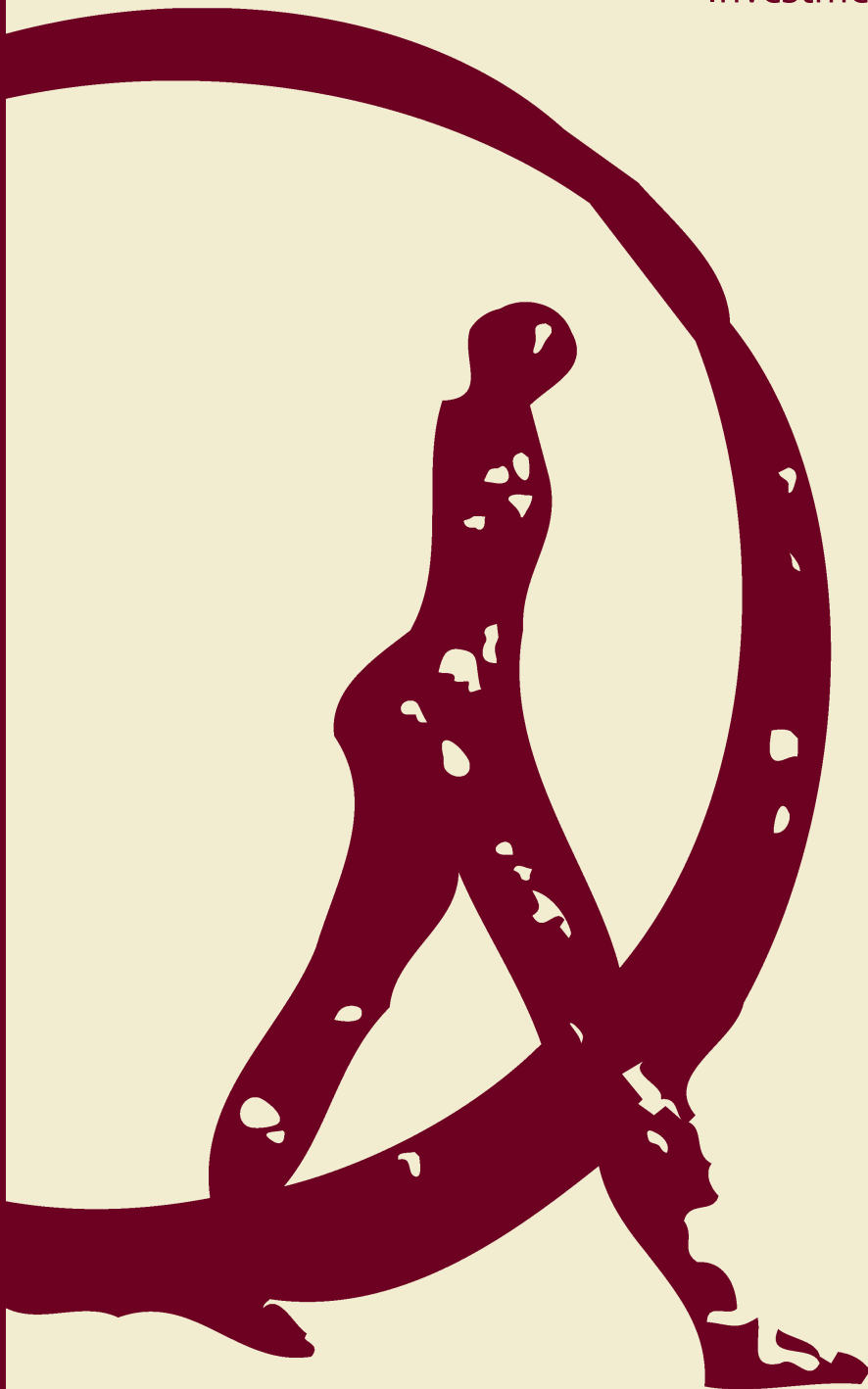


An Edhec Risk and Asset Management Research Centre Publication

Structured Forms of Investment Strategies in Institutional Investors' Portfolios

Benefits of Dynamic Asset Allocation Through Buy-and-Hold
Investment in Derivatives



CORPORATE &
INVESTMENT BANKING

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Abstract: It is perhaps surprising that institutional investors in general, and pension funds in particular, have been so dramatically affected by recent market downturns, given that an increasingly thorough range of investment vehicles have been developed over the past few years, allowing investors to tailor the risk-return profile of their portfolio in a more efficient way than simple linear exposure to the return on traditional asset classes. The focus of the present paper is to determine what fraction risk-averse institutional investors should optimally allocate to structured investment strategies (i.e., strategies involving a non-linear exposure with respect to underlying asset classes) in a general economy with stochastically time-varying interest rates and equity risk premium. We also study the impact of the presence of realistic levels of market frictions and heterogeneous expectations on volatility estimates. Our conclusion is that typical institutional investors, with a strict focus on risk management driven by the presence of liability constraints, should optimally allocate a significant fraction of their portfolio to structured investment strategies.

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About the Authors



Lionel Martellini PhD, is Professor of Finance at EDHEC Graduate School of Business and Scientific Director of the Edhec Risk and Asset Management Research Centre. Prior to joining Edhec, Lionel taught at the University of Southern California at Los Angeles as well as at the University of California at Berkeley, where he obtained a PhD in Finance. Lionel is a member of the editorial board of *The Journal of Portfolio Management* and *The Journal of Alternative Investments*. His expertise is in derivatives valuation and optimal portfolio strategies, and his research has been published in leading academic and practitioner journals. He has also co-authored several books in the area of fixed-income securities and alternative investment strategies.



Koray Simsek PhD, is associate professor of finance at Edhec, where he is responsible for teaching classes in financial modelling and quantitative methods in finance. He is also a member of the Edhec Risk and Asset Management Research Centre where he is co-head of the Asset-Liability Management (ALM) programme. Prior to joining Edhec, Koray taught at Princeton University where he obtained a PhD in Operations Research and Financial Engineering. Koray's specialist skills are in the area of decision making under uncertainty, with a particular focus on the theory and applications of stochastic programming in finance: equity and bond portfolio management, multi-stage financial planning, and asset-liability management for pension plans and insurance companies. He has co-authored numerous articles in these fields.



Felix Goltz is a research engineer with the Edhec Risk and Asset Management Research Centre and a PhD student in finance at the University of Nice Sophia-Antipolis. His research focus is on the use of derivatives in portfolio management and the econometrics of realized and implied volatility. He has studied economics and business administration at the University of Bayreuth, the University of Nice Sophia-Antipolis and Edhec. Felix is responsible for Edhec's asset management industry intelligence.

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Foreword

Structured Forms of Investment Strategies in Institutional Investors' Portfolios

Since it was set up, in 2001, the Edhec Risk and Asset Management Research Centre has made a point of conducting research that is both independent and pragmatic.

The concern to render our research work relevant and operational led us, in 2003, to publish the first studies on the policies of the European asset management industry.

The Edhec European Asset Management Practices survey allowed a comparison to be established between the academic state-of-the-art in the area of portfolio management and risks, and the practices of European managers.

This study was completed in the same year by a review of the state-of-the-art and the practices of European alternative multimangers, the Edhec European Alternative Multimangement Practices survey.

In drawing up the latter report, we were able to observe the gap that exists between the conclusions of academic research work and the practices of multimangers in measuring and reporting on the performance and risks of funds or portfolios of hedge funds. This observation led us to carry out research and a survey on this fundamental dimension of the relationship between investors and managers: the Edhec Funds of Hedge Funds Reporting Survey. This report was presented in February 2005 to pan-European institutional investors and professionals.

The new study that I have the pleasure of introducing today is part of this long line of contributions and is both scientific, because it is based on the academic state-of-the-art on the subject, and pragmatic, because it aims to respond to a key question for the asset management industry, and particularly for institutional investors: what place should be given to structured products in asset allocation?

In a universe in which financial engineering, imagination and sophistication are producing offerings that are more and more complex, and, one is obliged to point out, less and less transparent, the study carried out by our research centre seems to us to be a "reassuring" contribution. Going beyond the results in figures, and the optimisation techniques that take the non-linear character of these products into account, the "Structured Forms of Investment Strategies in Institutional Investors' Portfolios" study shows that the benefits of structured products are based on two fundamental results from modern portfolio theory:

- structured products accentuate diversification benefits through the access that they give to risks and returns that investors find difficult to manage, or indeed to find.
- structured products correspond to dynamic allocation forms which we know to be more general than, and therefore superior to, static allocation.

This study is part of a new research programme on ALM and Asset Management co-headed by Professors Koray Simsek and Philippe Touron. This programme studies the consequences for ALM of innovations in the area of asset management and of the evolution of the institutional and regulatory context for pension funds and insurance companies (IFRS and Solvency II). We hope that this initial publication will encourage you to monitor and participate in our considerations.

I would like to thank all the members of the Edhec Risk and Asset Management Research Centre team who helped to produce this research and, of course, Société Générale, who supported this research financially while leaving us the freedom to reach our conclusions on a subject that is nonetheless "sensitive" because it is a central part of Société Générale Corporate Investment Bank's activity.



Noël Amenc
Professor of Finance
Director of the Edhec Risk and
Asset Management Research Centre

Executive Summary



Executive summary

This research examines the proportion of buy-and-hold investors' portfolios that should optimally be allocated to structured products. It is the first ever academic study to explore this topic.

The findings, targeted to institutional investors, are relevant to all investors seeking to optimize the Risk-Return ratio of their portfolio.

Structured products and institutional investors

A growing market

- Institutional investors in general and pension funds in particular **have been affected by recent market downturns, some dramatically.**
- **An increasingly thorough range of structured products has been developed** over the past few years, which allows investors to tailor the risk-return profile of their portfolio in a more efficient way than simple linear exposure to traditional asset classes.
- As in the hedge fund industry, it is expected that institutional investors will follow the early lead by the private banks and that **significant inflows will occur over the next few years.**

Why Structured Products?

- Structured products: strategies involving long and short positions on equities, indices or funds, using derivatives and leveraging effects, in risk-free assets and packaged into investment vehicles that are easily accessible by investors.
- Through investing in structured products, it may be possible for institutional investors **to enjoy the benefits of dynamic asset allocation strategies while keeping the same investment throughout the period.**

A simple structuring methodology that can be generalised to a wider class of non-linear payoffs

Characteristics of the Guaranteed Structured Product (GSP) used for the study

1. A focus on the case of a **single underlying asset**, such as a stock index, with the following characteristics:
 - Guarantee of the capital invested at the initial date
 - Maturity of ten years
 - Pay off equal to the highest value reached by the underlying stock index (annual observation dates).
2. The study can also be applied to using more complex structured products in response to specific constraints.

The evaluation method

3. The **method compares the efficient frontier** (frontier which defines the limit of the maximum returns for a given series of risk levels) of an investment that includes stocks and bonds with one that includes in addition the Guaranteed Structured Product.
4. The risk is measured by the CVaR level. As opposed to Value-at-Risk (VaR) which describes a maximum loss that will not be exceeded (with a given confidence level), the CVaR measure summarises the distribution returns that are below this threshold.

Executive summary

Thus taking into account both the existence of fat tails in the return distributions and institutional investors' aversion towards taking on extreme risk.

Example of VaR and CVaR level

We take an investment with the following characteristics:

- Maturity of 10 years
- 10-year loss:
 - maximum 10% in 99% of cases
 - on average 12% in 1% of other cases

The risk is measured with the VaR and the CVaR method with a confidence level of 99%:

- VaR = 10%
- CVaR = 12%

5. We chose to proceed by defining continuous time processes for asset return dynamics and then simulated paths by using a discretisation scheme. On each path, we calculated the returns for stocks (including dividends), bonds and for the Guaranteed Structured Product. The scenarios for the total returns on each asset class were then fed to the optimization program, which allowed us to draw efficient frontiers.

6. The optimization criterion is minimizing a expected 10-year loss for a given level of expected 10-year return.

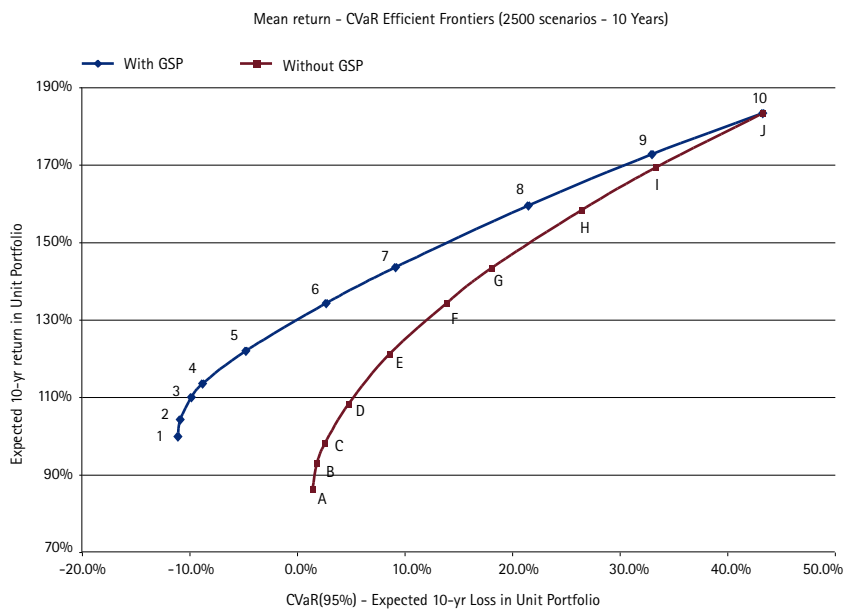
A significant improvement in efficient frontiers

Graph A

Efficient frontiers in an expected return – CVaR space with and without Guaranteed Structured Products (GSP). Note that negative CVaR values express negative losses, i.e., positive returns.

For example, point 1:

- Expected Return=100%
i.e. an investment of 100 euros/dollars will become 200 euros/dollars on average.
- CVAR(95%)= -10%
i.e. the average return of the 125 worst scenarios (5%*2500) is equal to +10%.



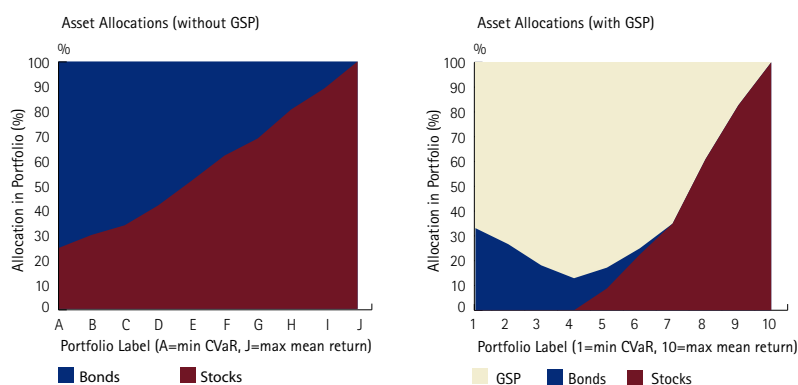
Our results show considerable improvement in the efficient frontiers depicting the risk return trade-off of the investor when the latter invests in the Guaranteed Structured Product.

Executive summary

Which asset allocation for structured products?

Graph B

One may observe the change in asset allocation with respect to the change in risk aversion. These allocations correspond to portfolios labeled on the two efficient frontiers above. The GSP helps risk-averse investors increase their returns by replacing the stock allocation in their portfolio and helps risk-seeking investors to decrease shortfall risk by replacing the bonds in their portfolio.



For investors with a strong aversion to risk (points 1–3) the weight of the structured product takes on values between 70 and 90 percent. On the other hand, risk-seeking investors (points 5–9) can actually decrease their shortfall risk exposure by replacing the bonds in their portfolio with a structured product. For this group of investors, optimal allocation to the structured product ranges from 10 to 70 percent. In fact, only the most risk-seeking investors (point 10) would have a zero allocation for a structured product and invest 100% in stocks.

Table C

Changes in the values of the objectives and the optimal asset allocation in the presence of weight constraints on GSP allocation. Numbers are not annualized. A 0% upper bound implies that GSP is not included in the opportunity set (point E in Graphs A and B).

Upper Bound on GSP Allocation	Allocation			
	Stocks	Bonds	GSP	Return / CVaR ratio
0%	51.69%	48.31%	00.00%	2.45
5%	47.15%	47.85%	5.00%	2.97
10%	42.65%	47.35%	10.00%	4.01
20%	36.54%	43.46%	20.00%	13.38

Robustness of diversification benefits

- It is important to consider the impact of:
 - market frictions
 - heterogeneous expectations on volatility estimates
 - fees

For the structured product, as a result of the fees, the stock and bond indices become more attractive and replace part of the allocation of the structured product. The allocation decrease of the structured product, however, is relatively small.

- Because of weight constraints for structured products, it is not reasonable to expect institutional investors to allocate a dominant fraction of their portfolio to structured products.
- We tested the impact of imposing an upper bound on the allocation to the product.

Overall, these results strongly suggest that adding even a limited fraction of the overall allocation to structured products allows for significant benefits, measured in terms of an increase in the return/CVaR ratio of the portfolio, a measure of risk-adjusted performance. (Table C).

Executive summary

Conclusion

An improvement in efficient frontiers

- Such products allow investors **to profit from the equity risk** premium without being fully exposed to the downside risk associated with investing in stocks.
- As a result, typical institutional investors, with a strict focus on risk management driven by the presence of liability constraints, **should ideally allocate a significant fraction of their portfolio to structured investment strategies.**

Prospects for the structured products market

- As institutional investors have high demands in term of risk management, a trend towards greater transparency, liquidity and cost control has already taken place in the structured investment management industries, and substantial amounts of assets are shifting from traditional investments to structured products.

Generalization of results

- While these results have been performed on the basis of a specific example, **they can be generalized to a wider class of nonlinear payoffs.** In particular, **products with convex payoffs are ideal for institutional investors** with a focus on the management of extreme risks.

Structured Forms of Investment Strategies in Institutional Investors' Portfolios

Benefits of Dynamic Asset Allocation Through Buy-and-Hold
Investment in Derivatives



Introduction

The institutional asset management industry has become an important feature of modern financial markets, with the scale of this business's importance readily apparent from the size of assets under management by different types of institutional asset managers. In particular, private and public sector pension plans have become key elements in the functioning of financial markets, as a significant percentage of stocks and fixed-income securities are held by these institutions.

Recent difficulties have drawn attention to the risk management practices of institutional investors in general and defined benefit pension plans in particular. A perfect storm of adverse market conditions over the past three years has devastated many corporate defined benefit pension plans. Negative equity market returns have eroded plan assets at the same time as declining interest rates have increased market-to-market value of benefit obligations and contributions. In extreme cases, this has left corporate pension plans with funding gaps as large as or larger than the market capitalization of the plan sponsor. For example, in 2003, the companies included in the S&P 500 and the FTSE 100 index faced a cumulative deficit of \$225 billion and £55 billion, respectively (Credit Suisse First Boston (2003) and Standard Life Investments (2003)), while the worldwide deficit reached an estimated 1,500 to 2,000 billion USD (Watson Wyatt (2003)).

That institutional investors have been so dramatically affected by market downturns is perhaps surprising given that an increasingly thorough range of investment vehicles have been developed over the past few years, which allow

investors to tailor the risk-return profile of their portfolios in a more efficient way. These investment vehicles, which allow their user to achieve a non-linear option-like exposure with respect to the return on traditional asset classes, are known as structured products. (In an attempt to clarify the terminology used throughout the paper it should be stated from the outset that we use interchangeably here the label "structured products" or "structured investment strategies" to refer to any contractual non-linear payoff resulting from a large variety of quantitative asset management techniques¹. The definition of "structured asset management", as a family of investment strategies, is actually quite large as it encompasses static asset allocation strategies as a trivial specific case, dynamic allocation strategies such as CPPI and OBPI, as well as payoffs resulting from the inclusion of exotic derivatives.²)

From a practical standpoint, the use of structured forms of asset management by institutional investors has found its roots in constant proportion portfolio insurance (CPPI) and other forms of portfolio insurance strategies. Since then, a wide range of structured products have been developed, allowing institutional investors to customize their exposure to equity markets so as to make it consistent with their preferences and liability-driven constraints³.

From an academic standpoint, it was recognized early on that structured products are natural investment vehicles for institutional investors, who have a particularly strong preference for non-linear payoffs because of the non-linear nature of the liability constraints they face (see for example Draper and Shimko (1993)).

1 - "Structured products" are typically perceived as standalone investments. That is the reason why it is perhaps preferable to refer to "structured asset management", as a family of quantitative asset management techniques.

2 - It should be noted that the use of "structured products" in this context could be confusing, given that it is more commonly used in the United States to refer to mortgage-backed securities and other forms of asset-backed securities

3 - See section 1 for more information on the market for structured products.

Introduction

Leland (1980) has shown in a fairly general context that investors whose risk tolerance increases with wealth more rapidly than the average will rationally wish to obtain portfolio insurance⁴. This is the case in particular for institutional investors whose portfolio value must at all cost exceed a given value, but thereafter can accept reasonable risks. From a pragmatic standpoint, taking for example the case of pension funds, it is clear that a small change in the probability of extreme contribution rates is typically considered much more important than an equal change in the probability of an extremely high refund. In other words, structured products allow investors to profit from the equity risk premium, which is often needed to match the returns on liability-driven benchmarks, without being fully exposed to the downside risk associated with investing in stocks. This is especially important for under-funded institutions, and in the presence of a low interest rate environment that raises concerns of mean-reversion towards higher interest rate levels, and hence lower bond prices, making a dominant exposure to bond markets an uncomfortable decision.

That institutional investment in structured products is bound to significantly increase in the future raises an interesting question that is the focus of the present paper: how to optimally mix structured products and non-structured products in an institutional investor's asset allocation? Generally speaking, two different approaches to risk management can be followed. For example in the context of market risk management, the first approach consists of risk diversification, i.e., reducing risk by optimal asset allocation techniques on the basis of

imperfectly correlated assets. The second approach consists of risk hedging, i.e., reducing risk by using some form of insurance contract (derivative instrument) on a given underlying asset, the packaging of both the underlying asset and the insurance contract being known as a structured product. Given that diversification and hedging are two different, and perhaps competing, forms of risk management, whether the benefits of these two approaches can be added and combined so as to generate even greater risk reduction benefits appears to be a rather nontrivial question. In particular, structured products are often seen as stand-alone products, already enjoying built-in risk reduction properties without the need for further risk diversification.

Fortunately, asset pricing theory allows us to better understand the nature of the relationship between allocation and structuring from a conceptual standpoint. It can be argued that a structured products approach to risk management, based on hedging, can be regarded as the most general, dynamic, as opposed to static, form of asset allocation. It is indeed well known, since Merton's (1973) replicating argument interpretation of the Black and Scholes (1973) formula, that non-linear payoff based on an underlying asset can be replicated by dynamic trading in the underlying asset and the risk-free asset⁵. As a result, it appears that an investor willing and able to engage in dynamic asset allocation strategies will be in a situation to generate the most general form of risk management possible, and this encompasses both static diversification and dynamic hedging. While the benefits of dynamic asset allocation strategies in a stochastically time-

4 - Leland (1980) also shows that investors whose return expectations are more optimistic than average will also rationally wish to obtain portfolio insurance.

5 - A previously mentioned example of the use of dynamic asset allocation techniques to generate a structured product type of payoff is the CPPI technique, where dynamic rebalancing between the risky and risk-free asset is performed so as to guarantee a minimum capital at maturity.

Introduction

varying environment have been recognized since the late 1960s (see Hakanson (1969, 1971) and Samuelson (1969), in a discrete-time setting, as well as Merton (1971), in a continuous-time setting, for the development of a multi-period approach to optimal asset allocation decisions), it is only recently that specific optimal asset allocation models that exhibit an explicit time-dependency in the presence of stochastic opportunity sets have been introduced⁶. With dynamic asset allocation, portfolio weights change through time, either as a response to changes in the investment opportunity set or in an attempt to generate non-linear payoff structures. This is opposed to a buy-and-hold strategy where weights evolve according to changes in prices, but also to a fixed weight strategy, where rebalancing is allowed to revert to the initial weights, hence severely restricting the kind of payoffs generated to simple linear functions of underlying asset classes⁷.

Given that a structured product can be viewed as a static packaging of some dynamic asset allocation strategy, institutional investors may be able to enjoy the benefits of dynamic asset allocation strategies through the use of buy-and-hold investment in structured products. In this paper, we focus on the benefits that can be gained by a typical myopic institutional investor who feels comfortable with making long-term asset allocation decisions, but is willing to expand the opportunity set so as to include structured products. The reason why we focus on the case of a myopic investor is twofold. Firstly, most institutional investors use static asset allocation schemes because they feel they lack the infrastructure and

expertise necessary for the implementation of dynamic asset allocation strategies and are reluctant to face the associated costs. Secondly, it is precisely the focus of this paper to show how institutional investors might be able to enjoy the benefits of dynamic asset allocation strategies, while staying within their comfort zone thanks to the use of static investment in structured products⁸.

Our paper is closely related to a number of papers that look at optimal portfolios when investors have access to a risky asset and options on this risky asset. Under a set of rather stringent assumptions (continuous trading, absence of transaction costs, leverage or short-sales constraints, constant volatility and drift of the underlying asset, symmetric information), it can actually be shown that the introduction of structured products would not improve the welfare of investors. In particular, in a Black-Scholes setting with continuous-time trading and no transaction costs, assets generating non-linear payoffs are redundant because they can be replicated by dynamic trading in the underlying assets and the risk-free asset. As a result the optimal allocation to such assets is indeterminate when an investor's expectation about parameter values matches that of the market, or infinite when the investor's beliefs lead him to view the derivative asset as under-priced. On the other hand, any violation of the aforementioned assumptions would involve a non-trivial demand for nonlinear payoffs. Because continuous trading is practically impossible and would be infinitely costly in the presence of transaction costs, several authors have examined the question of the optimal allocation to

6 - See for example Lynch (2000), Lioui and Poncet (2001), and Liu (2002), among others.

7 - A dynamic investor can also respond to conditioning information in order to exploit predictability of return dynamics or to hedge changes in the investment opportunity set (see recent papers by Brandt and Santa-Clara (2004) and Bansal, Dahlquist, and Harvey (2004) for interesting attempts at considering strategies that aim to exploit return predictability in the asset menu of a static investor.)

8 - Other advantages of structured products include privileged market access to derivatives (so-called "hidden assets") and mutualization benefits.

Introduction

derivatives for investors who are prevented from performing dynamic trading.

A first strand of the literature, initiated by Brennan and Solanki (1981), has examined the question of optimal positioning in derivative assets from the static investor's standpoint. The question there is to determine the general shape of the non-linear function of the underlying asset that would generate the highest level of expected utility for a given investor. Carr and Madan (2001) extend this early work by considering optimal portfolios of a riskless asset, a risky asset and derivatives on the latter in an expected utility framework. They solve the optimal asset allocation problems depending on the investor's preferences, beliefs and market prices for an investor with increasing concave utility over final wealth. In a first step, they derive the optimal wealth as a function of the price of the risky asset, and then obtain the portfolio of the three assets that delivers the optimal wealth function⁹. In that paper, the assumption of continuous-trading is replaced by the assumption of the existence of a continuum of call options with all possible strike prices, which allows static, as opposed to dynamic, replication of general non-linear payoffs¹⁰. A second strand of the literature (starting with Leland (1980) and extended by Benninga and Blume (2000)) has studied the question from a different angle: taking as given a set of standard derivatives, these papers examine the features of (static) investors' preferences that would support a rational non-zero holding of these contracts.

In this paper, we consider the question from a different perspective: we consider both the contract features and investors' preferences as given in an incomplete market setup and we are interested in quantitative estimates of the optimal allocation to non-linear contracts. In other words, the focus of our paper is positive in nature; given actual features of the investment world, our goal is to determine what fraction, if any, institutional investors should allocate to structured products¹¹. From a normative standpoint, it also provides a set of interesting insights into the robustness of optimal design analysis. If the optimal allocation to existing payoffs is significant even though such payoffs may not necessarily be optimal, it strongly suggests that the use of derivatives can significantly improve investors' welfare.

The rest of the paper is organized as follows. The first section gives an overview of structured products. The second section analyses the equivalence of a static position in a structured product to a dynamic trading strategy, while the third section presents the framework in which the diversification benefits derived from including such assets will be studied. The empirical results are presented in the fourth section. Conclusions and suggestions for further research can be found in the fifth section, while the details of the optimization program are relegated to a dedicated appendix.

9 - Driessen and Maenhout (2003), in addition to the expected utility case, consider specific derivatives strategies for a variety of non-expected utility investors.

10 - See also Bookstaber and Clarke (1983), Pelsser and Vorst (1995), and Chan, Ching, and Yun (2004) for the portfolio problem in the presence of multiple risky assets and options on those assets, and Carr, Jin and Madan (2003) and Liu and Pan (2003) for the case of a dynamic investor.

11 - Because our estimate for risk is a measure of downside risk, our work is also somewhat related to an early paper by Booth, Tehrani and Trennepohl (1985) who analyse stochastic dominance of portfolios using options. See also Ahn et al. (1999) for the optimal use of put options only or Dert and Oldenkamp (2000) for both put and call options in a downside risk context.

An Overview of Structured Products used in Institutional Money Management

The salient characteristic of structured products is the repackaging of strategies that involve long and short positions in derivatives and the underlying asset into an investment vehicle that is easily accessible by investors. In particular, since the investor does not have to access derivatives markets himself, he is not constrained to respect margin requirements. Historically, the first occurrence of a structured form of asset management was the introduction of portfolio insurance such as the simple option based portfolio insurance (OBPI) strategy. While any asset may be used in a structured product, this paper focuses on the case of a stock index as an underlying. In this first section, we provide an overview of the institutional aspects of structured products. We also review a range of structured products that are available to institutional investors.

Institutional Aspects of the Market for Structured Products

Like many financial innovations, structured products have become highly popular in recent years. Consequently, there are significant sums involved in the issuance of structured products. In the major European Union countries, the notional amount issued over the year 2004 for the retail market already reached €99.4 billion¹². Graph 1 below shows estimated amounts with a breakdown by country. This is to be compared with an inflow of €225.3 billion into European investment funds (UCITS) of all categories (stock, bond, balanced and money market funds) in 2003¹³.

As in the hedge fund industry, it is expected that institutional investors will follow this early lead by the retail market and significant inflows are expected to occur over the next few years.

The actual investment vehicle that is used in order to provide a structured product to an investor may take on various legal or organizational forms. These obviously depend on the legal framework in the given legislation. However, in principle, structured products may be organized as bonds or as investment funds. Bonds allow a straightforward implementation of structured products that involve a capital guarantee. The guarantee is achieved because the holder of the bond is entitled to the payment of the face value at maturity. The coupon payment may then be linked to the price of an underlying asset. This way, the bondholder effectively holds a portfolio of a zero coupon bond and a derivative asset. In other words, one component of the payoff at maturity is certain while the other part depends on the price of the underlying asset. While bonds that have a limited lifetime may seem to be a more natural vehicle, investment funds also allow structured products to be implemented. Investment funds pool investors' wealth into a portfolio whose management is delegated to a professional manager. The investment policy of the fund may then be defined as a strategy that achieves the desired payoff at maturity. In particular, the fund manager may make use of government bonds, an underlying asset and derivatives on the latter in order to achieve the desired payoff.

¹² - Estimates from Société Générale Corporate and Investment Banking.

¹³ - See FEFSI, 2003, Trends in the European Investment Fund Industry in the Fourth Quarter of 2003 and Results for Full-Year 2003. Results for 2004 are not available.

An Overview of Structured Products used in Institutional Money Management

Institutional Aspects of the Market for Structured Products

Since secondary markets exist for both bonds and investment fund shares, structured products may be exchanged between investors outside the subscription at the initial date and redemption at maturity. In fact, the issuer of the structured product often acts as a market maker for his product. It is typically argued that providers of structured products are more efficient than end-users in managing products involving non-linear payoffs for a variety of reasons including among others economies of scale, privileged market access (to the underlyings and to the derivatives), mutualization effect (managing a portfolio of options is cheaper than managing each option individually) and efficient financing (better access to money markets compared to conditions that would be faced by end-users if they had to borrow to implement the corresponding dynamic replication strategy).

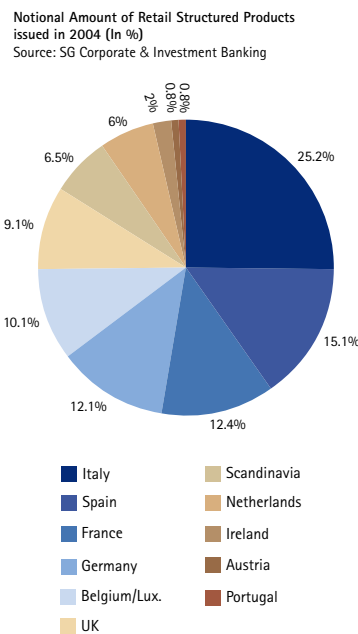
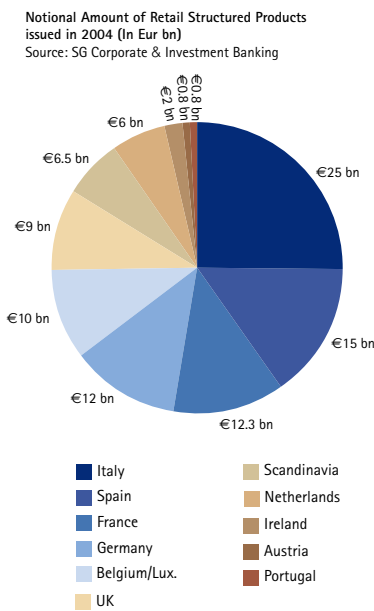
Typology of Structured Products

Historically, the first occurrence of a structured form of asset management was the introduction of portfolio insurance such as the constant proportion or option based portfolio insurance strategy. The next wave of structured investment strategies has led to the introduction of exotic derivatives in asset management. While the immense variety of structured products makes it difficult to provide a clear classification, existing products may be distinguished among several axes. From an investor's perspective, what matters is the type of payoff he obtains with the structured products. Therefore, the classification below builds on the payoffs and is partly based on a common way of classifying options.

Curvature

Structured products offer payoffs that are non-linear functions of the price of an underlying. According to the type of non-linearity, one can divide these products into those with concave and those with convex payoffs. Convex payoffs are typical for products that include a capital guarantee, typically offered by an investment bank. This may be the most obvious form of nonlinearity investors are looking for. Concave payoffs occur with discount certificates and other covered call writing strategies. These products allow investors to access limited upside potential of a given asset at a lower price than full upside potential. More complex structured products may mix both types of non-linearity.

Graph 1
Estimated notional issuance of retail structured products, 2004 (Billion Euros and Percentage)



An Overview of Structured Products used in Institutional Money Management

Typology of Structured Products

Vega

One of the reasons for investing in structured products is their exposure to changes in volatility of the underlying asset. Sensitivity (Vega) of the structured product may be positive or negative. Depending on their views on the evolution of volatility, investors may optimally select different types of exposure.

In this paper, we focus on a specific example (structured product involving a hindsight option written on a stock index), they can be generalized to a wider class of non-linear pay-offs (see the conclusion). For comparison purposes, we also discuss the standard OPBI in second section.

Path dependency

Rather than depending on the observed price of the underlying asset at maturity, the payoff may be a function of the extreme (lookback or hindsight option) or average (Asian option) price observed during the lifetime of the structured product. For example, path dependent products are used by investors who want to lock in the performance of a perfect timing strategy on a single asset (which corresponds to a lookback straddle strategy).

Type of underlying asset

The underlying assets involved in a structured product may be an index or a basket of stocks. A number of structured products involve pay-offs that depend on a large number of assets. Structures may include options to exchange one asset for another, or structures that pay the maximum return among a number of assets. Such products are used by investors who want to lock in the performance of a strategy that times perfectly among a number of assets.

Structured Products as Packaged Dynamic Asset Allocation Strategies

Because a structured product is nothing but a packaged version of dynamic trading in the underlying asset and cash, a portfolio including a structured product will effectively have time-varying weights. This section shows how a particular class of structured products is designed and tracks down their equivalence with dynamic trading strategies. This is done by looking at the replicating portfolios for structured products.

Stock index based products with a capital guarantee can be seen as an investment in a pure discount bond and in a delta-positive option on the underlying index. Investing the present value of a guaranteed amount in the pure discount bond assures protection against price decreases whereas the option position allows participation in price increases. The resulting delta of the entire structure is positive, distinguishing it from short selling strategies and straddle-like strategies. Furthermore, the long position in the option leads to vega-positivity and pay-offs are a convex function of the price of the underlying. Historically, such structures were based on plain vanilla call options, but the use of exotic options such as Asian, lookback, barrier or binary options constitutes a straightforward extension of the principle. This section looks at both plain vanilla and exotic structures in a simple setting where the assumptions of Black and Scholes (1973) hold. In a first step, we focus on the static portfolio of a zero coupon bond and a derivative that replicates the payoff of the structured product. In a second step, we make explicit the correspondence between a non-linear payoff and a dynamic asset allocation strategy.

Static Replication

Plain Vanilla Call

Starting with the example of a structure based on a simple European call option, we can write the payoff at maturity of the structured product as

$$S_0 + k \max(S_T - S_0, 0) .$$

In order to achieve this payoff, a portfolio P_0 consisting of the following two components may be created:

(i) Pure discount bond investment of $S_0 e^{-rT}$

(ii) European Call option investment of $k C_0$

Denote the Black-Scholes price of a European call option at time t as $C_t(S_t, \sigma, r, T)$. Then the participation in the upside potential is given by

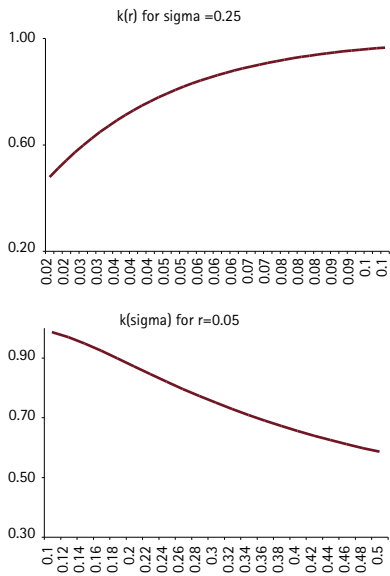
$$k = \frac{S_0 - S_0 e^{-rT}}{C(S_0, \sigma, r, T)} .$$

Therefore, for a given maturity and initial asset price, we can write k as a function of two parameters, σ which affects the option price and r which affects both option price and the present value of the guaranteed level. It is straightforward to see that k is a decreasing function of σ : as volatility goes up, the value of the option increases. Its dependence on r seems to be more ambiguous since this parameter influences both the option price and the present value of the guaranteed level. On the one hand, as the interest rate increases, the price of the pure discount bond decreases, and more cash can be invested in the option.

Structured Products as Packaged Dynamic Asset Allocation Strategies

Static Replication

Graph 2
Level of exposure to upside potential as a function of interest rate and volatility – The case of a standard option



On the other hand, the price of the call option increases with r . However, the first effect always dominates the second, so that k is an increasing function of r . Dependence of k upon parameter values can be shown by means of an example, as well as graphically or formally.

As an example, with $r = 5\%$ and $\sigma = 30\%$ and a 10- year contract, we have $S_0 e^{-Tr} = 60.65$, $C_o(S_0, \sigma, r, T) = 52.57$ and $k = 39.35 / 52.57 = 0.75$.¹⁴

Graph 2 shows k as a function of risk-free rate and volatility when other parameters are fixed at a level consistent with our example.

In addition to these examples, as long as the option price can be written in closed form, k may be written as an explicit function of parameter values:

$$k = \frac{S_0 - S_0 e^{-Tr}}{C(S_0, s, r, T)} = \frac{S_0 - S_0 e^{-Tr}}{S_0 N(d_1) - S_0 e^{-rT} N(d_2)}$$

where

$$d_1 = \frac{(r + \sigma^2 / 2)T}{\sigma \sqrt{T}}, d_2 = d_1 - \sigma \sqrt{T}$$

and $N(x)$ is the cumulative probability distribution function for a standardized normal distribution.

The following limiting cases as well as analysis of partial derivatives below show that $0 \leq k \leq 1$:

$$\lim_{r \rightarrow \infty} k(r, \sigma) = \frac{S_0}{S_0} = 1$$

$$\lim_{r \rightarrow 0} k(r, \sigma) = 0$$

$$\lim_{\sigma \rightarrow \infty} k(r, \sigma) = 0$$

$$\lim_{\sigma \rightarrow 0} k(r, \sigma) = \frac{S_0 - S_0 e^{-rT}}{S_0 - S_0 e^{-rT}} = 1$$

The partial derivatives of k with respect to these two parameters (for non dividend paying stock) read:

$$\frac{\partial k}{\partial \sigma} = \frac{-1}{(\sqrt{T} + N'(d_1))^2} = \frac{-1}{Vega^2}$$

$$\frac{\partial k}{\partial r} = \frac{S_0}{Rho^2} (e^{-rT} C_o - (1 - e^{-rT}) Rho)$$

$$\text{where } Rho = S_0 T e^{-rT} N(d_2)$$

One can verify that $\frac{\partial k}{\partial \sigma} < 0$ and,

for $r > 0$, $\frac{\partial k}{\partial r} > 0$.

Continuous Hindsight Option

A different way of capturing the upside of price movements in the underlying is to buy an option that pays off the maximum price that the underlying asset has reached during the period, i.e., $S_0 + k(S_{\max} - S_0)$.

¹⁴ - These numbers serve a mere illustration purpose and are not necessarily realistic. In particular while a 30% volatility is not unusual for individual stocks, typical estimates would be lower for equity index volatility.

Structured Products as Packaged Dynamic Asset Allocation Strategies

Static Replication

This specific kind of a lookback option is known as a hindsight option. This payoff allows investors to capture within maturity price rises as opposed to having a payoff that depends only on the price at maturity.

In order to achieve this payoff, a portfolio P_0 consisting of the following two components may be created:

- (i) Pure discount bond investment of $S_0 e^{-rT}$, which delivers a payoff at T of S_0 .
- (ii) Hindsight call option investment of $k HC_0$, which delivers a payoff at T of $k(S_{\max} - S_0)$.

We have to divide the difference between the initial outlay and the present value of the guarantee by the price of this payoff in order to get k . As stated above, the present value of the guarantee depends on r . The type of path dependency present in this product renders it more attractive and more expensive than the simple call option structure. With the same level of guaranteed capital, k will therefore be smaller than above. If we consider how k depends on the parameters σ and r , the reasoning for the structured product including a plain vanilla call carries over to this path dependent structured product. At initial date, the price of the hindsight option does not depend on S_{\max} since $S_{\max} = S_0$. In the context of a Black-Scholes economy, it can be shown (see Goldman, Sosin, and Gatto (1979)) that the price of the hindsight option, denoted as $HC(S_0, \sigma, r, T)$, is given by:¹⁵

$$HC(S_0, \sigma, r, T) = S_0 e^{-rT} \left(N(b_1) - \frac{\sigma^2}{2r} e^{Y_2} N(-b_3) \right) + S_0 e^{-rT} \frac{\sigma^2}{2r} N(-b_2) - S_0 e^{-rT} N(b_2)$$

where:

$$b_1 = \frac{\ln(S_{\max}/S_0) + (-r + \sigma^2/2)T}{\sigma \sqrt{T}},$$

$$b_3 = \frac{\ln(S_{\max}/S_0) + (r - \sigma^2/2)T}{\sigma \sqrt{T}},$$

$$b_2 = b_1 - \sigma \sqrt{T}, \text{ and}$$

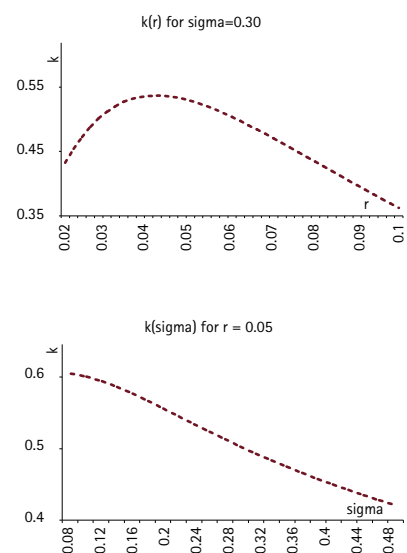
$$Y_2 = \frac{2(r - \sigma^2/2)\ln(S_{\max}/S_0)}{\sigma^2}.$$

Taking the example ($r=5\%$ and $\sigma = 30\%$) from above, we have that $S_0 e^{-rT} = 60.65$, and $k = 0.5$.

Graph 3 below shows k as a function of risk-free rate and volatility when other parameters are fixed at a level consistent with our example.

Graph 3

Level of exposure to upside potential as a function of interest rate and volatility – The case of a hindsight option



15 - Goldman, Sosin, and Gatto (1979) derive the price of a lookback put option, which delivers a payoff of $S_{\max} - S_T$. The hindsight call option may be created by adding a position in a forward contract and an amount of $(S_0 e^{-rT} - S_0)$ invested in the pure discount bond.

Structured Products as Packaged Dynamic Asset Allocation Strategies

Static Replication

As an explicit function for non-dividend paying stock, we have

$$k = \frac{S_0 - S_0 e^{-Tr}}{HC(S_0, \sigma, r, T)}$$

For partial derivatives, one can verify in this

case that $\frac{\partial k}{\partial \sigma} < 0$ whereas $\frac{\partial k}{\partial r}$ may take

positive or negative values.

Discrete Hindsight Option

The payoff of the product used in the fourth section is

$$S_0 + k \max(S_i - S_0)_{i=0, \dots, 10}$$

The only difference with the continuous version is that the price path is sampled at annual intervals. Therefore, intuition suggests that the price of this option is lower, hence k is higher than for the continuous version. At the same time, it is obvious that this type of option is more attractive than the simple call option from above; hence k is expected to be lower than with the plain vanilla call.

Broadie, Glasserman, and Kou (1999) introduce correction terms that allow the price of discrete options to be approximated using closed form solutions for their continuous counterparts. In our case the approximation error is large since the number of observations is small. Therefore, numerical procedures are preferable in order to estimate k (see the fourth section).

Market frictions

In the analysis above, pricing of the options is considered in a Black-Scholes setting. In practice,

providers of structured products have to account for model and parameter risks that cannot be dynamically hedged, in particular in the presence of transaction costs. This increases the price of the option component and thus leads to lower values for k . It should also be noted that implied volatilities are typically higher than estimated historical volatilities, which again increases the value of the option, and therefore decreases the fraction k of the upside potential, compared to a naïve estimate based on the Black-Scholes formula calibrated to historical returns. Moreover, the provider of the structured products would typically charge a fee, which results in offering a lower k or offering the option on the ex-dividend value of the underlying.

For example, the guaranteed payoff might be $100 + 0.7 \max(S_T - S_0, 0)$, whereas the fair value for k is 0.75. This means that

$$\frac{100 - 60.65}{0.7} - \frac{100 - 60.65}{0.75} = 3.75$$

is either spent by the bank in various market frictions and risk premiums on unhedgeable risks or included in the bank's profits.

A different pricing scheme for structured products of this type is to provide a payoff dependent on the ex-dividend value of the underlying asset. In other words, investors are offered to forgo the dividends on the stocks that form the index for the benefit of the capital guarantee.

Getting back to the example above (in the absence of market frictions), and assuming for example a 3% dividend rate, we obtain that taking dividends into account leads to a decrease in the value of the call option on the

Structured Products as Packaged Dynamic Asset Allocation Strategies

Dynamic Replication

underlying index from 52.57 to 31.8 because of the presence of dividends. This means that as much as $\frac{38.61}{31.8} = 121.41\%$ of the upside potential (based on ex-dividend returns) can be guaranteed, in the absence of costs and fees. The provider of the structured product would perhaps offer an access to a mere 100% of the upside potential, hence leaving $38.61 - 31.8 = 6.81$ for covering trading costs and insurance against unhedgeable risks and fees.

Dynamic Replication

From the derivation of the Black-Scholes formula, it is well known that the non-linear payoffs described above may be achieved by appropriate dynamic trading strategies in complete markets. Because of the presence of all sorts of frictions, dynamic replication of the non-linear payoff is typically not available to the institutional investors, which justifies the market for structured products that provides investors with an easy and static access to such payoffs¹⁶.

Even though institutional investors typically buy the options, as opposed to replicating them, it remains that holding the option is equivalent to holding a time-varying position in the underlying asset and the risk-free asset. The option pricing logic can be inverted in order to examine the equivalence of non-linear payoffs at a point in time with a dynamic trading strategy. Since a delta hedging strategy that replicates a simple call option supposes continuous rebalancing over time in a risk-free asset and the single risky asset, such a trading strategy leads to an asset allocation that is time varying.

Delta hedging the option involves a position of

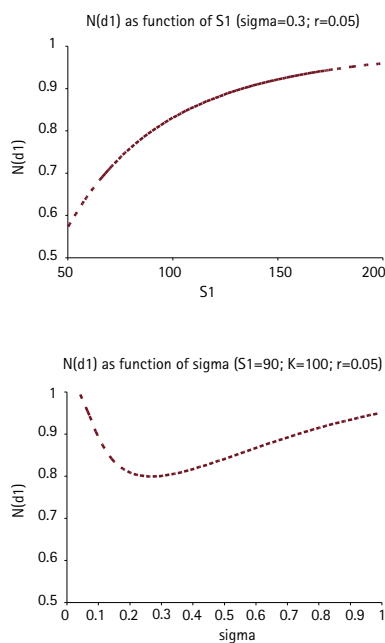
$$\frac{\partial C}{\partial S} = \Delta = N(d_1),$$

in the underlying asset and a position in the riskfree asset with notation from above.

Realized changes in the price of the underlying cause a change in its weight in the replicating portfolio. The top graph in Graph 4 shows that the allocation to the risky asset increases as its price increases. Therefore, a product including a static position in a call option effectively constitutes a trend following portfolio where price increases lead to buying more of the underlying asset. Price decreases, on the other hand, lead to a decrease of its weight in the replicating portfolio. The top graph in Graph 4 also shows that the dependence of allocation to the underlying asset upon the price of the latter is higher at low price levels.

The strength of the relationship also depends on the magnitude of the constant parameters (interest rates and volatility of the underlying). In practice, the time-varying nature of volatility is commonly taken into account when pricing options with the Black-Scholes formula though this is inconsistent theoretically. The bottom graph in Graph 4 shows the impact of expected changes in the volatility over the option's lifetime on the proportion invested in the underlying asset. It can be seen that if expected volatility rises or falls from intermediate levels of volatility (20% to 30%), changes in expectations will lead to a higher allocation to the underlying.

Graph 4
Hedge ratio (Delta) as a function of price of the underlying and volatility – The case of a plain vanilla call option



¹⁶ - An alternative to dynamic replication is static replication with standard plain vanilla options, as outlined in Carr and Madan (2001) for example. Replicating exotic options with a set of plain vanilla derivatives is in general only feasible under the very restrictive assumption of a constant volatility.

Diversification Effects of Structured Products in Institutional Asset Allocation

With the objective of being consistent with an institutional investor's constraint to follow static allocation strategies, we consider a static optimization program. More specifically, the stochastic optimization model that we intend to formulate will be a single-stage model based on two conflicting risk/reward objectives as in Markowitz's (1952) portfolio optimization framework.

Assumptions on Preferences

Our choice as the reward objective will not be different: the expected return on the portfolio, $\frac{1}{T} \ln \frac{P_T}{P}$, where P_t is the portfolio value at date t and T is the horizon taken equal to 10 years.¹⁷ However, as the risk measure, we choose Value-at-Risk (VaR) or Conditional Value-at-Risk (CVaR), as opposed to portfolio return variance. This is needed because our risk objective should account for the heavy tails present in asset returns, which legitimates the kind of focus on extreme risk management consistent with observed institutional investors' preferences. Rockafellar and Uryasev (2000) show that optimization of variance, VaR, and CVaR lead to identical solutions when the underlying asset returns are normally distributed. As the distribution becomes fatter in the tails, more focus should be paid to objectives that consider these unlikely events. Variance, as a risk objective, cannot provide this focus at all. CVaR will be our preferred risk objective, as it focuses on the left tail of the returns distribution beyond a threshold, as opposed to a certain quantile like VaR does.

Interestingly, CVaR would also be a preferred risk objective from an optimization perspective. VaR is difficult to optimize when it is calculated over scenarios because it leads to non-convex optimization problems; there are multiple extrema and the local optimization algorithms are unsuccessful whereas the global algorithms are inefficient. On the other hand CVaR can be optimized through stochastic *linear* programs (see Rockafellar and Uryasev (2000)).¹⁸

The aim of our exercise is to draw efficient frontiers with respect to the conflicting objectives and show how the optimal allocation decision and the risk/reward tradeoff are affected by the introduction of structured products. More specifically, we consider the following program:¹⁹

$$\text{Max}_w \frac{1}{T} \ln \frac{P_T}{P}$$

subject to $CVaR(T) \leq K$,

where $P_t = w_S S_t + w_B B_t + w_G G_t$ is the portfolio value at date t , and $w = (w_S, w_B, w_G)$, is the vector of portfolio weights (here the allocation in stocks S , bonds B and guaranteed structured product G).²⁰ In what follows, we focus on the case of the hindsight option (option on the maximum value of the underlying asset price – the stock index – observed at monthly time-intervals). Varying K allows us to span the whole efficient frontier in a mean return – CVaR space.

17 - In the optimization exercise, for practical reasons that are to be detailed later, we often replace this objective with the expected value of the portfolio at the horizon. For a single-stage model, this would be consistent with an expected return objective.

18 - At first, CVaR calculation seems to be complex, since it depends on the VaR itself, but through little optimization tricks you can formulate CVaR in an optimization problem without having to know the VaR value. And interestingly, optimization of CVaR will help you calculate VaR for that same level, as further explained in the Appendix (see Rockafellar and Uryasev (2000) for more details).

19 - For simplification, we drop the index that represents the uncertainty. Please see the Appendix for a complete formulation.

20 - In what follows, we use a 95% CVaR objective.

Diversification Effects of Structured Products in Institutional Asset Allocation

Assumptions on Asset Returns

Assumptions on Asset Returns

The issue of modeling the future uncertainty is critical to any stochastic optimization problem. A stochastic forecasting model consists of multivariate time series models. While the discrete time series models such as GARCH seem to be a perfect choice for stochastic programming, it is equally efficient to use stochastic differential equations (such as Geometric Brownian Motion) and to discretize them during the simulation of paths. The model selection is especially crucial since we want the asset returns to exhibit fat tails in the projected scenarios, which is also observed in the historical data. To this end, we avoid the Gaussian (normality) assumptions and suggest that the parameters of the stochastic differential equations (SDEs) be stochastic themselves. We also use a stochastic interest rate model, which will be used in the process of simulating a bond portfolio price path. Particular emphasis is laid on the fact that our model aims at capturing mean reversion of stock and bond returns over longer horizons. This is because the presence of mean reversion in returns as well as its impact on the asset allocation decision are well documented.²¹ Our data generating process is given by the following model with stochastic interest rate and stochastic volatility.²²

$$\begin{aligned}\frac{dS_t}{S_t} &= \mu_t dt + \sqrt{V_t} dW_t^S \\ dr_t &= a(b - r_t) dt + \sigma_r dW_t^B \\ dV_t &= \kappa(\theta - V_t) dt + \sigma_V \sqrt{V_t} dW_t^V\end{aligned}$$

where S_t is the value of the underlying asset (stock index) at date t , r_t is the (mean-reverting) value of the short-term rate at date t , V_t is the instantaneous variance of the log-returns for the stock, μ_t is the time-varying expected return on equity (to be specified below), W^S , W^B , and W^V are correlated Brownian motions.²³

It should be noted that in the stochastic volatility model the variance follows a square-root process, as in Heston (1993), and stochastic interest rates follow the Vasicek (1977) model. Our model is quite similar to the AFF1V model in Chernov et al (2003), which, with constant drift, corresponds to the Heston (1993) model.

The short-term rate process we use conveniently allows for bond prices to be obtained in closed-form. In particular, we have (see Vasicek (1977)):

$$\begin{aligned}B(t, T) &= \exp\left[-m(t, T) + \frac{1}{2}v(t, T)\right] \\ m(t, T) &= \beta(T - t) + (r_t - \beta) \frac{1 - e^{-a(T-t)}}{a} \\ v(t, T) &= \frac{\sigma_r^2}{2a^3} (1 - e^{-a(T-t)})^2 + \frac{\sigma_r^2}{a^2} \left((T-t) - \frac{1 - e^{-a(T-t)}}{a} \right)\end{aligned}$$

where β is the risk-adjusted long-term value of the short-term rate, given by

$$\beta = b - \frac{\sigma_r \lambda_B}{a}, \text{ with } \lambda_B \text{ being the risk-premium associated with interest rate risk.}$$

It should also be noted that the bond portfolio under consideration is a global bond index (e.g., the JP Morgan US Treasury bond index or the Euro MTS bond index), which can be modeled as a zero-coupon bond with constant time-to-maturity τ :

21 - Fama and French (1988) and Poterba and Summers (1988) were the first to provide empirical evidence of long-term mean reversion in stock returns. Barberis (2000), Wachter (2002), and Munk, Sorensen, and Nygaard (2004) derive solutions for optimal portfolio choice in the presence of mean reversion (driven by mean reversion in the dividend yield or the risk premium).

22 - In the presence of stochastic volatility, derivatives cannot be replicated by dynamic trading in the underlying asset and give access to new unspanned risk factors.

23 - For simplicity, we assume in what follows that W^B and W^V are independent Brownian motions.

Diversification Effects of Structured Products in Institutional Asset Allocation

Assumptions on Asset Returns

$$\begin{aligned}
 B_t^\tau &= \exp\left[-m_t^\tau + \frac{1}{2}v_t^\tau\right] \\
 m_t^\tau &= \beta\tau + (r_t - \beta)\frac{1 - e^{-a\tau}}{a} \quad (1) \\
 v_t^\tau &= \frac{\sigma_r^2}{2a^3}(1 - e^{-a\tau})^2 + \frac{\sigma_r^2}{a^2}\left(\tau - \frac{1 - e^{-a\tau}}{a}\right)
 \end{aligned}$$

We may now specify the dynamics for the drift of the stock index. We first write down the market price of risk vector:

$$\begin{pmatrix} \lambda_S \\ \lambda_B \end{pmatrix} = \begin{pmatrix} \sigma_t^S & \text{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau}, \frac{dS_t^\tau}{S_t^\tau}\right) \\ \text{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau}, \frac{dS_t^\tau}{S_t^\tau}\right) & \sigma_t^B \end{pmatrix}^{-1} \begin{pmatrix} \mu_t^S - r_t \\ \mu_t^B - r_t \end{pmatrix}$$

with $\sigma_t^S = \sqrt{V_t}$ the volatility of stock returns, and σ_t^B the volatility of bond returns (see below for more on the relationship between σ_t^B and σ_r).

Equivalently, we have:

$$\begin{pmatrix} \mu_t^S - r_t \\ \mu_t^B - r_t \end{pmatrix} = \begin{pmatrix} \sigma_t^S & \text{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau}, \frac{dS_t^\tau}{S_t^\tau}\right) \\ \text{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau}, \frac{dS_t^\tau}{S_t^\tau}\right) & \sigma_t^B \end{pmatrix} \begin{pmatrix} \lambda_S \\ \lambda_B \end{pmatrix}$$

from which we obtain:

$$\mu_t^S = r_t + \sigma_t^S \lambda_S + \text{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau}, \frac{dS_t^\tau}{S_t^\tau}\right) \lambda_B$$

What we are interested in is therefore the covariance between stock returns and bond returns. Using equation (1), the bond price can be equivalently written as the following exponential function of the short-term rate:

$$B_t^\tau = K_t^\tau e^{-L_t^\tau r_t}$$

$$\text{where } L_t^\tau = \frac{1 - e^{-a(r-t)}}{a}$$

Diversification Effects of Structured Products in Institutional Asset Allocation

Assumptions on Asset Returns

Using Ito's lemma, we find that:

$$\frac{dB_t^S}{B_t^S} = \left[\frac{\partial B}{\partial t} + \frac{\partial B}{\partial r} a(b - r_t) + \frac{1}{2} \sigma^2 \frac{\partial^2 B}{\partial r^2} \right] dt + \frac{\partial B}{\partial r} \sigma_r dW_t^B \equiv \mu_t^B dt + \sigma_t^B dW_t^B$$

$$\text{where } \sigma_t^B = \frac{\partial B}{\partial r} \sigma_r = -\frac{1 - e^{-a(r-t)}}{a} \sigma_r.$$

Finally, we have:

$$\mu_t^S - r_t = \sigma_t^S \lambda_S + \sigma_t^S \sigma_t^B \rho \lambda_B = \sigma_t^S \left(\lambda_S - \frac{1 - e^{-a\tau}}{a} \sigma_r \rho \lambda_B \right)$$

Therefore, assuming constant market prices of equity and bond price risk, we obtain that the expected excess return on the equity index is given as a mean-reverting process because it is a linear function of the volatility process, which itself is assumed to be mean-reverting. Our setup is somewhat similar to Wachter (2000) who also has mean reversion in excess stock returns. There are however a couple of differences between the two models. From the conceptual standpoint, Wachter assumes a constant volatility so that the mean-reverting stochastically time-varying excess drift over the risk-free rate can only be rationalized by assuming a stochastic market price for risk. We take an opposite route by assuming a constant price for risk (λ_S), and a stochastic quantity of risk (σ_t^S). On the technical front, so as to maintain the assumption of a complete market setting, Wachter assumes a perfect negative correlation between the drift term and stock return, while we consider a more general framework with a correlation not necessarily equal to -1 (we take that correlation equal to -.77 in our base case).

Diversification Effects of Structured Products in Institutional Asset Allocation

Discretization Scheme and Optimization Program

Discretization Scheme and Optimization Program

The next step is the time-discretization of these processes for simulating paths so that we can generate scenarios to represent the future uncertainty. We adopt a Milstein discretization scheme, as described in Glasserman (2004). As opposed to an Euler-scheme, it makes a difference in the discretization of the stochastic volatility process and helps us avoid negative variance values.

Once the paths are simulated, returns for each asset class are calculated on each path in order to generate the scenarios. Then, we compute the return scenarios for the guaranteed structured product. First, we determine the value of k that corresponds to these specific sets of scenarios. In other words, we determine the guarantee that can be offered given the expectations about the economy. Similar to the method described in section 2, we price the option on the paths we simulate, and find the matching value for k . Once this is done, we calculate the payoff of the guaranteed structured product (GSP) and its risk-neutral price at intermediate time points.

The scenarios for the total returns on each asset class are then fed into the optimization program (see the appendix). Our objective is to construct efficient frontiers that will correspond to the two conflicting objectives introduced earlier: expected portfolio return and the portfolio CVaR. Our strategy is to minimize a convex combination of the portfolio CVaR and the negative of the expected portfolio return. By varying the weight given to the CVaR (a parameter that would reflect the risk aversion level) in the objective function, we will run the optimization several times and trace the efficient frontier.

Empirical Results

We first conduct the base case experiment using given parameter values, and then perform a variety of robustness checks. We consider a typical allocation for an institutional investor's portfolio with stocks/bonds of 0.516 / 0.484. This was obtained as an average of allocations by European and U.S institutions.²⁴ We compare our solutions with this typical allocation at various stages of the analysis.²⁵

Base Case Results

The model we use for stock and bond returns involves quite a few parameters. One approach would consist of estimating these parameters by fitting the model to historical data. Alternatively, we have chosen to select parameter values consistent with the existing literature. Table 1 contains information about related papers and the parameter values they use.

using. The parameter values for the stochastic interest rate model are taken from Munk and Sorensen (2004) who use a Vasicek interest rate process that has been calibrated to fit historical bond return behavior. Finally, the market prices of risk are taken from Brennan and Xia (2002) whose definitions for these parameters are identical to ours. For comparison purposes, we also show in table 1 parameter values in two papers using similar models, Bakshi et al. (1997) and Duffie et al. (2000).

Using these parameter values, which are depicted on the right-most column of Table 1, we simulate 2500 paths. In the interest of representing the optimal behavior of a European institutional investor, we construct our bond index based upon the EuroMTS index. On the sample period ranging from January 1st 1998 to December 31 2003, the average sensitivity of the EuroMTS

Table 1

Parameter Values. For Bakshi et al (1997), the parameters are given on page 2018, under column All Options – SVSI. For Duffie et al (2000), the parameters are given on page 1363, under column SV. For Ait-Sahalia and Kimmel (2004), the parameters are given on page 38, under column Daily. For Munk and Sorensen (2004), the parameters are given on page 2000, in Equation 32. For Brennan and Xia (2002), the parameters are given on page 1221, under column Estimate. SV stands for stochastic volatility, CI stands for constant interest rates and SI stands for stochastic interest rates.

	Bakshi et al (1997)	Duffie et al (2000)	Ait-Sahalia and Kimmel (2004)	Munk and Sorensen (2004)	Brennan and Xia (2002)	This Paper
Type of Model	SV and SI	SV and CI	SV and CI	CV and SI	Just SI	SV and SI
Variance						
Speed of mean-reversion	0.98	6.21	5.0			5.0
Long-run mean	0.0408	0.019	0.045	0.0625	0.025	0.045
Sigma	0.42	0.61	0.48			0.48
Interest rate						
Speed of mean-reversion	0.58			0.15	0.105	0.15
Long-run mean	0.0345	0.0319	0.04	0.04	0.017	0.04
Sigma	0.03			0.015	0.013	0.015
Correlations						
Stock-Variance	-0.76	-0.70	-0.77			-0.77
Stock-Interest rate				-0.25		-0.25
Variance-Interest rate						0.00
Others						
Lambda_S					0.343	0.343
Lambda_B					-0.207	-0.207

For the parameters concerned with the variance process, we adopt the values in Ait-Sahalia and Kimmel (2004) who calibrate a Heston-type stochastic volatility model similar to the one we are

index was equal to 5.102. Given the mean-reversion parameter, α , in Table 1, this roughly corresponds to a constant maturity of 10 years and we generate the bond index returns accordingly.

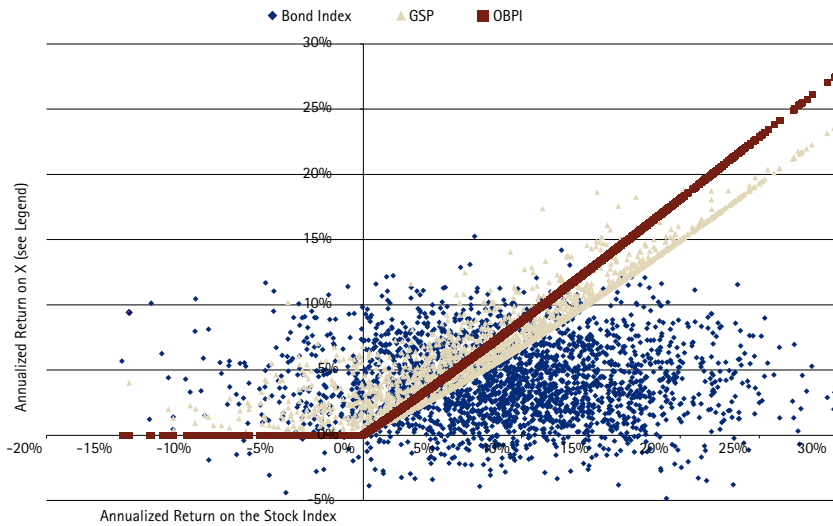
24 - Bank of International Settlements (September 2003).

25 - Different estimates can be found in the literature. According to Davis (2003), an average UK (US) pension fund holds 71% (67%) equity and 29% (33%) fixed income securities. According to a recent survey of UK pension funds by Mercer Investment Consulting, the equity/bond mix is 65%/35%.

Empirical Results

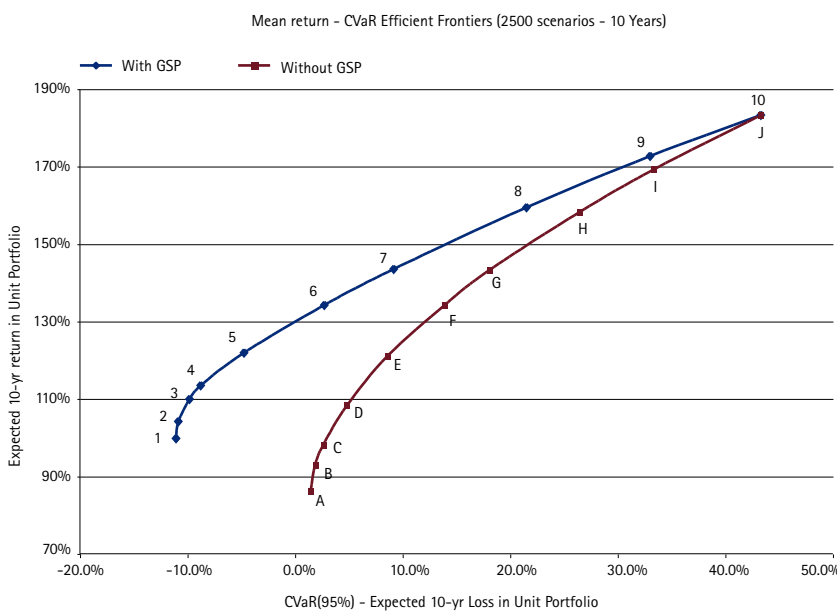
Base Case Results

Graph 5
Annualized returns of several assets versus the annualized return on the stock index



In this section (except for sub-section entitled Introducing Differences in Beliefs), before starting the optimal allocation exercise, we first calibrate the size of the payoff k to be consistent with the chosen parameter values. As a result, the structured product is always fairly priced. This value turns out to be 0.5790.²⁶

Graph 6
Efficient frontiers in an expected return - CVaR space. Note that negative CVaR values express negative losses, i.e., positive returns, corresponding to situations such that expected return beyond the VaR is positive



²⁶ - Since the guaranteed structured product can be described as a discrete hindsight option, it can be shown that this value is higher than the continuous hindsight and lower than the European call, respectively.

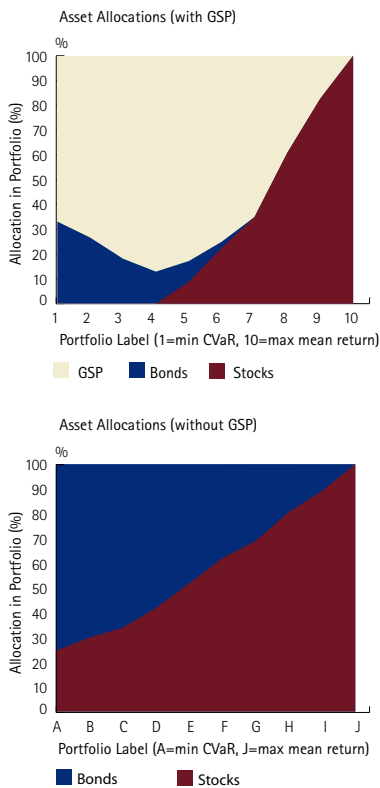
To give the reader an idea of what the return scenarios for the asset classes look like under our model and parametric assumptions, we plot for each scenario the return for different asset classes against the stock index returns in Graph 5. The horizontal axis corresponds to the annualized return for the stock index while the vertical axis corresponds to annualized returns for other asset classes. For comparison purposes, we also compute the payoff of the call-option based structured product that was introduced earlier in the second section (OBPI in Graph 5). As expected, OBPI exhibits the kind of piecewise linear convex payoff classic to call options. The GSP also exhibits a convex payoff indicating that capital guarantee is provided. On the other hand, it dominates OBPI under scenarios with severe drops in the stock index whereas OBPI can only provide a no-loss protection. Under optimistic scenarios however, GSP is not doing as well as OBPI. This is because of the higher cost, and hence lower upside potential, associated with the hindsight option, as opposed to the standard call option.

Next, we present the efficient frontiers that correspond to these scenarios in Graph 6. The curve labeled "Without GSP" is traced by restricting the allocation to GSP to zero. As expected, including GSP in the opportunity set improves the portfolio for almost any type of investor (i.e., lower values for the 5% CVaR value for the same level of return). The improvement is more remarkable for risk-averse investors. For instance, point E corresponds to the typical 51.6%/48.4% stock/bond allocation mentioned earlier. For the same risk-aversion level, the investor is able to reduce the risk while sacrificing some return (point 4 on the frontier labeled "With GSP").

Empirical Results

Extensions and Robustness Analysis

Graph 7



In Graph 7, one may observe the change in asset allocation with respect to the change in risk aversion. These allocations correspond to portfolios labeled on the two efficient frontiers above. The GSP helps the risk-averse investors increase their returns by replacing the stock allocation in their portfolio and the risk-seeking investors to decrease the shortfall risk they are exposed to by replacing the bonds in their portfolio.

In the following sections, we will perform various robustness analyses for the portfolio labeled 4 above. As mentioned earlier, this portfolio represents the improvement an average institutional investor can get at the same risk level over the typical allocation, which itself is represented by portfolio E.

Extensions and Robustness Analysis

Introducing Market Frictions

In the experiments that we have conducted so far, we have assumed away transaction costs and have chosen parameter values so as to make the product fairly priced (i.e., no profit for the bank) for a given size of the payoff (k). This has allowed us to focus on the optimal demand for structured products purely motivated by access to a non-linear payoff. As stated in the second section, typically, providers of structured products do not rely

on the Black-Scholes value of the option component and do charge a fee.

Because the introduction of market frictions leads to a decrease in the proposed level of exposure to the upside potential of the underlying index, we run the following stylized experiment.²⁷ To test for the impact of market frictions, we reduce k by a fixed percentage, starting from the base case value $k=0.579$ all the way down to $k=0.4053$ (30% decrease with respect to base case). Accordingly, we update the payoff for the guaranteed structured product. Finally, we optimize the weighted objective at the same risk aversion level as portfolio 4 for various fee levels. Table 2 summarizes this analysis. As a result of the market frictions, the allocation to the structured product decreases from 87.08% (base case) to 57.62% (after a 30% decrease in k).²⁸

Introducing Weight Constraints

Because it is not reasonable to expect institutional investors to allocate a dominant fraction of their portfolio to structured products, we test the impact of imposing an upper bound on the allocation to the GSP.

We present in Table 3 the optimal allocations and the values for the objectives under several weight constraints. A 0% upper bound implies that GSP is not included in the opportunity set (point E).

Table 2

Changes in the optimal asset allocation in the presence of frictions.

k	Allocation		
	Stocks	Bonds	GSP
0.579 (base case)	0.00%	12.92%	87.08%
0.55005 (5% decrease)	0.00%	14.11%	85.89%
0.5211 (10% decrease)	0.00%	15.07%	84.93%
0.4632 (20% decrease)	6.37%	20.20%	73.43%
0.4053 (30% decrease)	16.28%	26.10%	57.62%

Table 3

Changes in the values of the objectives and the optimal asset allocation in the presence of weight constraints on GSP allocation. Numbers are not annualized.

Upper Bound on GSP Allocation	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return
	Stocks	Bonds	GSP		
0%	51.69%	48.31%	0.00%	1.2110	8.6%
5%	47.15%	47.85%	5.00%	1.1864	6.27%
10%	42.65%	47.35%	10.00%	1.1623	4.05%
20%	36.54%	43.46%	20.00%	1.1512	1.13%

27 - Of course, in practice frictions are not an add-on cost that can be measured as a percentage of k as modeled here for simplicity, and investors can only observe k as quoted by the bank. The presence of parameter risk (addressed in the section entitled Introducing Differences in Beliefs), and model risk (not addressed in this paper) can actually explain that the actual participation in upside potential is higher, as opposed to lower, than what is consistent with the fair value perceived by the investor.

28 - While optimal allocation to the structured product remains relatively high, it is perhaps worth noting that the GSP being itself a packaged dynamic asset allocation, investments in the GSP eventually boil down to investments in the underlying index and the risk-free asset.

Empirical Results

Extensions and Robustness Analysis

Obviously, the weight constraint is binding because the optimal allocation in the base case was very high. Overall, these results strongly suggest that adding even a limited fraction of the overall allocation to structured products allows for significant benefits. For example, even a 10% allocation to structured products generates a significant reduction in portfolio CVaR over the ten-year-period, which decreases from 8.6% to 4.05% (neither value is annualized). On the other hand, as expected, the expected returns also decrease because of a lower allocation to stocks when the fraction invested in the structured product is increased.

Introducing Differences in Beliefs

In this section, we perform comparative analysis on parameter values to see how differences in opinions about these parameters impact investors' appetite for structured products. In the presence of stochastically time-varying parameters, it can be argued that an investment in structured products offers an added advantage to investors since it allows them to gain an active exposure to rewarded sources of risks potentially different from the one implicit in stock and bond returns. For example, if volatility is stochastically time-varying, the inclusion of a structured product in an institutional investor's asset allocation will induce a long position on volatility (position will make profit if volatility increases) that will provide useful diversification with respect to the short volatility position implicit in their stock holdings.

To address how active views on volatility (i.e. deviations from implied market volatility) can affect the optimal allocation to structured products, we follow a two-step process: First, we pick an optimal asset allocation when the structured product is fairly priced, and then we see how heterogeneous expectations on volatility between the investor and the structurer can induce a change in the allocation to these products.

For the first step, we choose our favorite portfolio, labeled 4 in Graph 5. For the second step, we re-generate all the scenarios using a new value for the parameter θ , the long-run mean for volatility. However, we skip the step where we calibrate the k value. It is as if the original scenarios were generated by the structurer (and therefore are used to determine k) and the new set were generated by the investor according to her expectations on volatility (and therefore is used to determine the optimal asset allocation). Table 4 summarizes our findings.

As expected, an investor endowed with a belief that the market's ex-ante perception of volatility is lower than it should be will allocate more to structured products than in the base case, because the structured products seem to be underpriced. For example, if an investor has an active view stating that the long-term mean value for volatility should be 25% (corresponding to $\theta = 0.0625$), as opposed to the base case value 21.21% (corresponding to $\theta = 0.045$), then the upside potential k is higher than what it should be based on the investor's prior, and the structured product becomes more attractive, which translates into an increase (from 87.08% to 89.91%) in the optimal weight.

Table 4
Changes in the optimal asset allocation in the presence of heterogeneous expectations on volatility.

θ	Allocation		
	Stocks	Bonds	GSP
0.03	0.00%	16.03%	83.97%
0.045 (base)	0.00%	12.92%	87.08%
0.0625	0.00%	10.09%	89.91%

Empirical Results

Extensions and Robustness Analysis

Introducing CVaR Constraints at Intermediate Dates

In practice, even if they are investing for the long-term, institutional investors face short-term constraints implied by frequent (quarterly or yearly) reporting requirements to outsiders, e.g., to trustees and plan sponsors. In an attempt to assess the impact of the presence of such short-term constraints, it would be interesting to see how the allocation would

In Table 5, we first adjust the long-term value for interest rates, either down from 4% (base case) to 3% or up to 5%. As a result of these changes, the optimal allocation to bonds is naturally affected: given the mean-reversion feature of interest rates, an increase (decrease) in the long-term value signals a higher probability of future increase (decrease) in interest rates and therefore future decrease (increase) in bond prices, with a decreased (increased) allocation to bonds as a result.

Table 5
Changes in the optimal asset allocation as a result of changes in long-term value of interest rate level.

b	No weight restriction on GSP					20% weight restriction on GSP					
	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return	b	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return
	Stocks	Bonds	GSP				Stocks	Bonds	GSP		
0.03	0.00%	14.73%	85.27%	0.9976	0.0813	0.03	34.28%	45.72%	20.00%	1.0416	0.0153
0.04 (base)	0.00%	12.92%	87.08%	1.1345	-0.0878	0.04 (base)	36.54%	43.46%	20.00%	1.1512	-0.0113
0.05	0.00%	11.80%	88.20%	1.2803	-0.0956	0.05	40.39%	39.61%	20.00%	1.2915	-0.0135

be affected by imposing a CVaR constraint not only at horizon, but also at intermediate dates.

The technical problem there is that it requires one to estimate the secondary market value of the structured products at each intermediate date, which involves re-generating thousands of scenarios at each point of each path process.²⁹ The computational complexity involved is rather formidable, and we leave this interesting question for further research.

Comparative Static Analysis

In what follows, we test for the impact of changes in the most important parameters, including speed of mean reversion, long-term volatility estimate, long-term risk-free rate, stock and bond risk premia, etc.

In the non-restricted case, the structured product absorbs all variations in optimal allocation to bonds and the optimal allocation to non-linear convex payoffs further increases in the case of expectations of increases in interest rates that make the bond index less attractive in a risk-return sense. On the other hand, in the presence of a (binding) 20% upper bound on the allocation to the structured product, the equity index allocation is evidently affected by changes in long-term interest rate levels.

29 - In principle it should not be just the investor doing it but it requires the accountants and regulators accepting those values.

Empirical Results

Extensions and Robustness Analysis

Table 6

Changes in the optimal asset allocation as a result of changes in long-term value of equity volatility level.

In Table 6, we conduct a similar experiment on the long-term value for stock index volatility (or actually variance), and adjust it either down from 4.5% (base case corresponding to 21.21% volatility) to 3% (i.e. 17.3205% volatility) or up to 6.25% (i.e. 25% volatility).

No weight restriction on GSP						20% weight restriction on GSP					
θ	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return	θ	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return
	Stocks	Bonds	GSP				Stocks	Bonds	GSP		
0.03	0.00%	14.37%	85.63%	1.0279	-0.1053	0.03	36.10%	43.90%	20.00%	1.0024	-0.0246
0.045 (base)	0.00%	12.92%	87.08%	1.1345	-0.0878	0.045 (base)	36.54%	43.46%	20.00%	1.1512	-0.0113
0.0625	0.00%	12.31%	87.69%	1.2317	-0.0736	0.0625	38.80%	41.20%	20.00%	1.3434	-0.0583

Table 7

Changes in the optimal asset allocation as a result of changes in speed of mean-reversion in equity volatility.

In Table 7, we conduct a similar experiment on the speed of mean-reversion to long-term stock index volatility (or actually variance), and adjust it either down from 4 (base case) to 3 or up to 5.

No weight restriction on GSP						20% weight restriction on GSP					
b (Kappa)	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return	b (Kappa)	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return
	Stocks	Bonds	GSP				Stocks	Bonds	GSP		
4.0	0.00%	13.02%	86.98%	1.1180	-0.0880	4.0	34.52%	45.48%	20.00%	1.1077	0.0075
5.0 (base)	0.00%	12.92%	87.08%	1.1345	-0.0878	5.0 (base)	36.54%	43.46%	20.00%	1.1512	0.0113
6.0	0.00%	12.82%	87.18%	1.1465	-0.0869	6.0	38.63%	41.37%	20.00%	1.1916	0.0181

Table 8

Changes in the optimal asset allocation as a result of changes in equity risk premium level.

In Table 8, we conduct a similar experiment on the long-term equity risk premium, and adjust it either down from 0.343 (base case) to 0.3 or up to 0.4.

No weight restriction on GSP						20% weight restriction on GSP					
λ_s	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return	λ_s	Allocation			Expected 10-Yr Return	CVaR (95%) of 10-Yr Return
	Stocks	Bonds	GSP				Stocks	Bonds	GSP		
0.300	0.00%	15.91%	84.09%	1.0094	-0.0791	0.300	31.32%	48.68%	20.00%	0.9885	0.0167
0.343 (base)	0.00%	12.92%	87.08%	1.1345	-0.0878	0.343 (base)	36.54%	43.46%	20.00%	1.1512	0.0113
0.400	0.00%	11.29%	88.71%	1.3452	-0.0946	0.400	51.35%	28.65%	20.00%	1.5479	0.0405

Overall, the impact of such changes does not appear extremely significant, which suggests a comforting level of robustness in the exercise conducted in the base case with respect to the choice of parameter values.

Conclusions and Suggestions for Further Research

That institutional investors have been so dramatically affected by recent market downturns is perhaps surprising given that an increasingly thorough range of structured products have been developed over the past few years, which allow investors to tailor the risk-return profile of their portfolios in a more efficient way than simple linear exposure to the return on traditional asset classes. The focus of the present paper is to determine what fraction a risk-averse institutional investor should optimally allocate to structured products in a general economy with stochastically time-varying interest rates and equity risk premium. We also study the impact of realistic levels of market frictions and heterogeneous expectations on volatility estimates. Our conclusion is that most institutional investors should optimally allocate a significant fraction of their portfolio to structured products, especially if they have a focus on extreme risk management.

While these results have been performed on the basis of a specific example (guaranteed structured product involving a hindsight option), they can be generalized to a wider class of non-linear payoffs. In particular, products with convex payoffs are ideal for institutional investors with a focus on the management of extreme risks. In light of the academic research referred to in the introduction (see in particular Leland (1980) as well as subsequent theoretical studies on the subject), it actually appears that a linear payoff being optimal is the exception rather than the rule (which of course is obvious from the fact that the set of non linear payoffs trivially encompasses the linear ones), not only for pension

funds and other liability-driven forms of portfolio management, but also potentially for mutual funds.

More generally, it can be argued that structured investment strategies, along with alternative investment strategies, are the modern counterpart of traditional long-only passive and active mutual fund strategies, respectively. While long-only strategies (both passive and active) can only generate a simple linear exposure to the return on underlying asset classes (go up and down with the indices), the main benefit of structured products and hedge fund strategies is that they allow for a non linear exposure with respect to stock and bond returns, in a passive (structured products) and active (hedge funds) way, respectively. The important issue, of course, from a pragmatic standpoint, is that institutional investors would need to see an improvement in transparency and liquidity of these products before they start investing massively in them. The same challenge applies to the hedge fund industry. We actually have reason to believe that such a trend towards greater transparency, liquid and cost control is already taking place, both in the alternative investment industry and the structured management industry, and massive flows of cash are shifting from traditional investment to these alternatives as a result of these changes (as well as a result of recent poor performance on traditional long-only passive and active strategies).

Our work can be extended in a number of ways.

Conclusions and Suggestions for Further Research

First, from an empirical standpoint, it would be interesting to compare in a unified environment the optimal allocation to various forms of structured products, including standard CPPI and OBPI structures, so as to determine under which economic conditions the value added by exotic options is most significant. Another possible extension would be to investigate the introduction to an objective different from the minimization of CVaR, e.g., by assuming some standard utility over terminal wealth displaying constant absolute or relative risk aversion. More importantly perhaps, a natural next step would consist of casting the optimal allocation problem in an asset-liability framework, as one expects the introduction to liability constraint to further enhance the attractiveness of nonlinear payoffs.

From a theoretical standpoint, a promising avenue for further research would involve extending the early work on optimal demand for portfolio insurance to path-dependent payoffs. In particular, one may attempt to determine the general shape of the function of the underlying asset's price path that would generate the highest level of expected utility for a given investor, a question that has been studied by Brennan and Solanki (1981) in a specific case where the payoff is a function of the underlying asset's terminal value only. Also, one may want to investigate the features of (static) investors' preferences that would support a rational non-zero holding of a set of exotic (i.e. path-dependent) derivatives, as opposed to plain-vanilla options (with a payoff that is a function of the underlying asset's terminal price only). Such an extension of the work initiated by Leland (1980) and Benninga

and Blume (2000) would allow us to rationalize the current strand in the industry of structured asset management, which is evolving beyond standard OBPI strategies towards a broad range of payoff-types.³⁰

30 - This extension can be cast within the traditional framework of expected utility maximization, or may involve more complex objectives. Intuitively, one might expect investors endowed with loss-aversion preferences to benefit significantly from the introduction of path-dependent derivatives such as the one studied in this paper.

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Appendix

For the mean-CVaR optimization model, we define the following parameters and variables.

Parameters:

T	Number of time periods
N	Number of scenarios
α	CVaR confidence level (0.95 or 0.99)
P_0	Initial value of the portfolio
β	Weight (between 0 and 1) given to CVaR in the objective function
$R_{t,i}^S$	Total return on the stock index in period t under scenario i
$R_{t,i}^B$	Total return on the bond index in period t under scenario i
$R_{t,i}^G$	Total return on the guaranteed structured product in period t under scenario i

Variables:

W_S	Portfolio weight of stocks at the beginning
W_B	Portfolio weight of bonds at the beginning
W_G	Portfolio weight of guaranteed structured product at the beginning
$P_{T,i}$	Value of the portfolio at the end of period T under scenario i
$r_{T,i}^P$	Rate of return for T periods on the portfolio under scenario i
ζ	Dummy variable that approximates VaR in the optimal solution
z_i	Dummy variable associated with CVaR constraint under scenario i

Given these definitions, the optimization model can be formulated as follows:

$$\text{Minimize } \beta \left(\zeta + \frac{\mathbf{1}}{N(\mathbf{1}-\alpha)} \sum_{i=1}^N z_i \right) - (\mathbf{1}-\beta) \left(\frac{\mathbf{1}}{N} \sum_{i=1}^N r_{T,i}^P \right) \quad (\text{A.1})$$

subject to

$$P_{T,i} = P_0 \left(w_S \prod_{t=1}^T R_{t,i}^S + w_B \prod_{t=1}^T R_{t,i}^B + w_G \prod_{t=1}^T R_{t,i}^G \right), \quad i = 1, \dots, N \quad (\text{A.2})$$

$$r_{T,i}^P = \frac{P_{T,i}}{P_0} - \mathbf{1}, \quad i = 1, \dots, N \quad (\text{A.3})$$

$$w_S + w_B + w_G = \mathbf{1} \quad (\text{A.4})$$

$$-r_{T,i}^P - \zeta \leq z_i, \quad i = 1, \dots, N \quad (\text{A.5})$$

$$z_i \geq \mathbf{0}, \quad i = 1, \dots, N \quad (\text{A.6})$$

$$\zeta \text{ is free,} \quad (\text{A.7})$$

$$w_S, w_B, w_G \geq \mathbf{1} \quad (\text{A.8})$$

Appendix

The objective function (A.1) is a convex combination of the two objectives. The expected value of the 10-yr rates of return is weighted by $(1-\beta)$, and the CVaR of these rates is weighted by β . Here, β can be seen as a degree of risk aversion. When $\beta = 1$, the investor is interested in minimizing risk with no interest in the return objective. When $\beta = 0$, the investor's only objective is maximizing the rewards. The minus sign before the return objective is because we minimize this weighted average. As this is a single-stage problem, we prefer the expected 10-yr rate of return to annualized rate of return as it is a linear function of the variables.

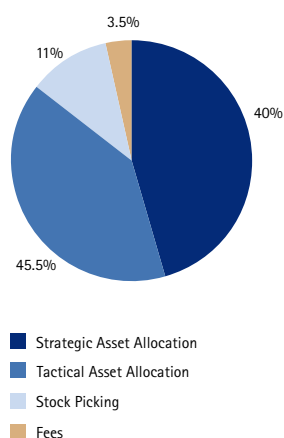
Constraints (A.2) and (A.3) represent how we define the ending portfolio value and the 10-yr portfolio rate of return, respectively, for each scenario.

We make sure that the portfolio weights add up to 1 through constraint (A.4). Short sales are disallowed by utilizing constraint (A.8).

Finally, constraints (A.5), (A.6) and (A.7) are needed to control the CVaR objective (the term in the objective that is multiplied by β). As shown by Rockafellar and Uryasev (2000), these constraints guarantee that the optimal value of this term in the objective gives CVaR and the corresponding optimal value of ζ (if it is unique) will be equal to VaR. If there are many optimal values of ζ , then VaR is the left end-point of the optimal interval.

Edhec Risk and Asset Management Research Centre

Edhec is one of the top five business schools in France owing to the high quality of its academic staff (over 100 permanent lecturers from France and abroad) and its privileged relationship with professionals that the school has been developing since it was established in 1906. Edhec Business School has decided to draw on its extensive knowledge of the professional environment and has therefore concentrated its research on themes that satisfy the needs of professionals. Edhec pursues an active research policy in the field of finance. Its "Risk and Asset Management Research Centre" carries out numerous research programs in the areas of asset allocation and risk management in both the traditional and alternative investment universes.



The choice of asset allocation

The Edhec Risk and Asset Management Research Centre structures all of its research work around asset allocation. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of active management based solely on stock picking as a source of performance. On the other, the appearance of new asset classes (hedge funds, private equity), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity in both conceptual and operational terms. This strategic choice is applied to all of the centre's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; measuring the performance of funds while taking the tactical allocation dimension of the alphas into account; taking extreme risks into account in the allocation; or studying the usefulness of derivatives in constructing the portfolio.

An applied research approach

In a desire to ensure that the research it carries out is truly applicable in practice, Edhec has implemented a dual validation system for the work of the Risk and Asset Management Research Centre. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the centre's orientation committee. This committee is composed of both internationally recognised researchers and the centre's business partners. The management of the research programmes respects a rigorous validation process, which guarantees both the

scientific quality and the operational usefulness of the programmes. To date, the centre has implemented six research programmes:

Multi-style/multi-class allocation

This research programme has received the support of FIMAT, Misys Asset Management Systems and SG Asset Management. The research carried out focuses on the benefits, risks and integration methods of the alternative class in asset allocation. From that perspective, Edhec is making a significant contribution to the research conducted in the area of multi-style/multi-class portfolio construction.

Performance and style analysis

The scientific goal of the research is to adapt the portfolio performance and style analysis models and methods to tactical allocation. The results of the research carried out by Edhec thereby allow portfolio alphas to be measured not only for stock picking but also for style timing. This programme is part of a business partnership with EuroPerformance (part of the Fininfo group).

Indexes and benchmarking

Edhec carries out analyses of the quality of indices and the criteria for choosing indices for institutional investors. Edhec also proposes an original proprietary style index construction methodology for both the traditional and alternative universes. These indices are intended to be a response to the critiques relating to the lack of representativity of the style indices that are available on the market. Edhec was the first to launch composite hedge fund strategy indices as early as 2003. The indices and benchmarking research programme is supported by AF2I, Euronext, BGI, BNP Paribas Asset Management and UBS Global Asset Management.

Edhec Risk and Asset Management Research Centre

Asset allocation and extreme risks

This research programme relates to a significant concern for institutional investors and their managers – that of minimising extreme risks. It notably involves adapting the current tools for measuring extreme risks (VaR) and constructing portfolios (stochastic check) to the issue of the long-term allocation of pension funds. This programme has been designed in co-operation with Inria's Omega laboratory. This research programme also intends to cover other potential sources of extreme risks such as liquidity and operations. The objective is to allow for better measurement and modelling of such risks in order to take them into consideration as part of the portfolio allocation process.

Asset allocation and derivative instruments

This research programme focuses on the usefulness of employing derivative instruments in the area of portfolio construction, whether it involves implementing active portfolio allocation or replicating indices. "Passive" replication of "active" hedge fund indices through portfolios of derivative instruments is a key area in the research carried out by Edhec. This programme is supported by Eurex and Lyxor.

ALM and asset management

This programme concentrates on the application of recent research in the area of asset-liability management for pension plans and insurance companies. The research centre is working on the idea that improving asset management techniques and particularly strategic allocation techniques has a positive impact on the performance of Asset-Liability Management pro-

grammes. The programme includes research on the benefits of alternative investments, such as hedge funds, in long-term portfolio management. Particular attention is given to the institutional context of ALM and notably the integration of the impact of the IFRS standards and the Solvency II directive project.

Research for business

In order to facilitate the dialogue between the academic and business worlds, the centre has recently undertaken four major initiatives:

- Opening of a web site that is entirely devoted to the activity of international research into asset management. www.edhec-risk.com is aimed at a public of professionals who wish to benefit from Edhec's analyses and expertise in the field of applied portfolio management research such as detailed summaries, from a business perspective, of the latest academic research on risk and asset allocation as well as the latest industry news assessed in the light of the results of the Edhec research programme. www.edhec-risk.com is also the official site for the Edhec Indices.
- Launch of **Edhec-Risk Advisory**, the consulting arm of the research centre focusing on risk management issues within the buy-side industry, and offering a wide range of services aimed at supporting fund managers and their service providers in the fields of operational risk, best execution, structured products, alternative investment due diligence and risk management system implementation.
- Launch of **Edhec Investment Research**, in order to support institutional investors and asset managers in implementing the results of the Edhec Risk and Asset Management

Edhec Risk and Asset Management Research Centre

Research Centre's research. Edhec Investment Research proposes asset allocation services in the context of a "core-satellite" approach encompassing alternative investments.

- Launch of **Edhec Alternative Investment Education**, which is the exclusive official CAIA association course provider for Europe.

The Team

The aim of the Edhec Risk and Asset Management Research Centre is to become the leading European centre of research into asset management in the coming years. To that end, Edhec has invested significantly to give the centre an international research team made up of both professors and permanent researchers, with whom professionals are affiliated in the capacity of research associates. To date, the Edhec Risk and Asset Management Research Centre has more than 28 members: 15 permanent members and 13 associates who are operating in firms that are reputed for their proficiency in asset management. This team is managed by Professor Noël Amenc, who has considerable experience in asset management as both an academic and a professional.

SG Corporate and Investment Banking

SG CIB, the corporate and investment banking arm of the Société Générale Group, serves corporate clients and investors in 45 countries. It is recognized for innovation and vast capabilities. Present on the European, US and Asian markets, SG CIB offers corporates, financial institutions and investors a tailored approach in meeting their financial needs.

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Your equity derivatives contacts

France

Tel: +33 (0)1 42 13 65 40

UK, Ireland & Nordic

Tel: +44 (0)20 7762 5026

Germany

Tel: +49 (0)69 71 74 650

Austria

Tel: +33 (0)1 42 13 50 52

Italy

Tel: +39 02 854 9396

Benelux

Tel: +33 (0)1 42 13 53 53

Spain

Tel: +34 91 589 80 50

Switzerland

Tel: +41 (0)58 272 33 00

Portugal – Greece – Middle East Mediterranean Rim

Tel: +33 (0)1 42 13 38 85

Central & Eastern Europe

Tel: +33 (0)1 42 13 38 85

North America

Tel: + 212 278 5909

South Africa

Tel: +44 (0)20 7762 5026

Tokyo

Tel: +81 3 55 49 52 52

Asia except Japan

Tel: +852 21 66 43 45

BEST EQUITY DERIVATIVES HOUSE 2004





**Edhec Risk and Asset Management
Research Centre**

393-400 promenade des Anglais

BP 3116

F-06202 Nice Cedex 3

Tel.: +33 (0)4 93 18 78 24

Fax: +33 (0)4 93 83 08 10