of beta and performance fee levels (we exclude management fees to focus more purely on the negative fee arbitrage associated purely with performance fees). The impact of paying fees for beta and then hedging it out is significant. In fact, it may be desirable to invest in highly hedged managers and to then supplement these primarily alpha exposures with cheap beta through long equity index futures positions, long CDX positions, etc. In this way, the FoF manager is paying performance fees and relatively high management fees only on non-directional and difficult-to-obtain exposures.

3. Estimation of Systematic Exposures in a Fund of Funds

Given limited transparency, the FoF portfolio manager should employ a variety of means to estimate the systematic and/or quasi-systematic exposures of a given FoF portfolio. The most widely used tool for estimating unknown systematic exposures is regression analysis. A rolling multiple regression analysis as shown in exhibit 1 can help to broadly identify the portfolio's systematic exposures,

8 - To simplify the analysis, we assume zero fees; the total expense ratio of the S&P 500 SPDRs is approximately 9.5 basis points.

while a rolling univariate analysis, as shown in exhibit 4, can help to isolate just one.[9] Using both multi-variate and univariate regressions, combined with good bottom-up information, enables the FoF portfolio manager to deduce a best estimate of a portfolio's systematic exposures. Once one has broadly determined the portfolio's systematic exposures, one can then determine whether hedging is necessary and which exposures should be hedged.

If it is decided that a particular exposure or set of exposures should be hedged, one then needs to estimate exposures again but this time only with reference to the exposures that will be hedged. With only one hedging instrument (e.g., equity futures) we should consider only one source of risk in estimating portfolio exposures. The use of a multi-factor approach will almost invariably create issues with multi-collinearity. As such, one cannot estimate several exposures simultaneously and then use the resulting output in order to hedge just one exposure. If we take a multi-factor approach to exposure estimation, then we must hedge using all of those factors. (We will consider the multi-factor approach more thoroughly in a section below.)

If, for instance, one completed the analysis shown in exhibit 1 on a FoF and obtained similar results, there may be a decision to hedge a portion of the FoF's equity exposure. The next step would be to perform a univariate analysis in order to observe the FoF's exposure to equities, whether arising from actual equity exposures or those implicit in long credit, short Treasury, etc.[10] (see exhibit 4). It is then this total exposure that must be considered for hedging purposes. In order to triangulate further on the expected systematic exposures or the portfolio, it is helpful to consider several rolling periods (12-month beta, 24-month beta, and 36-month beta, for instance) in order to get some context around quantitatively estimated beta exposures. An example of this analysis for some hedge fund indices is shown in exhibit 5 (using data through June 2008). It is clear that these exposures can differ substantially. As such, the manager may also want to get estimates from the FoF's bottom-up analysts. This might simply be a qualitative assessment as to how managers are generally positioned, or it might involve, for each manager, an estimate of the total net exposure along with the best fit equity benchmark. This would then allow for "conversion" from each best fit benchmark to the index represented by the hedging instrument,[11] and then aggregation to a portfolio level beta estimate.

4. Hedging a Single Exposure with Futures

Given the preponderance of overall industry exposures shown in exhibits 1 and 4, it is clear that a lot of factor risk can be removed from the average FoF portfolio by selling equity futures. This type of hedging program is particularly appropriate for activist, private equity, and long-biased managers, who typically take on significant market risk and whose skill set does not usually involve alpha shorting and hedging. Given that these managers tend to be predictably net long, a short futures position provides for a stable offset. The key issues for hedging with futures are establishing and updating a hedge target, the hedge rebalance process, basis risk, and capital support (margining).

Based on an analysis of the fund's exposures as described in the previous section, the FoF portfolio manager should set a hedge target for each portfolio. This may be based on a "mosaic" approach to determining a best guess range of portfolio beta and then setting a specific hedge level (e.g., a hedge target of -0.06 S&P 500 beta points). Alternatively, it may involve the determination of a single point estimate for current portfolio beta measured against a desired maximum beta (e.g., current estimated portfolio beta is 0.36, maximum beta is 0.3, so the hedge is -0.06).

9 - This analysis can be approached in two ways, the results of which should be the same. The first method is to generate a pro-forma FoF portfolio return stream (by taking current portfolio allocations applied to each manager's historical track record) and then perform the regression analysis on this pro-forma track record. The second is to run a regression on each individual manager currently in the FoF portfolio and to then aggregate the resulting exposures to the portfolio level using the manager's portfolio percentage allocation.

10 - The relationship between equities and credit is evident (both securities are dependent on company fundamentals; see also Fung, et al. 2008). A negative correlation between equities and Treasuries can become quite pronounced in volatile market environments (monthly correlation of -0.59 July 2006 through July 2008), as there is a "flight to quality" effect in Treasuries.

11 - This involves estimating the sensitivity of the best fit "benchmark" index to the index underlying the hedging instrument. We recommend the use of volatility ratios as an adjustment factor rather than the use of a standard beta. The beta of the manager to the hedging instrument, $\beta_{M,I}$ times the beta of the best fit index to the hedging instrument, $\beta_{I,H}$ gives us $\rho_{M,I}\frac{\sigma_M}{\sigma_I}\rho_{I,H}\frac{\sigma_I}{\sigma_H} = \rho_{M,I}\rho_{I,H}\frac{\sigma_M}{\sigma_H}$. Given that ρ is typically less than 1, this method will in most cases reduce beta. The approach biases beta lower. The volatility ratio approach does not behave this way: $\rho_{M,I}\frac{\sigma_M}{\sigma_I}\rho_{I,H}\frac{\sigma_I}{\sigma_H} = \rho_{M,I}\rho_{I,H}\frac{\sigma_M}{\sigma_H}$.

The amount of any hedge rebalance will be driven by a variety of factors, including inflows and outflows, portfolio performance, and hedge performance. A basic hedge rebalance tool is shown in exhibit 6, with the two approaches noted in the previous paragraph depicted in panels 1 and 2, respectively. The rebalance process is fairly straightforward in either case. The rebalance process can, however, become somewhat more complicated if one wishes to use several underlying equity indices to hedge. In this case, one must identify a "base" index for the purposes of describing portfolio beta, the hedge target, or the maximum portfolio beta. All hedge positions must be converted to units of this base index to ensure that the portfolio is being hedged to the appropriate degree. This conversion should be performed using the method shown in footnote 11. Exhibit 7 depicts a hedge rebalance tool that uses two underlying indices to hedge FoF portfolios. In this case we convert everything to S&P 500 equivalents to determine how much to hedge. We then (in columns I and J) determine the proportion of the rebalance that should go to each of the two underlying indices in use, and finally derive the contracts to trade as in the previous example.

One drawback of using equity futures within a FoF portfolio, particularly in the case of the hedge target method depicted in panel 1 of exhibit 6, is that of negative gamma. The hedge position is likely to move much more dramatically than the underlying portfolio in either a strong up or down market. In a strong up market, the notional value of the hedge will increase, thus leading to a reduction in the hedge position through the purchase of futures. In a strong down market, the notional will decrease, leading to further sales of futures contracts in order to maintain the hedge target. While a second order effect and not extraordinarily consequential, it is important to be aware of this negative gamma impact.

Another critical issue is basis risk. If one is hedging using an equity futures contract that is not representative of the equity exposure in the FoF portfolio, there is a substantial risk that the FoF investments could retreat while the hedge overlay provides little in the way of off-setting gains. Overall, it is incumbent on the FoF portfolio manager to qualitatively obtain a rough idea of the region, size, and style exposures that are, on average, most prevalent in the FoF portfolio and to then include this information when determining the most relevant hedging instruments.

Capital support can be an issue for a FoF given limited manager liquidity. With no leverage facility, the manager may be taking substantial margin call risk, given that the liquidity of the underlying hedge fund investments may require substantial notice in order to redeem. If the hedging program is quite large and the FoF manager would like to maintain substantial excess margin, then it may also be important to establish a leverage facility at the FoF level to ensure that the FoF can remain fully invested. A hedge of 30% with total margin of 15% would require 4.5% of AUM to support. Without a leverage facility, this would leave the FoF 95.5% invested, and even with this high level of over-margining there may be substantial margin call risk in a volatile environment. As such, a leverage facility is an important accompaniment for a futures hedging program.[12]

Another margining issue is the currency denomination of the futures contract. Margin must be posted in the currency of the contract being traded (S&P 500 contracts on the CME are supported by USD margin, while EuroStoxx 50 contracts traded on Eurex are supported by EUR margin). As such, hedging with contracts outside of the portfolio's base currency may induce currency risk unless this is hedged as well.

5. Hedging a Single Exposure with Vanilla Options

Again, given the overall industry exposures shown in exhibit 1, it is quite possible that some FoF portfolios, and especially those with a large number of relative value managers, might have both excessive equity exposure as well as short volatility exposure. The simplest way to offset this

12 - It should also be noted that this is one reason it can be difficult to use ETFs in order to hedge. Regulation T leads to capital requirements well in excess of those associated with a futures hedge.

combination is to buy vanilla at-the-money (ATM) or out-of-the-money (OTM) puts. The main considerations involve the best way to measure hedge exposure, the pricing of options (options tend to be most rich when they are most needed), and the extent to which spreads, straddles, or other combinations might be legitimate alternatives to an outright long put position.

Given the complexity of options, there are many possible ways to think about the exposures that result from their purchase. One could think about a desired dollar or percentage vega (portfolio gain for a 1% increase in implied volatility), the delta exposure inherent in the options contracts, a desired gamma profile (how much will the portfolio delta change as the market sells off?), or the total notional of the hedge (i.e., if the options are deep in the money, what percentage of the total portfolio is hedged?). The exposure measured and targeted should, of course, be related to the intended role of options in the portfolio. If, for instance, the intent is to hedge against changes in implied volatility, then vega exposure is critical. We assume here that the primary use of vanilla options in a FoF is simply to hedge downside exposure in a severe market sell-off. In this instance, the total notional of the hedge, combined with a consideration of moneyness, is most important. We might, for example, take comfort from knowing that we have a -0.10 beta hedge against equity moves more than 5% below current levels. This is a simplistic, portfolio insurance approach to the use of put options.

Given this approach to exposure measurement, it is important to get an idea of how much our "insurance" will cost and how this compares to the cost of buying the same amount of exposure historically. To do so, we calculate the cost of a one year S&P 500 put option for different "delta-points" or for different levels of moneyness. This analysis, shown in exhibit 8,[13] indicates the extent to which the cost, expressed in basis points of portfolio AUM, can vary based on various levels of market implied volatility. We can see, for example, that the 25 delta-point option[14] which hedges 10% of portfolio AUM might cost us anywhere from 16 to 82 basis points of portfolio value. One should consider this as an annual cost of insurance and determine the extent to which the FoF portfolio is able to bear this cost. The FoF might, for instance, have a policy of maintaining this insurance so long as the projected static cost is less than 50 basis points per annum, thereby preventing expensive volatility purchases.

It is also important to consider the potential additional costs of such a hedging program if one is in a perpetual bull market and is forced to roll the puts to higher strikes to ensure the appropriate level of protection. If the needed hedge exposure is always a certain notional amount struck x% OTM (e.g., notional 10% of portfolio AUM 9% OTM), then in a bull market existing hedge positions will quickly lose value and simultaneously need to be rolled to higher strikes. This would render the cost of hedging significantly higher than shown in the previous analysis. It also suggests that staggering option expiries may make sense, thus allowing the FoF portfolio manager to more easily roll into different strikes in keeping with movements in the underlying market.

This also leads to a consideration of the extent to which the use of put spreads, straddles, or strangles might meet the FoF's needs. Put spreads would serve to reduce the cost of the hedge position, but would also reduce the amount of protection in an extreme sell-off. Buying ATM or OTM calls in combination with ATM or OTM puts will guard against the costs that would be incurred in a steady bull market. These long straddles or long strangles move the position to that of a more classic long vol position, which will lose even more money in a stable, range-bound environment.

If one wanted to use both futures and options to hedge equity exposure, then it would be important that the hedging tool depicted in exhibit 7 incorporate the delta exposure of any options as well as the exposure inherent in the futures contracts.

13 - We use Bloomberg S&P 500 (SPX) delta point data in order to obtain implied volatilities. We then convert the delta into a strike and price the options using the Black-Scholes model (with other inputs obtained from Bloomberg).
14 - The 25 delta-point equates, on average, to a 9% OTM option.

6. Hedging a Single Exposure with Other Instruments

There are a seemingly infinite number of potential hedge instruments for hedging a FoF if one includes all forms of exotic instruments on various underlyings. We provide a brief overview of some of the less exotic and more obvious choices here—variance swaps, VIX futures, and portfolio credit derivatives (CDX and iTraxx). Exhibit 9 provides a summary of the advantages and disadvantages of each.

The first two instruments are equity volatility related. Given that hedge funds are commonly short volatility, buying these instruments may make sense. While they have no explicit directional exposure, these equity volatility instruments are highly inversely related to the level of equity market indices (monthly correlation July 2006 – June 2008 of -0.74 between the S&P 500 Index and the VIX Index, which is an index of one-month S&P 500 implied volatility), and they are likely to have an outsized reaction to equity market moves in a crisis. Variance swaps are in many respects ideal, as they are a pure play on volatility; they contain both gamma and vega[15] but do not have any explicit "delta". The drawback of variance swaps is that a long variance swap position has a highly negative risk premium (one is expected to lose money over the long term).[16] Gregory (2008) notes that the risk premium for variance swaps is particularly negative because of the way that variance swaps are priced (they take into account the full volatility skew) and therefore recommends the sale of variance swaps as a way to collect a hefty risk premium. VIX futures are somewhat different from variance swaps in that they have no gamma and thus no negative risk premium. They allow investors to hedge purely against changes in implied volatility. These futures contracts, however, are really forward contracts on the future level of the VIX, which is a one-month implied volatility index. The result is that they typically have more muted movements than the VIX itself. Additionally, in a normal market environment, volatility typically has an upward sloping term structure. As such, buyers of VIX futures often experience losses as their positions roll down the volatility curve.

CDX and iTraxx are portfolio credit derivatives that allow investors to hedge credit risk broadly. There are many series that an investor can choose from (e.g., high yield, investment grade, investment grade high volatility, sector indices, etc.). They are ideal in the sense that one can quickly and easily offset credit risk. Given that credit as a universe is one of the more diverse markets, the downside is tracking error. Portfolio credit derivatives contain only the most liquid underlying credits, while hedge funds might contain very idiosyncratic credit exposures.

One can also buy options on variance swaps, the VIX, or CDX indices, or engage in a number of other hedging transactions. For a discussion of other potential hedges see Gross (2006). The choice of instrument is highly dependent on the exposure that the FoF manager wishes to hedge and the extent to which the FoF manager may wish to tailor the hedge and its cost to very specific scenarios.

7. Hedging Standard and Alternative Beta with a Multi-Factor Approach

If one would like to broadly hedge a combination of the systematic and quasi-systematic exposures within a FoF, then a multi-factor approach makes the most sense. Such an approach is more pertinent in the context of hedging "inventory" positions related to structured products and perhaps a desire to broadly reduce risk exposures across a FoF.

This type of hedging program is simply the inverse of the recently much-discussed hedge fund replication, or cloning.[17] We formulate the FoF's replicating portfolio using multi-factor analysis and then sell it.

In its simplest form, one might produce a FoF *pro-forma* track record based on the managers currently in the FoF and then perform analysis as shown in exhibit 1 (using liquid and very tradable

15 - For a discussion of the vega and gamma characteristics of variance swaps, see Kuenzi (2005).
16 - See Kuenzi (2007), Carr and Wu (2004), and Bakshi and Kapadia (2003) for a full discussion of the risk premium associated with investments in volatility.
17 - For background on hedge fund replication and a consideration of some of the competitors in the space, see Della Casa, Rechsteiner, and Lehman (2008) and Géhin (2007).

instruments). The last points of the series represent the current exposures; trading the negative of these will theoretically hedge all systematic exposures in the FoF. Using only the main global asset classes provides for only a very coarse hedge, thus preventing us from picking up many of the nuances in hedge fund investing.

An approach that allows us to partially overcome this weakness is to map all hedge fund strategies to "multi-factor categories", or to collections of factors that make sense for that particular strategy. This mapping may be one to one, as perhaps for distressed debt, or it may be many to one, as for many equity strategies, as equity managers of all types may be mapped to the same set of factors. A schematic of this mapping is represented in exhibit 10.[18] Once we have completed this mapping, we regress each manager's returns against the pertinent set of factors and then aggregate to the portfolio level.

An example of this more granular approach is shown in exhibit 11, which shows a hypothetical four-manager portfolio along with the exposures of each manager to those factors represented in the pertinent multi-factor category. We then take a weighted sum of these to get total portfolio exposures. The next step is to disaggregate the factors into instruments and to group like instrument exposures. The large-cap exposure from the "small-large" factor, for example, needs to be combined with the S&P 500 exposure already identified. The result of this work is shown in the first column in exhibit 12. The final step is to multiply by -1 to obtain the desired hedge positions.

This more granular hedging is more likely to pick up nuances in hedge fund exposures and thus to provide a better hedge. There are, however, certain types of alternative beta that are very difficult to hedge. One example is the alternative beta related to statistical arbitrage.[19] During August 2007, broad systematic exposures were generally flat (MSCI World down -0.27%) while hedge fund managers performed quite poorly (HFRI Fund of Funds Composite down -2.18%). As described in detail in Rothman (2007), much of this was driven by dislocations among extraordinarily "rich" and "cheap" equities, as some highly levered statistical arbitrage managers were forced to delever, thus sparking a rally in "rich" names and a sell-off of "cheap" names. It is clear that this exposure can be accurately replicated only by creating a statistical arbitrage trading capability, an infeasible proposition for most.

Finally, all else equal, tracking error between the hedge portfolio and the underlying FoF will increase in more tumultuous markets. There are two important phenomena that drive this. The first is that mentioned in the previous paragraph—hedge fund beta that is not represented in the hedge portfolio can perform either extremely well or extremely poorly in these environments. The second is that during these environments, hedge fund managers may shift their net exposures very rapidly, while a factor-based approach will lead to only gradual shifts in factor exposures.[20] This can be observed in the differences between some of the replication products and hedge fund indices during volatile months. In March 2008, for example, the HFRI Fund of Funds Composite returned -2.71%, while replicators performed much better across the board (the Merrill Lynch Factor Model Index was down -0.51% and the Goldman Sachs Absolute Return Tracker was down -1.13%). This was likely due to credit market issues within many hedge funds but that were not easily captured by the instruments used in replication. Conversely, the month of June 2008 was only a marginally difficult month for hedge funds (the HFRI FoF Composite returned -0.82%), while replicators were hurt quite badly (the Merrill Lynch and Goldman Sachs products returned -2.54% and -2.15%, respectively). This was likely driven by the significant sell-off in equities, which hurt replicators (which track historical hedge fund equity exposures) more than hedge funds (which had likely reduced equity risk in the wake of the January sell-off). In sum, while a factor-model hedge is likely to work well on, say, a rolling six-month basis,[21] there may be significant month-to-month tracking error in tumultuous markets.

18 - Depending on the types of hedging instruments used, this type of hedging may be highly capital intensive and may therefore require the FoF to obtain a significant leverage facility.
19 - Statistical arbitrage strategies attempt to use quantitative factor-based models—using valuation, momentum, and other such factors—to identify stocks that are trading extraordinarily rich or cheap to the levels at which they should theoretically be trading.
20 - In some cases, these rapidly shifting positions may be captured by volatility-related factors. See Kuenzi (2008) and Kuenzi and Shi (2007) for more on this topic.
21 - See Jensen, Rotenberg, and Post (2008) for an example of the type of strong fit observed in a rolling six-month analysis.

Conclusion

The need to hedge in a fund of funds can be driven by a variety of considerations, including alpha opportunities among managers who are consistently net long (e.g., activist managers), by temporary increases in systematic risks, by the need to hedge structured products or FoF inventory, or by a need to broadly reduce risk before one is actually able to redeem from managers. Understanding these drivers and the implications of hedging on portfolio alpha is critical.

Once a decision to hedge has been taken, the focus shifts to exposure estimation and management. This shift is rendered particularly challenging by the limited transparency associated with investing in hedge funds and the often dynamic nature of underlying hedge fund exposures. As such, it is often helpful to consider the output of a variety of models in considering, implementing, and managing a hedging program. We have described many useful approaches to performing this analysis, both in terms of single exposure hedging and in the context of implementing a broad hedge for an entire FoF portfolio.

Overall, a hedging capability removes a significant constraint from FoFs, thus rendering them much more flexible in terms of the types of portfolios they are able to assemble. This should have the net result of improving alpha, allowing for more unique and idiosyncratic portfolios, and for more creative structured products.

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EXHIBIT 1
HFRI Fund of Funds Composite Directional Exposures

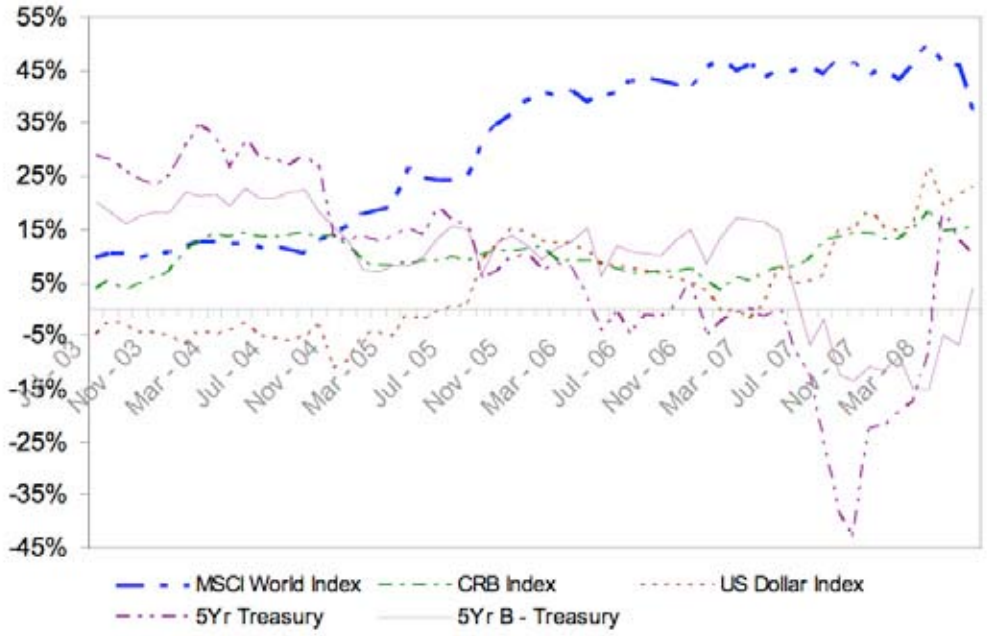

EXHIBIT 2
HFRI Fund of Funds Composite Factor Contribution to Portfolio Risk

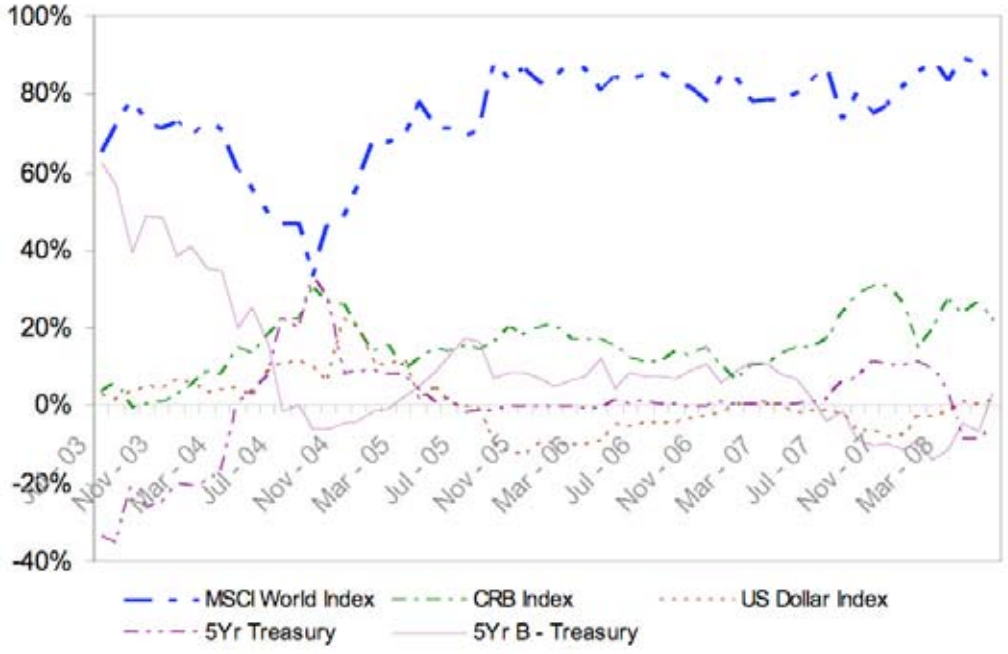

EXHIBIT 3
Required Alpha to Compensate for the Payment of Performance Fees on Standard Beta

Manager Beta	2/20 Fee Structure	2/25 Fee Structure	2/30 Fee Structure
0.25	1.31%	1.75%	2.25%
0.50	1.63%	2.17%	2.79%
0.75	1.94%	2.58%	3.32%
1.00	2.25%	3.00%	3.86%
1.50	2.88%	3.83%	4.93%
2.00	3.50%	4.67%	6.00%

EXHIBIT 4
HFRI Fund of Funds Composite Beta to MSCI World

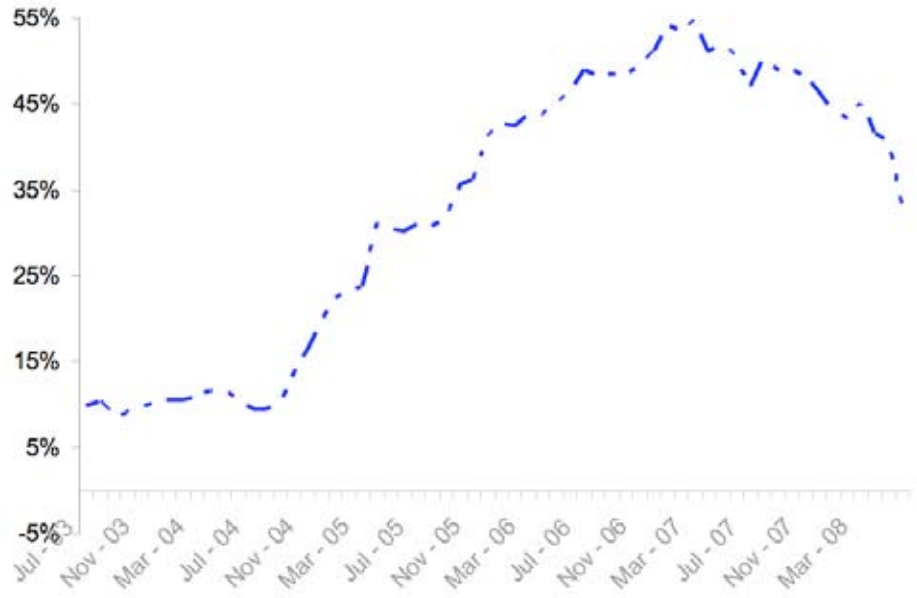

EXHIBIT 5
Betas of Various Hedge Fund Indices Taken Over Various Periods

	MSCI World Beta				S&P 500 Beta			
	1 Year	2 Year	3 Year	5 Year	1 Year	2 Year	3 Year	5 Year
HFRI Fund Weighted Composite Index	0.36	0.36	0.41	0.41	0.30	0.30	0.35	0.36
HFRI Fund of Funds Composite Index	0.32	0.34	0.38	0.35	0.25	0.27	0.31	0.30
HFRX Global Hedge Fund Index	0.28	0.34	0.37	0.35	0.22	0.29	0.33	0.31

EXHIBIT 6
Calculation of Hedge Rebalance Trades

Panel 1: Calculation of Hedge Rebalance Trades on a Specific Hedge Target

A	B	C	D	E = D - C	F = E*B	G = F / (P * 50)
	Portfolio Assets Under Management	S&P 500 Current Hedge Delta	S&P 500 Hedge Target	Amount to Hedge	Needed S&P 500 Dollar Delta	# of S&P 500 Future Contracts (Minis)
Portfolio 1	506,331,000	-0.063	-0.060	0.003	1,518,993	24
Portfolio 2	1,052,445,000	-0.039	-0.050	-0.011	-11,576,895	-179
Portfolio 3	353,807,000	-0.087	-0.080	0.007	2,476,649	38
Portfolio 4	4,709,806,000	-0.037	-0.045	-0.008	-37,178,938	-576
	S&P Price = P = 1290.52		(Multiplier = 50)			

Panel 2: Calculation of Hedge Rebalance Trades Based on an Estimated Beta vs. Maximum Portfolio Beta

A	B	C	D	E	F = min(E-D,0)	G = F*B	H = G / (P * 50)
	Portfolio Assets Under Management	S&P 500 Current Hedge Delta	Estimated Portfolio Beta	Maximum Portfolio Beta	Amount to Hedge	Needed S&P 500 Dollar Delta	# of S&P 500 Future Contracts (Minis)
Portfolio 1	506,331,000	-0.063	0.297	0.300	0.000	0	0
Portfolio 2	1,052,445,000	-0.039	0.311	0.300	-0.011	-11,576,895	-179
Portfolio 3	353,807,000	-0.087	0.193	0.200	0.000	0	0
Portfolio 4	4,709,806,000	-0.037	0.308	0.300	-0.008	-37,178,938	-576
	S&P Price = P = 1290.52			(Multiplier = 50)			

EXHIBIT 7
Calculation of Hedge Rebalance Trades Based on an Estimated Beta vs. Maximum Portfolio Beta Using Two Underlyings

A	B	C	D	E = C+ D*V	F	G
	Portfolio Assets Under Management	Current Hedge Delta		S&P 500 Equivalent	Estimated Portfolio Beta	Maximum Portfolio Beta
		S&P 500	Russell 2000			
Portfolio 1	506,331,000	-0.034	-0.024	-0.063	0.297	0.300
Portfolio 2	1,052,445,000	-0.021	-0.015	-0.039	0.311	0.300
Portfolio 3	353,807,000	-0.012	-0.063	-0.087	0.193	0.200
Portfolio 4	4,709,806,000	-0.025	-0.010	-0.037	0.308	0.300

H = min(G-F,0)	I	J	K = I/(I+J)*H*B	L = (J/(I+J)*H/V)*B	M = K/(P1*50)	N = L/(P2*100)
Amount to Hedge	Proportion of Rebalance		Needed Dollar Delta		# of Future Contracts	
	S&P 500	Russell 2000	S&P 500	Russsell 2000	S&P 500 (Minis)	Russell 2000 (Minis)
0.000	50%	50%	0	0	0	0
-0.011	100%	0%	-11,576,895	0	-179	0
0.000	100%	0%	0	0	0	0
-0.008	50%	50%	-18,589,469	-15,536,295	-288 (Multiplier=50)	-206 (Multiplier=100)

P1	P2	V1	V2	V = V2/V1
Price		Realized 52-Week Volatility		Volatility Ratio
S&P 500	Russell 2000	S&P 500	Russell 2000	1.20
1290.52	753.74	17.81%	21.31%	

EXHIBIT 8
Cost of Buying 1-Year S&P 500 Put Option with Notional of 10% of AUM

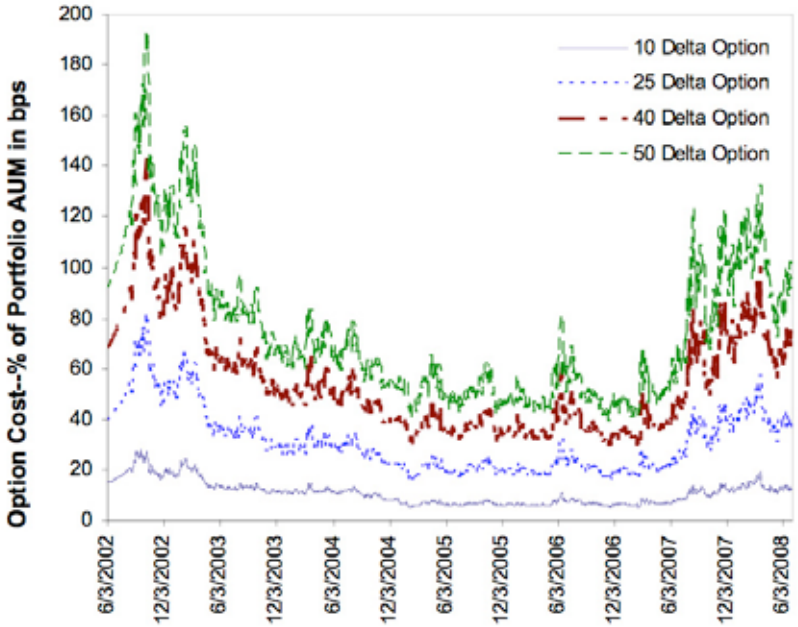

EXHIBIT 9
Other Possible FoF Hedging Instruments and Implications

Instrument	Advantages	Disadvantages
Variance Swaps	Provides an overall hedge to crisis risk. (Volatility responds strongly to sudden and unexpected crises.) Long vega and long gamma.	Significant risk premium payment. Long-term, one is expected to lose a significant amount of money in long volatility investments; this is arguably more pronounced in the case of variance swaps.
VIX Futures	Provides an overall hedge to crisis risk. Volatility responds strongly to sudden and unexpected crises. Long vega (but no gamma).	Essentially a forward contract on the future level of the VIX—captures smaller movements than the VIX itself. Investors can also experience negative returns by rolling down the term structure.
Portfolio Credit Derivatives (CDX and iTraxx)	Easily executed protection to a number of credit portfolios. Highly liquid instruments.	Significant potential tracking error as the indices capture only a very small component of the credit universe.

EXHIBIT 10
Schematic of Mapping of Hedge Fund Strategies to Multi-Factor Categories

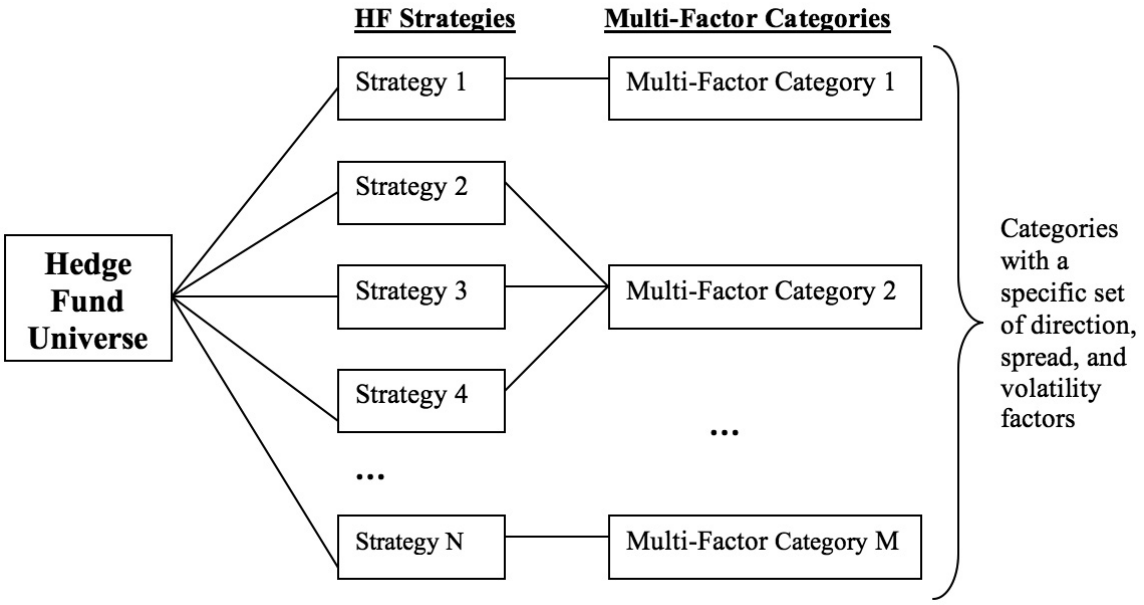

EXHIBIT 11
Example of FoF Exposures Using Different Sets of Factors for Each Hedge Fund Strategy

			Factor Exposures									
Manager	Hedge Fund Strategy	Portfolio Allocation	S&P 500 SPDR	5Yr Tsy Futures	Tech SPDR	Small - Large	Value - Growth	High Yld CDX	S&P 500 Call	S&P 500 Put	Tech Call	Tech Put
Manager 1	Equity Long Short	35%	40%			-20%	-15%		2%	0%		
Manager 2	Equity Mkt Neutral	20%	5%			-15%	-60%		-1%	0%		
Manager 3	Convertible Arb	25%	30%	10%				15%	2%	1%		
Manager 4	Tech Long Short	20%			25%	-30%					0%	-4%
	Weighted Totals:	100%	22.50%	2.50%	5.00%	-16.00%	-17.25%	3.75%	1.00%	0.25%	0.00%	-0.80%

EXHIBIT 12
Example of a FoF Replicating and Hedge Portfolio

	Relicating Portfolio	Hedge Portfolio
S&P 500 SPDR	38.50%	-38.50%
5Yr Tsy Futures	2.50%	-2.50%
Tech SPDR	5.00%	-5.00%
Russell 2000 iShares	-16.00%	16.00%
Russell 1000 Value iShares	-17.25%	17.25%
Russell 1000 Growth iShares	17.25%	-17.25%
High Yld CDX	3.75%	-3.75%
S&P 500 Call	1.00%	-1.00%
S&P 500 Put	0.25%	-0.25%
Tech Call	0.00%	0.00%
Tech Put	-0.80%	0.80%