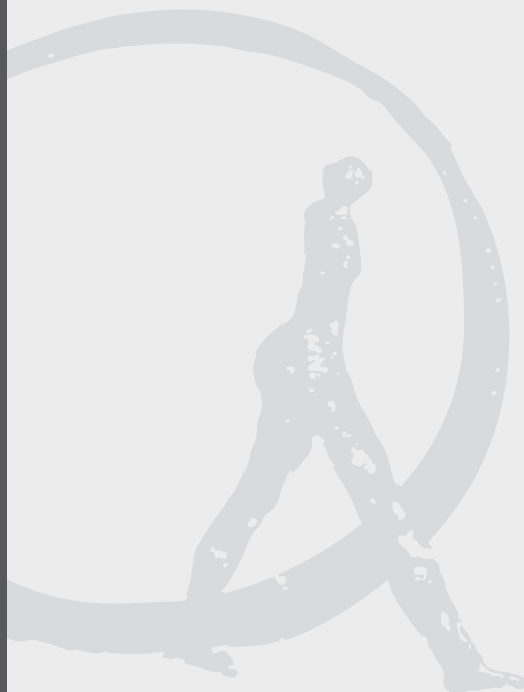


# Derivatives Strategies for Bond Portfolios



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## **Abstract**

In this paper, we examine how standard exchange-traded fixed-income derivatives (futures and options on futures contracts) can be included in a sound risk and asset management process so as to improve risk and return performance characteristics of managed portfolios. Our results show that the non-linear character of the returns on protective option strategies offers appealing risk reduction properties in the pure asset management context. Consequently, such strategies should optimally receive a significant allocation, especially when investors are concerned with minimising extreme risks. In an asset liability management context, we also show that fixed-income derivatives in general, and recently launched long-term futures contracts in particular, offer significant shortfall risk reduction benefits. These results have potentially significant implications in the context of an increased focus on matching liability portfolios.

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## Introduction

Interest in the use of fixed-income derivatives in asset management has perhaps never been so high, both from a pure asset management perspective and from an asset-liability management perspective. On the one hand, from a pure asset management perspective, historically low levels of long-term yields, combined with the prospect of increases in short-term rates by central banks in Europe as well as in the United States, are a strong incentive to implement strategies that aim to deliver downside protection by combining a long position in bond markets with a long position in put options. On the other hand, recent pension fund shortfalls have drawn attention to the risk management practices of institutional investors in general and defined benefit pension plans in particular, and have provided new incentives for institutional investors to use fixed-income derivatives in the management of their liability risk.

"A perfect storm" of adverse market conditions around the turn of the millennium has devastated many corporate defined benefit pension plans. Negative equity market returns have eroded plan assets at the same time as declining interest rates have increased the mark-to-market value of benefit obligations and contributions. In extreme cases, this has left corporate pension plans with funding gaps as large as, or larger than, the market capitalization of the plan sponsor. For example, in 2003, the companies included in the S&P 500 and the FTSE 100 index faced a cumulative deficit of \$225 billion and £55 billion respectively, (as per surveys by Credit Suisse First Boston [2003] and Standard Life Investments [2003]), while the worldwide deficit reached an estimated 1,500 to 2,000 billion USD (Watson Wyatt [2003]). That institutional investors in general and pension funds in particular have been so dramatically affected by recent market downturns has emphasized the weakness of risk management practices. In particular, it has been argued that the asset allocation strategies implemented in practice, which were heavily skewed towards equities, in the absence of any protection with respect to their downside risk, were not consistent with sound liability risk management. In this context, new approaches, referred to as *liability-driven investment* solutions, have rapidly elicited interest from pension funds, insurance companies and investment consultants alike, following recent changes in accounting standards and regulations that have led to an increased focus on liability risk management. Such strategies, which can add significant value in terms of liability risk management, lead to an increased focus on matching investors' portfolios via cash instruments (long-term real and nominal bonds) or via derivative instruments.

From an academic perspective, while an impressive amount of research is available on issues related to bond and fixed income derivative pricing and hedging, relatively few results can actually be found on how these instruments can be used to improve an investor's welfare in the context of portfolio strategies. This paper, as it happens, focuses on illustrating how various types of derivative securities can be used to shift the risk associated with investing in fixed income securities.

We take an asset allocation perspective and assess in more detail the benefits of including bond futures and options on these futures in a fixed income portfolio. In addition to linear instruments such as futures, options have non-linear payoffs that allow investors to construct funds with skewed or dissymmetric return profiles. In comparison to simple bond index exposure, these may offer an efficient way to limit the extreme values of a portfolio return distribution.

As such, our paper is closely related to a number of papers that look at optimal portfolios when investors have access to an asset and options on this asset. Based on Merton's (1973) replicating portfolio interpretation of the Black and Scholes (1973) option pricing model, one could argue that options are redundant assets, and as a result the optimal allocation to such assets should be either indeterminate (when an investor's expectation about parameter values match those of the markets), or infinite (when investors' believe the derivative asset is underpriced). It should however be recognized that this result depends on the concept of dynamic completeness (see Radner [1972] for a formal analysis), which can only be justified under a set of rather stringent assumptions that are never met in practice (continuous trading, the absence of transaction costs, leverage or short-sales constraints, constant volatility and drift of the underlying asset, symmetric information, etc). When any of these assumptions are relaxed, it can actually be shown that the introduction of derivatives would improve the welfare of investors. A first strand of the literature, initiated by Brennan and Solanki (1981), has examined the

question of optimal positioning in derivative assets from the static investor's standpoint. The question there is to determine the general shape of the non-linear function of the underlying asset that would generate the highest level of expected utility for a given investor. Carr and Madan (2001) extend this early work by considering optimal portfolios of a riskless asset, a risky asset and derivatives on the latter in an expected utility framework. A second strand of the literature (starting with Leland [1980] and extended by Benninga and Blume [1985]) has studied the question from a different angle: taking as given a set of standard derivatives, these papers examine the features of (static) investors' preferences that would support a rational holding of these contracts.

This paper, which sets itself apart by focusing exclusively on bond derivatives, complements existing literature by focusing on the use of derivatives, not only in asset management, but also from an asset-liability management perspective. In an attempt to focus on plain vanilla products that can readily be used by investors, we take as objects of study a set of exchange-traded fixed-income derivatives traded on Eurex, the world's leading futures and options market for euro denominated derivative instruments. The use of futures and options will be organized according to the investment process and objectives and we will examine the value of introducing derivatives both in a pure asset management context and in an asset-liability management context.

The rest of the paper is organized as follows: section one introduces the theoretical model and shows how we implement it. Section two details the implementation of the investment strategies and assesses their benefits in a pure asset management context. Section three examines similar questions but from an asset-liability management (ALM) perspective.

## 1. The Framework

In order to assess the different uses and benefits of derivatives in fixed-income portfolios in the context of a full-blown numerical experiment, one needs to generate stochastic scenarios for asset prices. The scenario generation process will be similar for the whole paper, with additional assumptions on investors' preferences needed for some sections. In this introductory section, we therefore describe our assumptions with respect to how fixed income markets are modelled.

### 1.1. Choice of a Model

The issue of modelling the future uncertainty is critical to any investment management problem. A stochastic forecasting model consists of multivariate time series models.

While generating stochastic scenarios for stock prices can be achieved in a straightforward manner, a specific challenge related to simulating bond prices is the need to ensure the absence of arbitrage between the prices of bonds with different maturities at a given point in time. In other words, one needs to impose some restrictions on the cross-section of bond prices. This can be achieved by using a consistent model of the term structure of interest rates. Term structure models imply that changes in yields of bonds of different maturities can be expressed as a function of changes in some underlying factors.

The first option would be to use an equilibrium model of the term structure. This type of model starts with diffusion processes for some state variables and specify endogenously a dynamic process for the short rate as well as a zero coupon curve. The obvious drawback of this class of model is that the simulated bond prices do not automatically fit those observed in the market. Arbitrage models, on the other hand, take as given the observed term structure of interest rates, which is then regarded as the underlying asset (see Ho and Lee [1986] for a discrete-time example, or Heath, Jarrow and Morton [1990,1992] for a general formulation in continuous-time). In our context, since we do not deal with the problem of minimizing pricing errors in a practical hedging exercise, the drawback of equilibrium models does not seem to be a relevant one. It actually turns out that the calibration of popular arbitrage models would involve too much complexity given our specific requirements for the effects that the model should capture.

More specifically, the requirements are, 1) stochastic volatility for bond prices, 2) non-redundancy of multiple bonds, and 3) the availability of closed-form pricing formulas. Particular emphasis is placed on the fact that our model aims at capturing the stochastic volatility of bond prices, which is an important ingredient in the analysis of options available on bond and bond futures contracts. Further motivation for using a stochastic volatility model is related to the fact that the presence of time varying volatility of asset prices has been well documented in empirical studies (see e.g. Bollerslev et al [2000], Reilly et al., and [2000] and Jones et al.[1998]). A further issue is that we need to avoid redundancy of multiple bonds, which rules out single factor term structure models, where bonds with different maturities are perfectly correlated. Finally, our task of testing different investment strategies will be greatly eased in terms of computational burden and complexity if we have closed form solutions.

The above considerations led us to choose the multifactor term structure model by Longstaff and Schwartz (1992), LS henceforth. Their model, 1) has stochastic volatility of bond prices which stems from the stochastic volatility of the process for the short rate; 2) incorporates more than one factor, which means that bonds of different maturities are not necessarily (dynamically) redundant assets; and 3) is tractable in the sense that it allows one to obtain an explicit closed-form solution for bonds of different maturities and for options on these bonds.

The LS model is a two factor extension of the Cox, Ingersoll and Ross general equilibrium model. It can be seen that the dynamics of the short rate are similar to those in the CIR model, except for an additional factor of uncertainty given by the volatility of the short rate. The above equations allow writing the processes for the short rate  $r$  and the volatility of the short rate  $V$  as:

$$dr_t = \left( \alpha\gamma + \beta\eta - \frac{\beta\delta - \alpha\xi}{\beta - \alpha} r_t - \frac{\xi - \delta}{\beta - \alpha} V_t \right) dt + \alpha \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} dZ_t^{(1)} + \beta \sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}} dZ_t^{(2)}$$

$$dV_t = \left( \alpha^2\gamma + \beta^2\eta - \frac{\alpha\beta(\delta - \xi)}{\beta - \alpha} r_t - \frac{\beta\xi - \alpha\delta}{\beta - \alpha} V_t \right) dt + \alpha^2 \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} dZ_t^{(1)} + \beta^2 \sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}} dZ_t^{(2)}$$

where  $Z^{(1)}$  and  $Z^{(2)}$  are standard Brownian motions, and  $\alpha, \beta, \gamma, \delta, \eta, \xi$  are constant parameters. LS obtain the price of a zero coupon bond with time to maturity  $\tau$  as functions of  $r$  and  $V$ .

$$B_t(\tau, r_t, v_t) = A^{2\gamma}(\tau) B^{2\eta}(\tau) \exp(\kappa\tau + C(\tau)r_t + D(\tau)v_t)$$

where

$$\begin{aligned} v &= \lambda + \xi, & A(\tau) &= \frac{2\phi}{(\delta + \phi)(\exp(\phi\tau) - 1) + 2\phi} \\ \phi &= \sqrt{2\alpha + \delta^2} & B(\tau) &= \frac{2\psi}{(v + \psi)(\exp(\psi\tau) - 1) + 2\psi} \\ \psi &= \sqrt{2\beta + v^2} & C(\tau) &= \frac{\alpha\phi(\exp(\psi\tau) - 1)B(\tau) - \beta\psi(\exp(\phi\tau) - 1)A(\tau)}{\phi\psi(\beta - \alpha)} \\ \kappa &= \gamma(\delta + \phi) + \eta(v + \psi) & D(\tau) &= \frac{\psi(\exp(\phi\tau) - 1)A(\tau) - \phi(\exp(\psi\tau) - 1)B(\tau)}{\phi\psi(\beta - \alpha)} \end{aligned}$$

and  $\lambda$  is the market price of risk

LS also obtain the price of a call option on the zero coupon bond with payoff at expiration of  $\max(0, F(r, V, T) - K)$  as

$$C(r, V, \tau, K, T) = F(r, V, \tau, T) \Psi(\theta_1, \theta_2; 4\gamma, 4\eta, \varpi_1, \varpi_2) - KF(r, V, \tau) \Psi(\theta_3, \theta_4; 4\gamma, 4\eta, \varpi_3, \varpi_4),$$

where

$$\theta_1 = \frac{4\zeta\phi^2}{\alpha(\exp(\phi\tau) - 1)^2 A(\tau + T)}$$

$$\theta_2 = \frac{4\zeta\psi^2}{\beta(\exp(\psi\tau) - 1)^2 B(\tau + T)}$$

$$\theta_3 = \frac{4\zeta\phi^2}{\alpha(\exp(\phi\tau) - 1)^2 A(\tau)A(T)}$$

$$\theta_4 = \frac{4\zeta\psi^2}{\beta(\exp(\psi\tau) - 1)^2 B(\tau)B(T)}$$

and

$$\varpi_1 = \frac{4\phi \exp(\phi\tau) A(\tau + T)(\beta r - V)}{\alpha(\beta - \alpha)(\exp(\phi\tau) - 1)A(T)}$$

$$\varpi_2 = \frac{4\psi \exp(\psi\tau) B(\tau + T)(V - \alpha r)}{\beta(\beta - \alpha)(\exp(\psi\tau) - 1)B(T)}$$

$$\varpi_3 = \frac{4\phi \exp(\phi\tau) A(\tau)(\beta r - V)}{\alpha(\beta - \alpha)(\exp(\phi\tau) - 1)}$$

$$\varpi_4 = \frac{4\psi \exp(\psi\tau) B(\tau)(V - \alpha r)}{\beta(\beta - \alpha)(\exp(\psi\tau) - 1)}$$

$$\zeta = \kappa T + 2\gamma \ln A(T) + 2\eta \ln B(T) - \ln K$$

The function  $\Psi(0_1, 0_2; 4\gamma, 4\eta, \omega_1, \omega_2)$  is the joint distribution

$$\int_0^{\theta_1 \theta_2 - \theta_2 u / \theta_1} \int_0^u \chi^2(u; 4\gamma, \varpi_1) \chi^2(v; 4\eta, \varpi_2) dv du,$$

where  $\chi^2(\cdot; p, q)$  denotes the non-central chi-square density with degrees of freedom parameter  $p$  and non-centrality parameter  $q$ . We rewrite this as a single integral by applying the Fubini theorem and solve the resulting integral numerically.

## 1.2. Scenario Generation

The model we use for interest rate dynamics involves quite a few parameters. One approach would consist of estimating these parameters by fitting the model to historical data. Alternatively, we have chosen to select parameter values consistent with the existing literature. Table 1 contains information about the parameter values we use, and how they relate to existing literature.

We adopt the parameters in Jensen (2001). These were estimated using the efficient method of moments from a sample of T-Bill rates that spans a recent period. The parameters are estimated from data with weekly frequency, which corresponds to the frequency we choose for our discretisation. In addition to the parameters in Jensen, we have to specify the risk aversion parameter  $\lambda$ . We choose  $\lambda$  equal to  $-0.4$  which is in the range of parameters estimated by Navas (2004). In addition, our choice of  $\lambda$  allows us to obtain returns for the bonds that correspond to recent data for German government bonds. For example, the iBoxx Euro Sovereign Germany index for long maturity bonds shows an annualized return of 5.12% since its inception in 1999, which is in line with the mean return of 4.96% which we obtain in our scenarios.

Table 1: Parameter Values.<sup>1</sup>

	Jensen (2001)	Longstaff and Schwartz (1992)	Longstaff and Schwartz (1993)	Navas (2004)	Munk (2002)	This Paper
Alpha	0.0001	-0.0439	0.001149	2.6693E-06	0.01	0.0001
Beta	0.00186	0.0814	0.1325	8.3633E-05	0.08	0.00186
Gamma	70.3324	-	3.0493	12.3990	0.1	70.3324
Delta	1.0872	0.3299	0.05658	9.0490E-0.4	0.33	1.0872
Eta	8.7990	-	0.1582	0.0418	16	8.7990
Xi	0.8081	-	3.998	4.8949E0.3	14	0.8081
Lambda	-	-	-	-0.2 to -0.4	0	-0.4
Nu =Xi+Lambda	-	14.4227	-	-	14	

Using these parameter values, which are depicted in the Far-right column of Table 1, we simulate 1000 paths for the short rate  $r$  and its volatility  $V$  according to the equations for  $dr$  and  $dV$  in the LS model. The next step is the time-discretisation of these processes for simulating paths so that we can generate scenarios to represent the future uncertainty. We generate paths with 52 observations per year over a horizon of one year, and chose starting values for interest rate and interest rate volatility to be equal to their long-term mean given our chosen parameter values. We obtain the long-term mean by determining the equilibrium point of the differential equations for  $r$  and  $V$  above. With this approach, we obtain an initial value of 2.1% for the interest rate and of 1.654% for interest rate volatility.

Once the paths for interest rates and interest rate volatility are simulated, returns for different asset classes are calculated on each path in order to generate the scenarios. We describe the assets in the following subsection.

### 1.3. Asset Universe

For reasons outlined in the introduction, we use as benchmark instruments Eurex futures contract on notional debt instruments issued by the Federal Republic of Germany with different remaining terms, as well as options on these futures contracts.

To model the price evolution of these instruments, we generate scenarios for the price of a zero coupon notional bond with a long-term maturity. The position in this fictitious bond is rolled over every three months. As a result, the maturity of the strategy does not decline constantly, but is instead reset every three months. We also consider options on this notional bond.<sup>2</sup>

#### 1.3.1. Futures

In this section, we consider the futures contract on a long-term instrument with a six percent annual coupon (Bund Futures). In section three, we will introduce an additional contract on very long term bonds with a four percent annual coupon (Buxl Futures).

We want to ensure that the instruments we model have a sensitivity to interest rate changes that correspond to the underlyings of the Eurex futures. Since the duration of a pure discount bond is equal to its maturity, we use the calculated duration of the underlyings to specify the maturity of the bonds in our simulation model. For the sake of simplicity, we calculate duration as

$$D = \frac{\sum_{t=1}^T \frac{tC(t)}{(1+r)^t}}{P_t}$$

where  $P_t$  is the bond price at expiration of the future,  $C(t)$  is the cash flow in period  $t$  and  $r$  is the discount rate.

<sup>1</sup> - The parameter values from Jensen (2001) are from Table 5. Longstaff and Schwartz (1992) only estimate four parameters, since this is sufficient if bond maturity is assumed to be given. Parameter estimates are from their Table II. Parameter values from Longstaff and Schwartz (1993) are from Exhibit 3. Those from Navas (2004) are from table 3 and Figure 2. Parameter values used by Munk (1999) are from page 173.

<sup>2</sup> - In practice, using futures contracts and options on futures is usually considered a cheaper alternative to the use of bond and bond options. We have chosen to model bond and bond option prices, as opposed to futures and futures option prices, because of the increased tractability required for the numerical exercises performed below. In particular, in the Longstaff and Schwartz framework outlined above, we are able to use closed-form solutions for zero coupon bonds and options on these bonds. It should be stressed that this is done for modelling reasons only and has no conceptual consequences. Because of only minor differences in the contractual payoff of these contracts, our conclusions can be considered as directly carrying over to the use of bond futures and options on these futures.

For the purpose of calculating duration, we set  $P_t$  to the nominal value and  $r$  to the coupon rate.

Table 2 gives an overview of the contract specifications and the calculated duration.<sup>3</sup>

Table 2: Contract specifications

Contract	Remaining term defined in contract	Remaining term assumed by us	Calculated duration used in this paper
Euro-Bund Futures (FGBL)	8.5 to 10.5	10	7.8017
Euro-Buxl Futures (FGBX)	24 to 35	30	17.984

### 1.3.2. Options on Futures

We consider the Eurex options on the Bund futures contract. These options are available with expiry dates up to 6 months in the future. Available expiry dates are the three nearest calendar months, as well as the following quarterly month of the March, June, September and December cycle thereafter. A number of strike prices are available with intervals of 0.5 points or 50 ticks. For each contract month, there are at least nine call and nine put series, each with an at-the-money strike price as well as four in-the-money and four out-of-the-money strike prices. There are no options available on the Buxl futures contract currently.

## **2. Derivatives Strategies in Fixed Income Portfolio Management**

The management of fixed-income portfolios comprises a variety of tasks and involves both passive (indexing) and active (bond timing and bond picking) strategies. All these tasks may effectively be facilitated by using derivatives. In particular, futures on fixed income instruments help investors or managers to neutralize biases in terms of factor exposure (changes in level or slope of the yield curve) that may result from bond picking bets. Conversely, an investor may decide to protect himself from market risk in order to conserve only the return linked to the active bets taken by a manager. Another natural use of bond futures is for timing strategies between different maturity segments of the bond market. In addition, it is possible to create leveraged positions through futures, following arbitrage strategies between the futures contract and the underlying or hedging interest rate risk.

We focus below on how the use of options can further improve risk management thanks to their ability to generate non-linear, convex payoffs that offer downside risk protection. Most fixed-income strategies rely on static exposure to the returns of the different instruments. In fact, it can be argued that the implementation of dynamic strategies face a number of barriers stemming from the particular characteristics of bond markets (infrequent pricing and low liquidity compared to stock and currency markets). Under these circumstances, using derivatives is an alternative to the direct implementation of dynamic trading strategies. In fact, from the derivation of no arbitrage pricing formulas, it is well known that a payoff that is a non-linear function of the price of the underlying asset may be achieved by appropriate dynamic trading strategies in complete markets. This logic can be inverted in order to examine the equivalence of non-linear payoffs at a point in time with a dynamic trading strategy over time (see in particular Haugh and Lo [2001]). From the investor's perspective, it is interesting to assess which benefits may be expected from investing in such instruments.

In a first sub-section, we analyze the benefits of option-based strategies on a stand-alone basis. In the next sub-section, we consider the same question from a portfolio standpoint.

### **2.1. Option Strategies**

#### 2.1.1. Description of Strategies

A favorite strategy with investors and asset managers alike is a protective put buying (PPB) strategy. This strategy consists of a long position in the underlying asset and a long position in a put option, which is rolled

over as the option expires. It should be underlined that PPB is different from portfolio insurance, since the put is rolled over in each sub-period. Therefore, the payoff at the end of a total period with multiple sub-periods does not simply correspond to a guaranteed minimum payoff, as in the case of portfolio insurance. At the end of every sub-period, however, the long position in the put option offers a protection against downside risk, which leads to avoiding the left tail of the returns distribution.

The PPB strategy has been widely studied in the context of equity portfolio management (see Merton et al. [1982], Figlewski et al. [1993]). However, such an option strategy has not been analysed in any detail for the case of fixed-income derivatives. This may be surprising since downside risk is also very significant in fixed-income markets. In the theoretical setup that we have chosen, changes in the short rate and changes in interest rate volatility lead to fluctuations of bond prices. This reflects the fact that, in practice, these two factors are important determinants of the value of bond portfolios. We introduce the PPB strategy in our simulation analysis in order to assess the benefits that investors can expect from such a strategy by comparing the strategy to a position in the Bund futures contract.

Denote  $\tau$  the time until expiration of the option and  $T$  the remaining term of the bond at the initial date. We model the position in the Bund futures as a position in the bond that is held from  $t_0$  to  $t_0 + \tau$ . Hence  $T - \tau$  is the remaining term of the discount bond (i.e., the duration of the coupon bond) at option expiration. At  $t_0$  the maturity of the bond is equal to  $T$ . At  $t_0 + \tau$  the bond with remaining maturity  $T - \tau$  is sold and a new position is taken up in a bond with maturity equal to  $T$ , which is held up to  $t_0 + 2\tau$  and so forth. This corresponds to a futures position that is held for three months and then rolled over. In our simulations, we take  $\tau = 3$  months and  $T - \tau = 7.8017$ .

We construct the PPB strategy in the following way, which is similar to Merton et al. (1982, p. 35). The portfolio held is made up of a number  $N$  of the underlying bond plus the same number of put options. We then scale the initial investment to be equal to an amount  $I$ , say €100. Put options with time to expiration equal to  $\tau$  are bought at  $t_0$  so that they expire at  $t_0 + \tau$ . Hence, an option pricing formula is only needed to establish the premium at the initial investment, not for the payoff at expiry. We choose the strike price  $K$  of the put option as a function of the price of the bond  $F(r, V, T)$  at the date when the option is bought.

One outstanding question is the choice of the strike price of these options. Given the fact that exchange traded options are issued with strike prices rather close to the current price of the underlying asset, an investor will choose from this proposed range of strike prices and end up with options that are not too far in- or out-of-the-money. The typical range of moneyness considered in the literature on options strategies is from 10% out of the money to 10% in-the-money. Hence, Merton, Scholes and Gladstein (1982) consider moneyness of -10%, 0% and 10%. Likewise, Figlewski, Chidambaran and Kaplan (1993) consider moneyness of -10%, -5%, 0%, 5% and 10%. As outlined in Merton, Scholes and Gladstein (1982), there is no single best alternative for the strike price. Instead, the choice depends on the preferences of the investor, such as his risk tolerance. In what follows, we decide to set the strike price to 10% out of the money in our base case so that  $K / F(r, V, T) = 0.9$ . This corresponds to an investor who is willing to take on some downside risk in any sub-period and is concerned over decreasing profitability of the strategy when the strike price is increased (see Macmillan [2000, Chapter 17]). Fabozzi (1996, Chapter 16) highlights that in the context of bond portfolio management protective puts are usually implemented with out-of-the money puts on bonds or futures.

We generate scenarios for the position in the Bund futures and for the PPB strategy over one year with rebalancing taking place at the beginning of months 1, 4, 7, and 10.

### 2.1.2. Regulatory Issues

Since the use of derivatives faces regulatory control in practice, it is important to discuss the regulatory constraints that investors and asset managers may have to face. Starting with a concern for investor protection, regulatory authorities may indeed apply different regulations for asset managers or pension fund managers to restrict risk exposure, limit leverage or estimate risk and pass this information to investors.

For the collective investment funds in the European Union that operate under the rules of the UCITS directive the latest revised version of the directive (commonly referred to as UCITS III) applies. The so called Product Directive from January 21st, 2002, specifies some rules for investment in financial derivatives. At present, most of the member states have already transposed it into their local regulations. Under the Product Directive, investment in derivatives is permitted to be a part of the investment policy of a UCITS to improve the financial performance or hedge the risk, subject to a requirement of risk measurement.<sup>4</sup> It is worth mentioning that uncovered sales are prohibited for UCITS.

With respect to pension funds, the regulations on the use of derivatives in different countries vary widely, although there are still some common points. Generally, investing in derivatives that are not traded on a regulated market, as well as using derivatives for purposes of "speculation", is prohibited. In addition to the above mentioned explicit regulations, fund managers are also required to obey the "prudent man" principle, which means that they should use derivatives for risk control and cost control.

The PPB strategy that we describe above involves buying a put option with a long position in the underlying securities, which means that the position is fully covered and the downside risk of the investment is reduced, thus meeting the requirements stated in the UCITS directive. The motivation is essentially better risk control is a requirement for pension funds. In addition, we focus only on the derivatives that are traded in a regulated market. This means that the prohibition on the investment in OTC derivatives for pension funds does not apply. In sum, the practical implementation of this type of strategy seems to be feasible for both collective investment funds and pension funds. Further discussion of the regulation related to the use of derivatives has been relegated to an appendix.

### 2.1.3. Results

We first conduct the base case experiment using given parameter values, and then perform a variety of robustness checks.

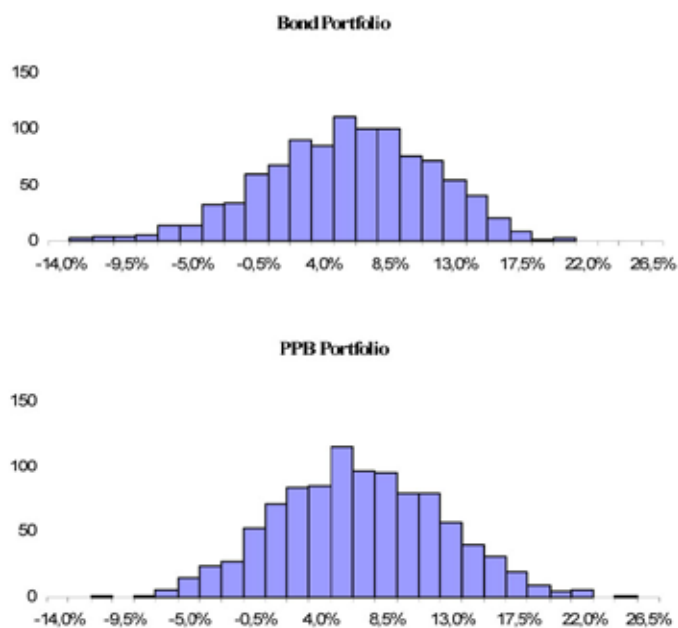
Table 3: Risk and return of a Bund futures strategy and Protective Put Buying Strategy for a one year investment horizon. The left part of the table indicates the percentiles of the return distribution over 1000 scenarios that we generated. The right part of the table indicates some standard performance measures based on this distribution. The information ratio is calculated with respect to the Bund futures strategy. "Bond" denotes the Bund futures strategy. "PPB" denotes the protective put buying strategy.

Performance statistics		
	Bond	PPB
Mean	4.96%	5.98%
Sharpe-Ratio (2%)	0.52	0.70
VaR (95%)	4.59%	3.20%
Skewness	-0.28	0.14
Information Ratio	0.00	0.41

In order to assess the results for the PPB strategy with Bund futures and options on Bund futures compared to the position in the Bund futures only, we look at the portfolio returns after one year. Table 3 shows both the percentiles of the returns distribution and typical performance statistics. It should be underlined that all these statistics are based on the distribution of the final portfolio value across the 1000 scenarios that we generate. This is different from calculating such statistics from a time-series of asset returns, as is done in empirical studies.

Examining the performance statistics leads to the conclusion that the PPB strategy is largely favourable. In particular, the mean return also increases. This stems from the fact that the put option is exercised in scenarios with strongly negative returns. Consequently, the left tail of the returns distribution is cut off, which increases the mean return. This effect is equally apparent from the higher skewness of the PPB strategy. Figure 1 below shows the return distributions for both the futures strategy ("Bond") and the PPB strategy.

Figure 1: Return distributions for both the futures strategy ("Bond") and the PPB strategy.



Inspection of the probability distribution functions of annual returns confirms the results in table 3. Focusing on negative returns below 7%, it can be seen that the PPB strategy has less frequent losses of this magnitude.

It is not true, however, that "PPB" dominates "Bond Only" for each single scenario. For a given scenario, the return of "PPB" could be lower than that of "Bond Only". Intuitively speaking, one may actually expect that the cost of purchasing the downside protection will have an impact on the performance. The following table (table 4) is insightful in this respect since it actually shows that the probability of underperforming the bond futures strategy is as high as 73.20% when the investor purchases the puts. This results from paying the option price at the beginning of every three-month period. However, the negative effect of paying the option price does not outweigh the benefits of avoiding the most negative drawdowns, as can be seen from the complete returns distribution. In other words, the PPB does under-perform slightly when bond markets are doing well, and outperforms significantly when bond markets are doing poorly.

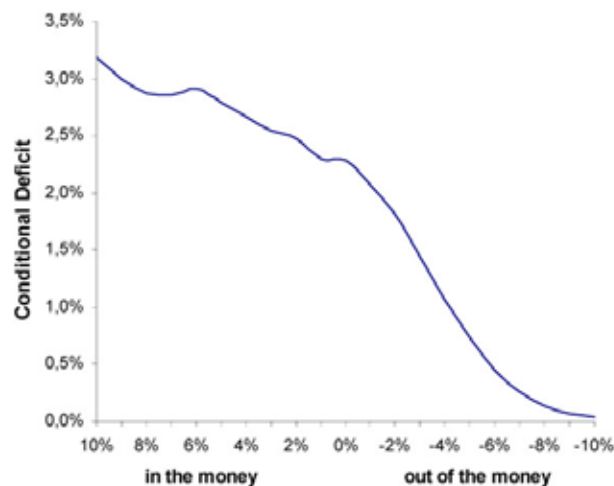
Table 4: Probability of under/outperformance.

	Return(PPB)<Return(Bonds)	Return(PPB)>Return(Bonds)
Probability	73.20%	26.80%

Since we are concerned that our results may be driven by our choice of strike price, we vary  $K$  and compare the PPB strategy to the simple futures strategy for various values of  $K$ . From a different standpoint, one can argue that rather than choosing a given value for  $K$ , an investor would be interested in choosing the optimal value. This issue is also addressed by our robustness analysis.

Changing the values of  $K$  does not lead to a significantly different evaluation of the PPB strategy. Across a wide range of strike prices, the strategy actually stays favorable when compared to the simple futures strategy. The graph in figure 2 below shows the evolution of the conditional relative deficit, i.e., average under-performance of PPB across scenarios that are such that PPB out-performs the bond only portfolio, as a function of the degree of moneyness chosen for the put options. (As recalled above, our base case corresponds to a level of moneyness equal to -10%.)

Figure 2: Conditional under-performance of the option strategy for different strike prices



The cost of the protection is apparent from this graph. It can be seen that with rising levels of protection (i.e., as the strike for the put option is chosen more and more into the money), the underperformance increases. Again, it should be noted that this is the magnitude of underperformance conditional on the fact that PPB underperforms, i.e., given that the put option is not exercised and thus that the returns of PPB are inferior to the case where no option is bought. Our choice of a level of protection consistent with a -10% level of moneyness, which is consistent with related papers, is supported by this analysis since it leads to a conditional deficit converging to zero.

## 2.2. Optimal Allocation

The previous simulations considered stand alone investments into either the options strategy or the bond futures strategy. It appears that the PPB strategy clearly dominates the bond futures strategy. More precisely, the PPB strategy has lower downside risk, while achieving returns that are considerably above those for the bond futures strategy. Investors looking for capital preservation would naturally favor such an investment. The assessment of the stand-alone benefits would ultimately suggest that an investor should replace its bond portfolio with a suitably designed option strategy.

However, instead of looking at choices between single assets, a more relevant question is to ask what benefits arise from investing into the option strategy in a portfolio context, as an addition to the bond futures strategy. This subsection turns to this issue.

### 2.2.1. Description of the Optimization Problem

Benefits of the asymmetric characteristics of options are expected to arise in a framework that explicitly takes into account the asymmetry of the returns distribution, such as in the context of quintiles of the return distribution (such as Value-at-Risk). Since long positions in put options can truncate the returns distribution at a given level, they are expected to constitute a significant portion of the optimal portfolios of investors with downside risk preferences.

Our portfolio choice problem is between the bond futures strategy and the PPB strategy described above. We test two optimization objectives: Our first risk measure is the Value-at-Risk (at 95% confidence) and the second one is the variance of portfolio returns. The time horizon is one year, as in the section above.

### 2.2.2. The Results

The aim of our exercise is to show the risk/return characteristics of minimum risk portfolios with respect to the two risk measures. We also want to analyse the resulting optimal allocation decision, i.e., the weights given to protective put buying and the simple bond futures strategy. It should be kept in mind that the PPB strategy itself contains a position in the bond futures and just adds the long position in the put option.

Table 5 below shows the risk and return characteristics of the minimum risk portfolios. In addition, the last two lines of the table show the improvement over the case where the portfolio 20 consists of 100% invested in the bond futures strategy. It can be seen that the reduction of the volatility (standard deviation of returns across simulated portfolio wealth at a horizon of one year on our 1,000 paths) is present, but not very significant in economic terms. However, the reduction of Value at Risk is very pronounced, and this even in the case of the minimum variance portfolio, which reduces the Value at Risk by 15%. The minimum Value-at-Risk Portfolio even reduces the Value-at-Risk by 32%.

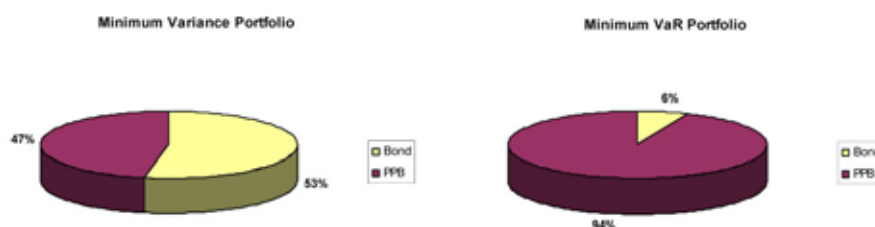
Table 5: Risk and return characteristics of minimum risk portfolios

	Min VaR	Min Variance
Mean	5.92%	5.44%
Std Dev	5.64%	5.52%
VaR (95%)	3.20%	3.90%
Sharpe Ratio (2%)	0.69	0.62
Skewness	0.11	-0.10
Reduction of Volatility	0.04%	2.21%
Reduction of VaR	30.36%	15.13%

These results clearly show that assessing the value of the option strategy in terms of standard deviation is misleading, since the standard deviation does not fully describe the risk in the case of asymmetric return distributions. The main benefit of including the option strategy comes from the fact that it introduces positive asymmetry in the portfolio. This only becomes fully apparent when looking at the Value-at-Risk, which captures the asymmetry of the returns distribution.

In addition to assessing the resulting risk return characteristics, we are interested in what percentage an investor would allocate to the option strategy given that one is averse to normal risk (standard deviation) or extreme risks. The graphs in figure 3 below show that the PPB strategy is included with a significant weight in both the minimum variance and the minimum Value-at-Risk portfolio. However, the allocation in the latter case is far greater. This is not surprising given that the optimization objective in the Value-at-Risk case leads to a preference for the asymmetry implicit in the PPB strategy. On the other hand, it appears that even under the volatility objective, the weight attributed to the PPB strategy is significant.

Figure 3: Composition of minimum risk portfolios



In sum, this section concludes that option strategies are a useful supplement to a pure linear exposure to fixed-income instruments. It should be noted that rather than designing an optimal options strategy, we define the strategy ex-ante, which suggests that even higher benefits may be obtained by an investor if one achieves an optimal design. In other words, the fact that the optimal allocation to the PPB strategy is significant even though such payoffs may not necessarily be optimal strongly suggests that the use of derivatives can significantly improve investors' wealth.

In light of the academic research (see in particular Leland [1980] as well as subsequent theoretical studies on the subject), it actually appears that a linear payoff being optimal is the exception rather than the rule (which of course is obvious from the fact that the set of non linear payoffs trivially encompasses the linear ones). While this is true in a pure asset management context (such as an investment process of a mutual fund), it is advisable to test whether non-linear payoffs will also be attractive for liability-driven forms of portfolio management. We now turn to an assessment of the strategy used in this section in an asset-liability management context.

### 3. Managing Liabilities with Interest Rate Derivatives

While in the first section, we have considered the case of an investor who does not face liability constraints (or does not explicitly take them into account in his investment decisions), here we turn to an asset-liability decision framework.

As recalled in the introduction, the recent pension fund crisis and the emergence of new regulatory and accounting standards have induced a heightened level of scrutiny of the risk management practices of institutional investors. A paradigm change is currently taking place at asset management that puts the relative performance in relation to the institution's liabilities in the forefront of the decision-making process. In the face of the dramatic impact of equity bear markets at the beginning of the new millennium, combined with an environment of decreasing interest rates that has led to a sharp increase in liabilities, the recognition by asset managers of the specific nature of the investment decision of such investors has led to the emergence of liability driven investment solutions.

In particular, it is widely recognized that institutional investors, such as pension funds, have a particularly strong preference for non-linear payoffs because of the non-linear nature of the liability constraints they face (see for example Draper and Shimko [1993]). From an academic standpoint, Leland (1980) has shown in a fairly general context that investors whose risk tolerance increases with wealth more rapidly than the average will rationally wish to obtain portfolio insurance. In Leland's words, this is particularly true for "safety-first investors": "..., [such as] pension or endowment funds which at all costs must exceed a minimum value, but thereafter can accept reasonable risks". However, rather than purchasing portfolio insurance over the investment horizon, an institution also has the possibility to use strategies with exchange-traded derivatives (options), such as those described in this paper. This will not provide perfect insurance, but achieves a truncation of the returns distribution which may be favorable in an asset-liability management context. In either case, the motivation for the use of such non-linear instruments is to generate some surplus return while achieving a protection against shortfall risk.

A different approach to asset-liability management, explicitly focusing on risk management, is a simple matching strategy, where the investor tries to hold assets that replicate his liabilities. If exact replication of the cash flows is achieved, the assets will perfectly match the liabilities and there is no risk of shortfall, as well as no chance for surplus. Let us assume for example that a pension fund has a commitment to pay out a monthly pension to a retired person. Leaving aside the complexity relating to the uncertain life expectancy of the retiree, the structure of the liabilities is defined simply as a series of cash outflows to be paid, the real value of which is known today. It is possible in theory to construct a portfolio of assets whose future cash flows will be identical to this structure of commitments.

Instead of perfectly matching the cash flow, institutional investors may take a duration matching approach, where the interest rate sensitivity of the assets matches that of the liabilities. While OTC contracts such as inflation and interest-rates swaps can prove useful in the implementation of liability-matching portfolios, we argue in this section that exchange-traded alternatives, such as futures, can be regarded as natural cost-efficient alternatives. In a nutshell, interest-rate swaps are naturally fitted for implementing cash-flow matching strategies, which involves ensuring a perfect static match between the cash flows from the portfolio of assets and the commitments in the liabilities, while exchange-traded futures should be the instruments of choice in the implementation of immunization strategies, which allow the residual interest rate risk created by the imperfect match between the assets and liabilities to be managed in a dynamic way.

We analyze in more detail this latter technique. In order to do so, we first look at a simple example. We then turn to duration matching in the context of our simulation study. The same derivatives strategies as above will be used in the simulation framework, with the addition of a very long term bond futures contract such as the Buxl futures.

### 3.1. Duration Gap Analysis

A typical pension plan is exposed to significant interest rate risk emanating from a duration mismatch between assets and liabilities. This exposure is permanent and represents a large, unacknowledged, strategic bet on interest rates and the mismatch in duration exposes the plan to uncompensated risk.

Assuming that most pension plans have an average duration between 10 and 15 years, it is obvious that there is an opportunity cost and dramatic risk/reward difference of using government bonds with a duration that will be significantly lower than that of the liabilities. A given decrease in the level of interest rates will have a more dramatic impact on the value of the liabilities than on the value of the assets, with a dramatic decrease of surplus size as a consequence

In what follows, we first formalize the argument that in an asset-liability management perspective (ALM), what matters is the gap between the investor's portfolio and the investor's liabilities, as opposed to the gap between the investor's portfolio and the proxy for market portfolio. For example, if an investor has a long-maturity liability, holding a broad-based bond or bond future exposure can introduce a (relative) risk. After reviewing the basics of duration gap analysis, we show how maturity sub-indexes can be used as completeness portfolios to neutralize unintended biases in the allocation induced by the presence of liability constraints.

A mismatch between assets and liabilities leads institutions to enjoy an exposure to interest rate risk. For example, banks or insurance companies experience interest rate risk if changes in market interest rates cause bank profits to fluctuate. If a bank has more rate-sensitive liabilities than assets, a rise in interest rates will reduce bank profits and a decline in interest rates will raise bank profits.

Let us define the gap as the difference between the present value of interest rate-sensitive assets and the present value of interest rate-sensitive liabilities. The interest rate risk with respect to the gap is then equal to the change in interest rate multiplied by the gap.

It is well known that duration measures the elasticity of the assets' (or liabilities') market value (MV) with respect to a change in the interest rate. Duration is the weighted sum of the maturities of the payments in the financial instruments, where the weights are equal to the present value of the payment divided by the present value of the asset or liability.

This allows us to measure the effects of interest rate changes on an institution's (say a bank) net worth NW:

$$\Delta NW = \Delta A - \Delta L = \left( \frac{\Delta A}{A} \right) A - \left( \frac{\Delta L}{L} \right) L$$

where A denotes bank assets and L bank liabilities.

We now recall that \$ duration can be used to compute an approximation of the absolute profit-and-loss of portfolio  $V_t = \sum_{i=1}^m \frac{F_i}{[1+y]^{t_i}}$  with cash-flows  $F_i$  for a small change  $\Delta y$  of the yield to maturity  $y$ , as can be seen

from the following relationship:  $Absolute\ P\ \&\ L \approx NV \times \$Dur \times \Delta y$

where  $\$Dur = -\sum_{i=1}^m \frac{(t_i - t)F_i}{(1+y_i)^{t_i-t+1}}$ , the derivative of the bond value function with respect to the yield to

maturity is known as the \$ duration of portfolio. On the other hand, modified duration, or sensitivity, can be used to compute the relative profit-and-loss of portfolio with cash flows, as can be seen from the following relationship:

$$\text{Relative } P \ \& \ L \approx N_V \times MDur \times \Delta y$$

where

$$MDur = \frac{1}{1+y} Dur = \frac{\sum_{i=1}^m \frac{(t_i - t)F_i}{(1+y_i)^{t_i-t+1}}}{V_t} .$$

By the definition of duration as an elasticity measure, we therefore have

$$\frac{\Delta A}{A} \cong -Dur_A \left( \frac{\Delta y}{1+y} \right)$$

and

$$\frac{\Delta L}{L} \cong -Dur_L \left( \frac{\Delta y}{1+y} \right)$$

where  $Dur_A$  and  $Dur_L$  represent duration measures for assets and liabilities, respectively.

$$\text{Hence, we finally get } \Delta NW \cong -(Dur_A A - Dur_L L) \left( \frac{\Delta y}{1+y} \right)$$

The terms in the first set of parentheses (scaled by assets A) represent the duration gap faced by the institution; that is,

$$\text{Duration Gap} = Dur_A - Dur_L \left( \frac{L}{A} \right)$$

So, we have

$$\Delta NW \cong -[(Duration \ Gap)A] \left( \frac{\Delta i}{1+i} \right) \tag{1}$$

A positive duration gap indicates that assets are more interest rate sensitive than liabilities, on average. Thus, when interest rates rise (fall), assets will fall proportionately more (less) in value than liabilities and the market value of equity will fall (rise) accordingly. On the other hand, a negative duration gap indicates that weighted liabilities are more interest rate sensitive than assets. Thus, when interest rates rise (fall), assets will fall proportionately less (more) in value than liabilities and the market value of equity will rise (fall).

The fiduciary is responsible for managing the net asset-liability interest rate position. Most fiduciaries maintain a mismatch between what are typically short-duration assets and long-duration benefit obligations. In an upward-sloping yield curve environment, a plan implicitly pays a cost of carry to bet on rapidly rising rates—a strategy that has proven detrimental in the past several years. On the level of the overall company, this duration mismatch may be exacerbated because the sponsor typically has medium- to long-duration debt opposite interest-insensitive (zero-duration) assets.

Table 6: This table shows the balance sheet of a hypothetical insurance company. The numbers below are expressed in millions of euros.

		Maturity	Market Value	Duration
<b>Assets</b>				
	Cash		100	0.00
	Earning assets			
	• 3-yr Treasury bond	3	700	2.69
	• 6-yr Treasury bond	6	200	4.99
	• Total Earning Assets		900	2.88
	Total assets		1000	2.88
<b>Liabilities</b>				
	Interest bearing liabilities			
	• Short term liabilities	1	220	0.25
	• Life insurance	10	700	8.81
	• Total liabilities		920	6.76
Total equity			80	
Total liabilities & equity			1000	

In table 6, we show the balance sheet of a hypothetical institution (insurance company). Please note that the numbers and assumptions used in that example are not necessarily realistic, as they merely serve an illustration purpose, and a similar example can be built for any type of institutional investor (bank, pension fund, etc.).<sup>5</sup>

From this example, we can calculate the duration gap.

$$Dur_A = \left(\frac{700}{1000}\right) \times 2.69 + \left(\frac{200}{1000}\right) \times 4.99 = 2.88$$

$$Dur_L = \left(\frac{220}{920}\right) \times 0.25 + \left(\frac{700}{920}\right) \times 8.81 = 6.76$$

The duration gap is then  $2.88 - (920/1000) \times 6.76 = -3.34$  years. The duration of assets is significantly lower than the duration of liabilities. As a result, liability value changes more than asset value. This is an implicit bet on an increase in interest rates. Indeed, if interest rates increase, the value of the liabilities will decrease faster than the value of the assets, which will increase the surplus level.

This bet can be beneficial if it is consistent with the manager's view about interest rates. On the other hand, in the absence of any active view on interest rates, the implicit bet induced by the mismatch between assets and liabilities introduces an undesirable element of luck in the management process.

Assuming that the yield curve is flat at an 8% level, the modified duration gap is  $-3.34/1.08 = -3.09$ . Hence, by equation (1), if interest rates decrease by 1%, the surplus, or net worth of the company, will lose value by 3.09% of the assets.

### 3.2. Duration Gap Management using Interest Rate Futures and Options

The example above emphasizes the importance of interest rate risk measurement for an institution with liability constraints. In the context of our simulation model, we evaluate the benefits of different derivatives strategies by observing the impact of price fluctuations of fixed income instruments. In order to analyse derivatives strategies in an asset liability management framework, we have to simulate paths for liabilities in addition to the paths for asset prices from section two.

5 - See Amenc et al. (2005) for more perspective.

### 3.2.1. The Investor's Liabilities

On the liability side, in an attempt to focus on a stylized institutional investor, we model the liabilities as a short position in a global bond index, which can be represented by a zero-coupon bond with constant time-to-maturity. The Longstaff-Schwartz framework allows us to easily price liabilities modelled in this manner. Our way of representing liabilities reflects that most institutional investors' liabilities are impacted by changes in interest rates and changes in interest rate volatility. As a result, liabilities would be perfectly correlated with return on a bond index. In practice, there are certainly other factors, such as actuarial uncertainty, that determine the returns on liabilities. This is the reason we introduce a disturbance term in our liability process  $l_t$ , as a convenient reduced-form way to achieve a target correlation between liabilities and the discount bond. In our model, liabilities then correspond to the returns of the discount bond with price  $L$  plus a Gaussian white noise:

$$\frac{dl_t}{l_t} = \rho \frac{dL_t}{L_t} + (1-\rho)a_t \quad \text{with } a_t \sim N(\mu_l, \sigma_a^2)$$

where  $a$  is orthogonal to  $\frac{dL_t}{L_t}$ , the return process of the bond, and where we take  $\rho = 0.8$ ,  $\sigma_a = 0.05$ , and

we set  $\mu_l$  to be equal to the mean of  $L_t$ .<sup>6</sup>

The duration of the discount bond is chosen to be equal to 10 years in a first case and equal to 15 years in a second case. This choice corresponds to the typical pension plan liability duration of 10 to 15 years (see Farley [2003]). According to van Dootingh (2004), the average duration of liabilities is approximately 15 years. In the following subsections, we consider the effect of investing in different derivatives strategies on the performance relative to liabilities. In order to isolate the effects of short-term changes in the interest rate and interest rate volatility, we assess the gap between assets and liabilities after a three-month horizon.

### 3.2.1. Bund Futures and Options

We will first look at our base case, where the investor just holds a position in the Bund future. We assess the outcome of investing 100% of the assets into this strategy in terms of shortfall measures. Since the duration of the Bund future is lower than that of the liabilities we model, it can be expected that the shortfall risk will be significant. A major problem in management of liability risk is actually that most institutional investors face liabilities that have very long maturities (as in the example of pension funds). While in principal the liability position of such an investor can be associated with a short position in a bond, the bond market may not offer any instruments of such long maturity. Therefore, it is difficult to find an appropriate instrument for hedging purposes (see next sub-section for the benefits of using futures available on longer-term bonds in that perspective).

An alternative to managing expected shortfall on the asset side may lie in using options on interest rate futures, as in the PPB strategy introduced earlier in this paper. Due to the convex nature of the payoffs to the strategy, one may expect a reduction in expected shortfall from employing the PPB strategy.

Table 7 shows the percentiles and the mean of the difference between assets and liabilities at the three-month horizon.

Table 7: Surplus/Deficit between assets and liabilities after a three-month period for the Bund futures strategy and Protective Put Buying Strategy. The percentiles of the surplus/deficit distribution over 1000 scenarios that we generated are shown. The results are for liabilities with 15-year duration.

	Bund	PPB
Minimum	-0.77%	-0.78%
5%	-0.56%	-0.56%
25%	-0.16%	-0.17%
Median	0.09%	0.08%
Mean	0.34%	0.70%
75%	0.33%	0.32%
95%	0.74%	0.76%

6 - It should be noted that the return on liabilities is usually considered higher than that of a bond with similar duration. Since the difference in performance is typically compensated by additional contributions, which we do not model here, we have chosen to abstract away from this added complexity in this paper.

Max	1.14%	6.97%
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The shortfall risk is significant, as can be seen from the fact that with 25% probability, the assets will be lower than the liabilities both for the simple futures strategy and for the PPB strategy. This has been anticipated, as there is an important duration gap. It can also be seen that, somewhat surprisingly, the PPB strategy does not reduce the shortfall risk, but actually increases significantly the performance of the portfolio. This latter statement can be verified when looking at the mean surplus and the maximum surplus. When thinking about the impact of an isolated shock to interest rates, these results become more intuitive.

Consider a steep rise in interest rates. This will lead to a decline in the value of the Bund futures and to an even steeper decline in the value of the liabilities, due to their longer duration. Therefore, we will observe a surplus after the interest rate shock for the Bund futures strategy. For the PPB strategy, the surplus will be even more positive. This comes from the fact that the decline in value of the PPB portfolio will be limited due to the protection from the put options. Next consider a steep decline in interest rates. The value of the Bund futures strategy will rise. Liabilities will rise even more in value, so that we will observe a shortfall for the Bund futures strategy after the interest rate rise. If the investor holds the PPB strategy, the value of his/her assets will also rise, but less than for the simple futures strategy. The difference is given by the option premium that the investor has to pay.

In summary, the PPB strategy actually serves as a return enhancer, while being exposed to roughly the same risk of drawdown relative to liabilities upon a negative interest rate shock (decline in the interest rate). While it may be interesting to look at alternative option strategies in the context of asset-liability management, the most straightforward way to manage the shortfall risk is to pursue simple duration matching strategies.

### 3.2.2. Introducing the Buxl Contract

As highlighted above, having long-term maturity instruments for hedging purposes at their disposal is a critical need for institutional investors since the convexity of the price-yield relation for a bond increases with longer maturities. The significant shortfall risk for the two strategies based on the Bund futures actually stems from the lower duration compared to the liabilities. This problem is all the more significant in times of low interest rate coupons which also have a positive impact on convexity. Given these problems, we propose to assess the usefulness of a 30-year bond future for hedging purposes. This contract corresponds to a duration of roughly 18 years which is actually higher than the duration for the liabilities in our model (10 years and 15 years), while the duration of the Bund contract is lower in both cases.

Figure 4 below shows the distribution of the difference between assets and liabilities at the three-month horizon. Again, this allows us to assess the impact of interest rate shocks on the investor's situation, given that he/she faces liability constraints. The left-hand histograms show the surplus/deficit distribution in the case where the liabilities correspond exactly to a short position in a bond index. The histograms on the right show the surplus/deficit distribution in the case where the liabilities only have a correlation of 0.8 with the bond index, i.e., where we introduced a white noise perturbation. The upper graphs show the case where the investor holds the simple Bund futures strategy, the middle graphs correspond to the Buxl futures strategy, and the lower graphs show the case where the investor tries to match the duration of liabilities by mixing both strategies. It can be seen that the Buxl futures strategy leads to a lower variability of the distribution, while the duration matching strategy achieves the lowest distribution. In addition, it appears that even if the liabilities deviate significantly from a zero coupon bond, the duration-matching technique is useful, i.e., the main risk stems from the interest rate changes.

We obtain the weights of the Bund and Buxl strategy by minimizing the shortfall variance. We favor this approach to simple duration calculations, which rely on certain assumptions, notably that the yield curve is only affected by small parallel shifts, which may not hold in our setup.

Figure 4: Surplus/Deficit between assets and liabilities after a three month period. 10 year duration for liabilities

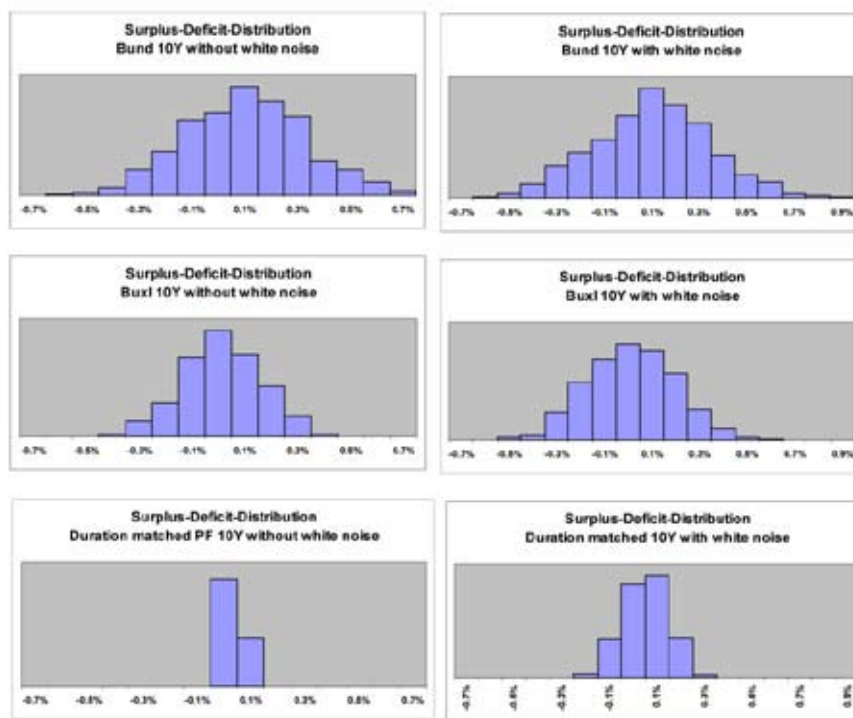


Figure 5 shows the results for the case, where liabilities have a duration of 15 years. The Buxl futures allows for even more significant reduction in this case, which is not surprising given that its duration is close to those for the liabilities.

Figure 5: Surplus/Deficit between assets and liabilities after a three month period. 15 year duration for liabilities

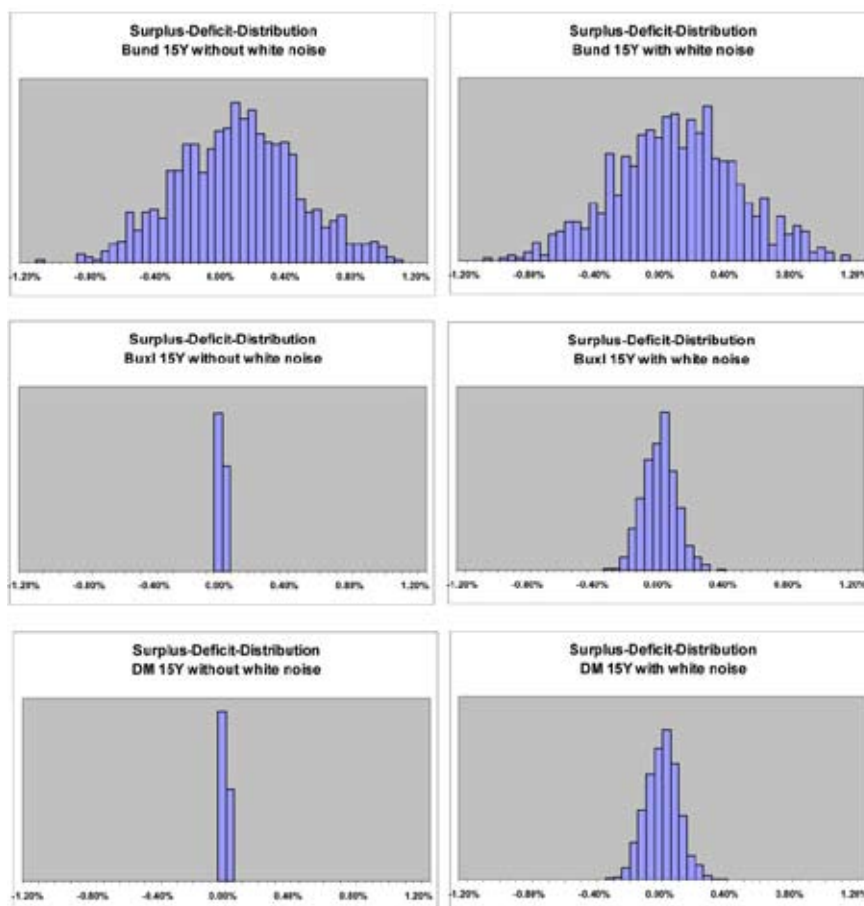


Table 8 shows the weights that we obtain for the different cases. The positive contribution of the Buxl future is obvious. The longer duration of these futures contracts leads to significant reduction of the surplus/deficit variance, allowing investors with liability constraints to achieve enhanced matching compared to the case where the Buxl futures is not available. This is reflected in the significant allocations the Buxl futures strategy obtains in the duration-matching portfolios, which range between 60 and 98.8%.

Table 8: Composition of duration matching portfolios

	Duration of Liabilities	Bund	Buxl
Without	10Y	39.7%	60.3%
white	15Y	3.8%	96.2%
noise			
With	10Y	39.2%	60.8%
white	15Y	1.2%	98.8%
noise			

A comment on the limits of our ALM risk management analysis is in order. In practice, simultaneous management of interest rate risk and other risk factors is known to be extremely difficult (see Le Vallois et al. [2003]). This is captured in our model by the error term which reflects non-interest rate related risk. It can be seen, as expected, that results with the white noise perturbation are less favorable than in the case without the white noise perturbation.

Another, probably more important, disadvantage of the risk minimization strategy considered here (as well as its more standard and specific applications such as cash-flow matching strategies, or immunisation strategies) is that it represents a positioning that is extreme and not necessarily optimal for the investor in the risk/return space. In fact, we can say that the surplus risk minimization approach in asset-liability management is the equivalent of investing in a low risk asset in an asset management context. It allows tight management of the risks. However, the lack of return, related to the absence of risk premia, makes this approach very costly, which leads to an unattractive level of contribution to the assets. The PPB strategy presented above may be a partial answer to this concern.

## Conclusion

In this paper, we assess the value of using bond futures and options on these futures. We examine the value of introducing derivatives both in a pure asset management context and in an asset liability management context. We conduct a simulation study based on the Longstaff-Schwartz (1992) interest rate model by generating stochastic scenarios for instruments that correspond to Eurex contracts on long-term German government bonds. In particular, we look at the Bund futures and Buxl futures contracts, as well as options on the Bund futures contract. Our results show that the non-linear character of the returns on put option buying strategies offers appealing risk reduction properties in the pure asset management context. Consequently, such strategies obtain a high weighting in an optimal portfolio, especially when investors are concerned with minimizing the extreme risks. In an asset-liability management context, we show that such strategies offer surplus benefits. In order to achieve significant shortfall reduction, the investor typically needs to invest in very long-term contracts that reflect the long duration of most liability constraints. Our results underline the importance of such contracts for asset liability management.

Future research may take a similar perspective in order to assess the value of different types of options strategies. In this paper we have limited the investor's asset menu to derivatives strategies that are defined *ex ante*. An extremely relevant research topic, both from a practical and from a theoretical topic, is the question of optimal product design, given the preferences of asset managers or institutional investors.

## References

- Amenc, N., J. R. Giraud, P. Malaise, and L. Martellini, 2005, "Be active with your bond trackers", *Journal of Bond Trading and Management*, forthcoming.
- Bollerslev, T., J. Cai, and F. Song, 2000, Intraday periodicity, long memory volatility, and macroeconomic announcement effects in the US Treasury bond market; *Journal of Empirical Finance*, Vol. 7, pp. 37-55.
- Benninga, S. and M. Blume, 1985, On the Optimality of Portfolio Insurance, *Journal of Finance*, 40, pp. 1341-52.
- Brennan, M. J., and R. Solanki, 1981, Optimal Portfolio Insurance, *Journal of Financial and Quantitative Analysis* 16, 279-300.
- Carr, P., and D. Madan, 2001, Optimal Positioning in derivative securities, *Quantitative Finance*, 1, 19-37.
- Cox, J. C., J. E. Ingersoll and S. A. Ross, A theory of the term structure of interest rates, *Econometrica*, 53, 385-407 (1985).
- Haugh, M., and A. Lo, 2001, Asset Allocation with Derivatives, *Quantitative Finance*, 1, 45-72.
- Fabozzi, F. J., 1996, Bond Portfolio Management, Frank J. Fabozzi Associates.
- Farley, D., 2003, Selecting a "Liability Appropriate" Fixed Income Management Approach, State Street Global Advisors <<http://www.ssga.com/library/povw/Danfarleyselectingaliabilty20031215/page.html>>, December 15th.
- Figlewski, S., N. K., Chidambaran, and S. Kaplan, 1993, Evaluating the Performance of the Protective Put Strategy, *Financial Analysts Journal*, Vol. 49 (4), pp. 46-57.
- Heath, D., Jarrow, R. and A. Morton, 1990, "Bond Pricing and the Term Structure of Interest Rates: A Discrete Time Approximation." *Journal of Financial and Quantitative Analysis* 25 (4), pp. 419-440.
- Heath, D., Jarrow, R. and A. Morton, 1992, "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation." *Econometrica*, 60, pp. 77-105.
- Ho, T. S. Y. and Lee, S. B. " Term Structure Movements and Pricing Interest Rate Contingent Claims." *Journal of Finance*, 41 (December 1986), pp. 1011-1029
- Jensen, M. B., 2001, "Efficient Method of Moments Estimation of the Longstaff and Schwartz Interest Rate Model", working paper, Aalborg University
- Jones, C. M., O. Lamont, O., and R.L. Lumsdaine, 1998, "Macroeconomic news and bond market volatility", *Journal of Financial Economics*, vol. 47(3), pp. 315-337.
- Leland, H., 1980, "Who Should Buy Portfolio Insurance?", *Journal of Finance*, 35, 581-94.
- Liu, J., and J. Pan, 2003, "Dynamic Derivative Strategies", *Journal of Financial Economics*
- Longstaff, F. A. and E. S. Schwartz, 1992, "Interest Rate Volatility and the Term Structure: a Two-Factor General Equilibrium Model", *Journal of Finance*, vol. 47, (4), pp. 1259 - 1282.
- Longstaff, F. A. and E. S. Schwartz, 1993, Implementing the Longstaff - Schwartz Interest Rate Model, *Journal of Fixed Income*, pp. 7 - 14

- McMillan, L. G., 2001, *Options as a Strategic Investment*, Pearson Professional Education.
- Merton, R. C., M. S. Scholes and M. L. Gladstein (1982):" The Returns and Risks of Alternative Put-Option Portfolio Investment Strategies", *Journal of Business*, 55, pp. 1-54.
- Munk, C., 1999, "Stochastic Duration and Fast Coupon Bond Option Pricing in Multi-Factor Models", *Review of Derivatives Research*, 3, 157–181.
- Navas, J. F., 2004, Yield Curve Fitting with Term Structure Models: Empirical evidence from the Euro Market, *Revista de Economia Applicada*, 36(12), pp. 1-28.
- Radner, R., 1972, "Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets", *Econometrica*, 40, 289-303.
- Standard Life Investments, 2003, Bridging the pensions gap, Global Bytes, [http://uk.standardlifeinvestments.com/content/strategy/strategy\\_index.html](http://uk.standardlifeinvestments.com/content/strategy/strategy_index.html). van Dootingh, S, 2005, The Implications of the FTK, *European Pensions & Investment News*, April 25th.
- Watson Wyatt, 2003, Global asset study (ongoing); as cited by "Finanz und Wirtschaft" (28/01/2004), <http://www.finanzinfo.ch>.

## Appendix: The Investment Regulation on Derivatives

In order to provide an overview on the regulations related to derivatives. In this part, we would like to further discuss the regulations regarding the investment on derivatives for UCITS. These regulations include the limitation on the risk exposure involved with derivatives and the evaluation of the risk – market risk and leverage risk. In addition, there is also a brief introduction on how the use of derivatives in pension funds are regulated in UK, Germany and France.

### UCITS

According to the Product Directive of UCITS III and the "Commission Recommendation on the use of financial derivative instruments for UCITS", financial derivative instruments can be used to reach its financial target or control risk within the following limitation:

The global exposure relating to the derivative instruments must not exceed the total NAV of the UCITS Fund. Therefore; the UCITS' overall risk exposure may not exceed 200% of the NAV on a permanent basis and under any circumstances (for example, with a temporary borrowing), the UCITS' overall risk exposure may not exceed 210% of the NAV.

The total value of the investment in derivative instruments together with transferable securities and money market instruments, certain bonds and deposits should not exceed 35% of the NAV of a UCITS fund. When a transferable security or money market instrument embeds a derivative, the latter must be taken into account. However, it was also suggested that the Member States may allow that index-based financial derivative instruments need not to be combined to the limits if they fulfil the following criteria :

- The composition is sufficiently diversified
- The index is a representative benchmark for the market to which it refers
- The index is published in an appropriate manner.

In most cases, uncovered sales are forbidden. It is strongly recommended for the UCITS to hold the underlying assets or other highly liquid assets – cash, liquid debt instruments or alternative underlying financial instruments.

In order to evaluate the risk exposure, the commitment approach, which means that the derivative positions of a UCITS are converted into the equivalent position in the underlying assets embedded in those derivatives, is recommended for "non-sophisticated UCITS". For the "sophisticated UCITS", which has more complex derivative positions, a VaR approach together with stress tests is recommended for the measurement of market risk and leverage although for the latter the commitment approach is also allowed.

However, it's worth mentioning that the above mentioned are for derivatives that are traded on regulated markets. For OTC derivatives, more restrictions and more complex regulations will be applied.

At present, the Product Directive is fully implemented in UK, Germany and France, while the principles in Recommendation are partially adopted in Germany and France.

### Pension Funds

At present, there are only some general requirements on this issue in the UK. For example, according to the Pension Act 1995 it is required that trustees should consider the diversification and suitability of the investment when they exercise such discretion. Under the Pension Act 2004, it was required that the assets should be invested predominantly on "regulated markets". In addition, the trustees or fund managers should also comply with the prudent man rules. A regulation related to the investment in occupational pension schemes is expected to be issued in the near future.

In Germany, both Pensionskassen (pension insurance funds) and Pensionsfonds (pension funds) should comply with the insurance supervision act (Versicherungsaufsichtsgesetz-VAG). Under §7(2) VAG, the investment in derivatives is only for the purposes of hedging, preparation of purchase of assets and additional yields out

of existing investments, arbitrage and speculation are forbidden. In detail, only the operation under hedging, acquisition-preparation, yield-enhancing, combined strategies and Index operations are allowed. The investment under acquisitionpreparation operation should not exceed 7.5% of the total asset portfolio and only 7.5% of the asset portfolio can be applied for yield-enhancing operations. In addition, the investment of derivatives on index operations can only be accepted where the index is proven to be almost completely correlated with the hedged portfolio.

In France, the investment activities of pension funds are regulated by ARRCO-AGIRC. According to "Reglement Financier de L'ARRCO" Part VI, pension funds can only invest in derivatives which are traded on regulated markets in the EU and only covered positions are allowed.

### References on Regulation

- Official Journal of the European Communities, 13 February 2002, "Directive 2001/108/EC of the European Parliament and of the Council" , L41/35
- Official Journal of the European Union, 30 April 2004, "Commission Recommendation on the use of financial derivative instruments for undertakings for collective investment in transferable securities (UCITS)", L 144/33
- The Committee of European Securities Regulators, July 2005, "Implementation of the European Commission's Recommendations on UCITS, Report of the review conducted by CESR", Ref: CESR/05-302b
- Financial Service Authority, 2004, "The Collective Investment Scheme Information Guide", < <http://www.fsa.gov.uk/pubs/foi/collguide.pdf>>
- Financial Service Authority, March 2004, "The CIS sourcebook- A new approach, Feedback on CP185 and made text", <[http://www.fsa.gov.uk/Pages/Library/Policy/Policy/2004/04\\_07.shtml](http://www.fsa.gov.uk/Pages/Library/Policy/Policy/2004/04_07.shtml)>
- Autorité des Marchés Financiers, 2005, "Asset management companies – Adopted directives being transposed into French Law", <[http://www.amffrance.org/documents/general/6360\\_1.pdf](http://www.amffrance.org/documents/general/6360_1.pdf)>
- Autorité des Marchés Financiers, last amended on 15 April 2005, "AMF General Regulations, Book IV - Collective investment products", <[http://www.amffrance.org/documents/general/6274\\_1.pdf](http://www.amffrance.org/documents/general/6274_1.pdf)>
- Freshfields Bruckhaus Deringer, October 2004, "German Investment Modernisation Act - Changes to German investment fund and investment fund taxation law", < <http://www.freshfields.com/practice/corporate/publications/pdfs/8858.pdf>>
- Retraite Complémentaire des Salariés (L'ARRCO), 2001, "Règlement Financier de L'ARRCO",
- the German Federal Financial Supervisory Authority Office (BaFin), 19 October 2000, "Circular R 3/2000", < <http://www.bafin.de>>