EDHEC RISK AND ASSET MANAGEMENT RESEARCH CENTRE

1090 route des crêtes - 06560 Valbonne - Tel . +33 (0)4 92 96 89 50 - Fax. +33 (0)4 92 96 93 22
Email : research@edhec-risk.com – Web : www.edhec-risk.com

Derivatives in portfolio management
Why beating the market is easy

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François-Serge Lhabitant
Head of Quantitative Risk Management at Union Bancaire Privée
Associate Professor at EDHEC

EDHEC
BUSINESS SCHOOL
Lille - Nice

Abstract:

Under the efficient market hypothesis, overwriting calls or purchasing insurance should not improve risk-adjusted portfolio returns. A proper analysis should show that if options are traded at a fair cost, the risk-reward characteristics of an option position would fall on the efficient market line. In this paper we show that, due to several limitations of mean-variance analysis, this is not the case in practice. We quantify and identify the nature of the resulting biases for performance evaluation, and explain why alternative measures such as semi variance do not help in avoiding such biases.

INTRODUCTION

Derivatives and options in particular have progressively established themselves as common tools in portfolio management for asset allocation, hedging, diversification, and/or leverage. Their non-linear payoffs enable investors to create return profiles that are unachievable by simple combinations of conventional investment vehicles such as stocks and bonds. Consequently, their increased presence has been accompanied by a variety of claims regarding their ability to simultaneously enhance rewards *and* reduce risks. For instance, covered call writing is often presented as an efficient way to increase income of stock ownership through the received option premium and convert the prospects for uncertain future capital gains into immediate cash flows. Similarly, protective put buying allows its users to ascertain a minimum value of their portfolio, limiting downside risk while preserving the upside potential. Although desirable, this win-win suggested behavior violates the efficient market hypothesis, since risk-adjusted portfolio returns should not be improved by the systematic use of options.

Aside from the specific characteristics inherent to option valuation, several authors have examined option management in a portfolio context. Unfortunately, neither the academic studies nor the nostrums for market success offered by practitioners to help investors are really conclusive. The lack of consistency among their results and their strong focus on the U.S. market does not allow for complete confidence in their conclusions. In fact, most of the option-based strategies have not yet received a careful theoretical analysis. Even worse, they have often received an analysis based on inadequate theoretical tools. Traditional portfolio performance evaluation relies on the dominating efficiency paradigm in financial economics, namely mean-variance analysis. It assumes a buy and hold strategy with normally distributed returns over a specific time-horizon, and compares the results obtained with those of an efficient market index. These assumptions are no longer valid when options are involved in a portfolio. Hereafter, our goal is to illustrate and understand the biases of traditional performance measures when applied to portfolios containing options. As we will see, these biases are both wholly measurable and rectifiable when the option strategy is known, but cannot be solved by using alternatives performance measures such as semi-variance or downside risk.

LITERATURE OVERVIEW

Contrary to common belief, several studies exist on optioned portfolio performance prior to the Black-Scholes-Merton pricing formula, and even prior to the 1973 trading of listed options in Chicago. The early studies were primarily based on small samples of OTC quotes from brokers[1]. They generally concluded that writing options was a better strategy than buying them, and frequently observed that writing calls against a diversified portfolio would increased portfolio returns while lowering risk. Their results can be attributed to their choice of a particular time frame, to market inefficiencies and/or to the lack of an effective valuation model.

After 1973, most of the studies still focused on covered call writing, but considered longer time horizons and used listed option prices as well as theoretical option premiums. Their results were mixed. On one hand, Bookbinder [1976], Kassouf [1977], Pounds [1978], Grube and Panton [1978] or Yates and Kopprasch [1980] observed that covered call writing still produced substantially larger returns than the traditional buy and hold, even though the risk level of the optioned portfolio (measured by the standard deviation of returns) was lower. On the other hand, Merton, Scholes and Gladstein [1978, 1982] obtained opposite results, producing evidence of a more favorable situation for option buyers than for option sellers.

The strong U.S. bull market of 1982 heavily penalized covered call writing with respect to an indexed strategy and reduced popularity of the strategy. Research then focused on static (protective put strategies) and dynamic portfolio insurance. Both became extremely popular until the failure of the latter in the 1987 crash. Once again, interest was reduced, particularly after the publication of new studies based on more complex tools (such as stochastic dominance) that concluded that there was no real dominant strategy; see for instance Booth, Tehranian and Trennepohl [1985], Clarke [1987], or Brooks, Levy, and Yoder [1987].

However, motivated by the strong increase of stock market volatility, interest for option strategies has recently resurged. But there is still no clear evidence regarding the long-term dominance of a systematic option-based strategy on traditional buy and hold positions. The ex-post results depend on the period, the market and the strategies considered, and with a few exceptions, are often U.S. specific; see for instance Austin [1995], Lhabitant [1998], Rendleman [1999], Green and Figlewski [1999], or Ineichen [2000]. Therefore, there is a strong need to take a proper ex-ante approach to assess the *expected* performance of these strategies.

THEORETICAL FRAMEWORK

In our model, an investor is assumed to allocate his wealth among a single stock or index (henceforth stock), a long put and a long stock (protective put), or a short call and a long stock (covered call writing)[2]. The returns of the underlying stock are assumed to follow a lognormal distribution with a mean and variance of the annual logarithmic return μ and σ^2. No dividends are paid on the stock, and we assume that put and call options on the stock are traded at their fair Black-Scholes values. We denote by P_t and C_t their price at time t, by K their exercise price, by T their time to maturity, and by r the annual continuously compounded rate of interest.

Traditional modern portfolio theory tells us that our investor should select the strategy with the highest Sharpe [1966] ratio – in other words, the strategy that is expected to give him the highest expected return per unit of risk. We will see what happens if this path is followed.

By going through simple but lengthy algebraic equations, it is possible to write the moments of the return distribution for each strategy as a closed-form expression depending only of the six parameters K, T, S_0, r, μ and σ (see appendix). Since the Sharpe ratio considers only the first two moments of a distribution (mean return and volatility), it is also possible to write the Sharpe ratio of an optioned portfolio as a closed form expression of the above mentioned parameters. This solves our strategy selection problem. Moreover, this allows us to assess the performance of each strategy for different values of the parameters μ and σ -- that is, for different underlying stochastic processes (i.e. different assets), different time-to-maturity (T) and degree of moneyness (K), with the latter two being controlled by the investor.

EMPIRICAL RESULTS

We performed various simulations. While the results would differ depending on the assumptions, they share a common nature. As a result only the results for one set of simulation (μ=10%, r=5%, σ=15%, T=1 year, S_0= 100) will presented from this point on.

Figure 1 and 2 show the expected return and volatility for the three strategies as a function of the option moneyness. As one expects, in a "normal" environment, the selection of a higher strike price should increase both expected return and volatility for covered call writing. Deeply in the money covered call writing is equivalent to holding a risk-free bond, and deeply out of the money covered call writing is similar to holding a naked stock. Conversely, in the case of protective put buying, a higher strike price should decrease both expected returns and volatility. Deeply out of the money protective put buying is similar to holding a naked stock, while deeply in the money protective put buying is equivalent to holding a risk-free bond.

Let us now mix both the expected return and the risk effects into a performance measure. Figure 3 shows the Sharpe ratio for the three strategies as a function of the option moneyness. When increasing the exercise price of a protective put position, we move from a pure stock to a pure risk-free bond position, and the Sharpe ratio decreases accordingly. As a consequence, the protective put has a Sharpe ratio that is always lower than the Sharpe ratio of a long stock position. When increasing the exercise price of a covered call position, we move from a pure bond to a pure stock position, and the Sharpe ratio increases accordingly. However, for a set of exercise prices that are just out of the money, the Sharpe ratio of the covered call exceeds the Sharpe ratio of the long stock position. This phenomenon is more important for long term options than for short-term ones[3].

The consequences for our traditional investor are twofold. First, he will not be interested in buying puts without having specific expectations on the market moves, as the protective put position is dominated. Second, his optimal strategy will consist of selling slightly out-of-the-money covered calls, since this dominates the pure stock strategy. When the stock is actually a market index, this implies that, *ex-ante*, some covered call positions form a dominating efficient frontier in the mean-variance space. Moreover, the same phenomenon will be observable when measuring performance with any mean-variance or CAPM based performance measure, such as the Treynor ratio or the Jensen alpha. Consequently, it becomes possible and easy to beat an efficient market without any timing ability: given some market parameters (μ, σ), use the closed-form expression for the Sharpe ratio to find the optimal exercise price. Then, sell covered call options on the market with this exercise price and the longest maturity available. This strategy appears as superior in the mean-variance space.

DISCUSSION AND EXTENSION

Fortunately, this apparent dominance arises from the inadequacy of the Sharpe ratio. Despite its popularity, the Sharpe ratio suffers from a series of methodological problems.

First, the Sharpe ratio ignores asymmetric reduction of the variance, therefore not presenting the whole picture. For instance, let us consider a protective put with an underlying asset's log-returns that are normally distributed[4]. The resulting distribution combines two effects. First, we truncate the underlying distribution at the point S_T=K, and replace the left portion of the equation with a Dirac delta function having a weight equal to the probability that the stock price is lower or equal to the exercise price. Next, we translate the new distribution to the left, in order to incorporate the put premium payment influence. For the covered call strategy, we can apply the same methodology, except that we truncate the right portion and that we translate on the right, as the premium is received. As clearly evidence in Figure 4, the option strategies will have highly skewed distributions, with an asymmetric volatility reduction. Call writing truncates the right-hand side of a distribution and results in negative skewness (undesirable), while put buying truncates the left-hand side of a distribution and results in positive skewness (desirable). Therefore it is natural that the compensation for risk reduction varies between the two strategies, as a reward for holding a portfolio with a skewed return distribution is required. As the Sharpe ratio ignores the asymmetric reduction of the variance, its application to optioned portfolios effectively overstates the performance of call writing and understates the performance of put buying.

Second, the Sharpe ratio is a static tool, whereas the option strategies considered previously are dynamic in nature. It can be shown that the volatility of an optioned portfolio will change over time, even though the volatility of the underlying stock returns is constant (see Appendix 2). This is possible because of the time elapsing, and because of stock price variations, which result in a change of the option's delta as well. But the Sharpe ratio does not account for this changing risk over time. Nor do measures based on semi-variance or downside (static) risk, such as the Sortino [1991] ratio.

Note that these observations are not only limited to portfolios containing options, but also to investment managers implementing dynamic trading rules (such as repurchasing securities after market declines, selling after market increases, etc.) as well as to some hedge funds whose returns exhibit non-linear option-like exposures to standard asset classes[5]. When the exact nature of the dynamic strategy is known, it is possible to compute the benchmark "Sharpe ratio", which differs from the market Sharpe ratio. However, this makes performance measurement strategy-dependant. Considering the wide variety of option strategies employed in the fund management industry, this can creates obvious challenges.

CONCLUSIONS

Several managers have successfully implemented systematic option based strategies; one example would be the $700 million Gateway Fund (GATEX), who has sold covered calls since 1977, or the large number of portfolio insurance funds. Most institutions perceive these strategies as efficient. In particular, they view covered call writing as a means of augmenting portfolio returns and protective put buying as a solution to avoid downside risk. Our paper shows that these option strategies produce systematic biases when evaluated using traditional performance measures. For instance, when applied to the market portfolio, some option strategies are expected to beat the market from the point of view of the Sharpe ratio.

In reality, financial theory suggests that these strategies will partially hedge market risk at the expense of a reduced portfolio return, the latter being the market price of risk reduction. Consequently, when options markets are efficient, the returns and risk reduction derived from the sale or purchase of efficiently priced options, when combined with the risk and returns of a diversified market portfolio used as a collateral, will be offsetting, and yield no excess risk-adjusted portfolio returns. Observing positive excess performance when using options violates the efficient market hypothesis or implies that there is a reward for holding a portfolio with a skewed return distribution or a non-static risk profile. Therefore, beating the market is easy, but only when using inadequate performance measures!

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APPENDIX 1

The methodology for computing the moments of the return distribution for an optioned portfolio is detailed in Lhabitant (1998). In particular, for the moments of order one (mean return) and two (variance), we have:

For the long spot (S) returns:

$$E(R_S) = e^{\mu T} - 1$$
$$\sigma^2(R_S) = e^{2\mu T}(e^{\sigma^2 T} - 1)$$

For the protective put (PP) returns:

$$E(R_{PP}) = \frac{KL_{(0)} + U_{(1)}}{S_0 + P_0} - 1$$
$$\sigma^2(R_{PP}) = \frac{K^2 L_{(0)} + U_{(2)} - (KL_{(0)} + U_{(1)})^2}{(S_0 + P_0)^2}$$

For the covered call (CC) returns:

$$E(R_{CC}) = \frac{KU_{(0)} + L_{(1)}}{S_0 - C_0} - 1$$
$$\sigma^2(R_{CC}) = \frac{K^2 U_{(0)} + L_{(2)} - (KU_{(0)} + L_{(1)})^2}{(S_0 - C_0)^2}$$

The functions $U_{(n)}$ and $L_{(n)}$ are called n^{th} order upper and lower partial moments of the stock price and are equal to

$$L_{(n)} = S_0^n e^{n(\mu+(n-1)0.5\sigma^2)T} N\left(\frac{\ln\left(\frac{K}{S_0}\right) - \left(\mu + \frac{(2n-1)\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \right)$$

and

$$U_{(n)} = S_0^n e^{n(\mu+(n-1)0.5\sigma^2)T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + \left(\mu + \frac{(2n-1)\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \right)$$

where N() denotes the cumulative normal function.

APPENDIX 2

To obtain the instantaneous volatility of the (continuously compounded) returns of an option-based portfolio, one needs to apply Ito's lemma to the value of the portfolio. For a protective put position, for instance, we obtain that its conditional volatility is equal to:

$$\frac{(1+\Delta_P)\sigma S_t}{S_t + P_t}$$

where Δ_P is the delta of the put option. Even when σ is constant, this expression will change over time, particularly when the option is at the money.

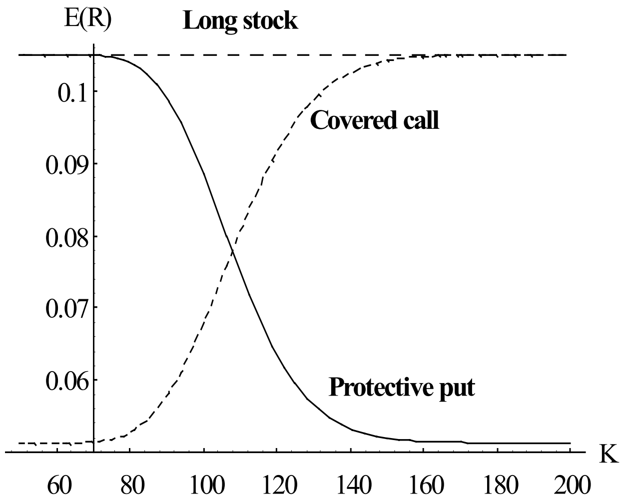

Figure 1: Expected returns for the three strategies

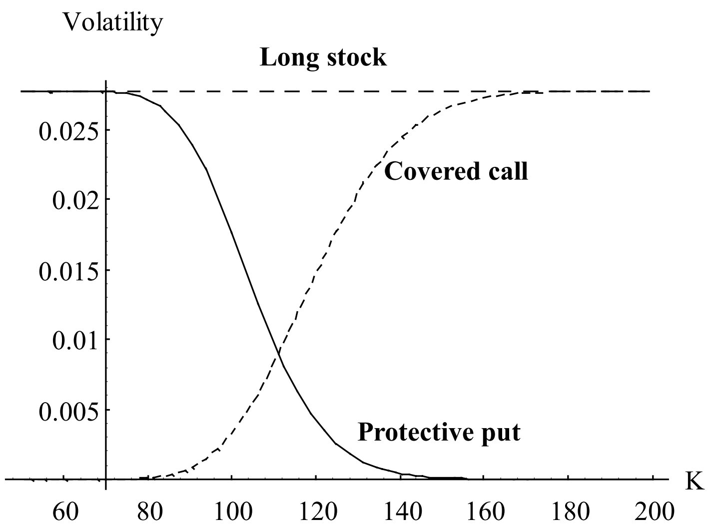

Figure 2: Expected volatility for the strategies

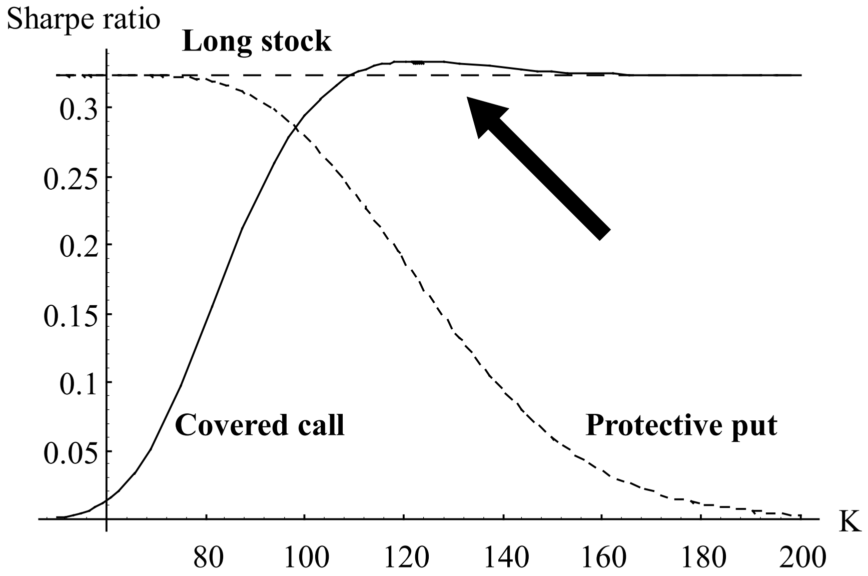

Figure 3: SHARPE ratios for the strategies

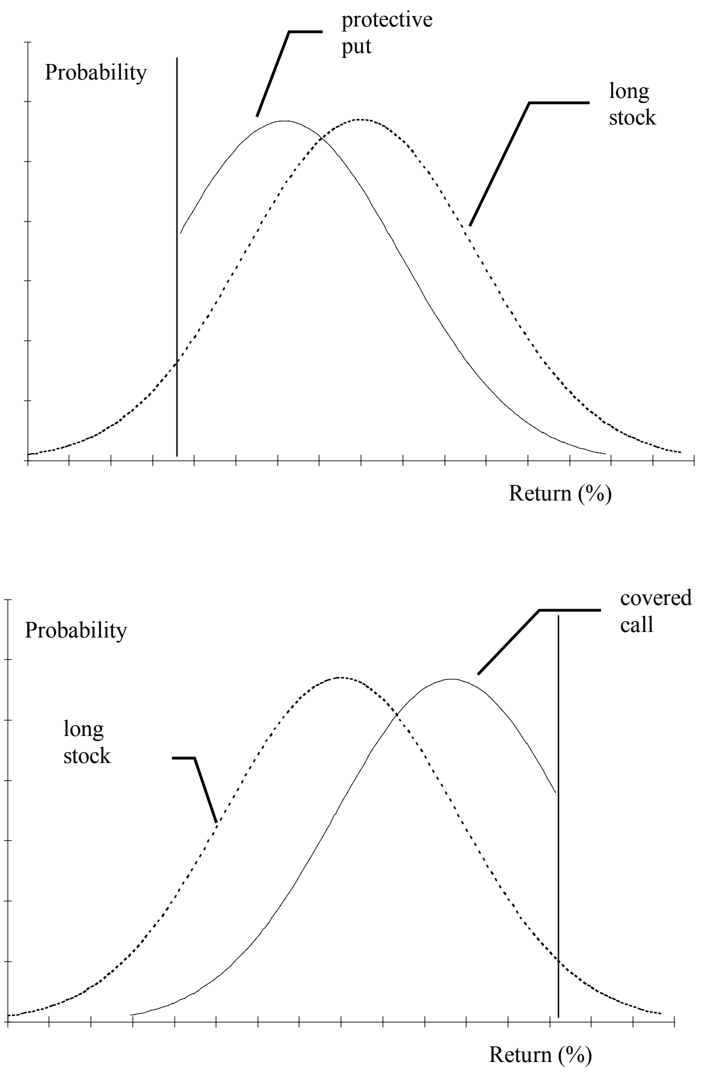

Figure 4: Log-return distribution of a protective put and a covered call

Notes

[1] See for instance Kruizenga [1964], Boness [1964], Rosett [1967], Malkiel and Quandt [1969] or Malkiel [1972].

[2] Other option strategies could easily be added here. However, these three strategies are the most popular among institutional investors.

[3] The remark is still valid if the underlying asset log-returns are not normally distributed.

[4] Note that considering skewness in the performance measure does not solve anything. For instance, Lhabitant [1997] provides examples in which rational risk-averse investors select a portfolio with the lowest mean, highest volatility, and lowest skewness. Considering kurtosis will again only move us one step forward. All higher order moments should be considered.

[5] See for instance Michell and Pulvino [2000], who evidence that that "risk arbitrage" strategy payoffs are similar to those obtained from writing an uncovered put option on the market.