

The Brave New World of Hedge Fund Indices

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Abstract

The fact that hedge funds have started to gain widespread acceptance while remaining a somewhat mysterious asset class enhances the need for better measurement and benchmarking of their performance. One serious problem is that existing hedge fund indices provide a somewhat confusing picture of the investment universe. In this paper, we first present detailed evidence of strong heterogeneity in the information conveyed by competing indices. We also attempt to provide remedies to the problem and suggest various methodologies designed to help build a “pure style index”, or “index of the indices” for a given style. We first explore a statistical approach to the construction of pure style indices, using Kalman filter techniques for the estimation of an unobservable factor from competing index return observations. Because it is desirable that a pure index can be regarded as a portfolio of existing indices, and hence a portfolio of underlying funds, we also suggest a portfolio approach to the problem. In particular, we suggest to use principal component analysis to extract the “best possible one-dimensional summary” of a set of competing indices, and to use minimum variance analysis to extract the “least biased portfolio” from a set of competing indices. Finally, we also provide evidence of the ability of pure style indices to improve benchmarking of hedge fund returns. Our results can easily be extended to traditional investment styles such as growth/value, small cap/large cap.

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In this paper, we attempt to emphasize the need for a better understanding of investment style indices by focusing on the alternative investment universe where the problems are most visible. Beyond traditional styles such as *growth/value*, *small-cap/large-cap*, alternative investment styles such as equity *market neutral*, *event driven*, *long/short*, etc., have recently become popular. Alternative investment strategies actually come in many different shapes and forms (see Appendix A for a classification of hedge fund strategies), and can be regarded as a natural extension of the logic of traditional style investing. Interest in hedge fund investing, in particular, is undoubtedly gathering pace, and the value of the hedge fund industry is now estimated at more than 700 billion US dollars, with more than 7,000 funds worldwide while new hedge funds are being launched every day to meet the surging demand.¹

The fact that hedge funds have started to gain widespread acceptance while remaining a somewhat mysterious asset class enhances the need for better measurement and benchmarking of their performance. One serious problem is that existing hedge fund style indices provide a somewhat confusing picture of the investment universe. There are at least a dozen competing hedge fund index providers (see table 1) which differ in terms of selection criteria (examples of selection criteria include length of track record, assets under management and restrictions on new investment), style classification (manager's self-proclaimed styles versus objective statistical-based classification), weighting scheme (equally-weighted versus value-weighted) and rebalancing scheme (e.g., monthly versus annually). As a result of such differences in the construction methods, competing index providers offer a very contrasted picture of hedge fund returns, and differences in monthly returns can be greater than 20%! For example, Zürich reports a 20.48% return on *long/short* strategies in February 2000, while EACM reports a -1.56% return in the same month! As further evidence of the contrasted information conveyed by hedge fund indices, we also find that the correlation between two competing indices for the same strategy can be as low as .16 (correlation between Hennessee and MAR *equity market neutral* indices). It can even reach a negative -0.1901, as in the case of EACM and Zürich *long/short* indices, thus reflecting a sharp difference in net exposure of these two competing indices. (See table 5 in section 2 for more details).

These difficulties raise the question of what lies behind the concept of a "good" index. From a normative standpoint, there are two major shortcomings from which all existing indices potentially suffer (see figure 1 for an illustration). One problem is that existing indices do not fully represent the whole universe; roughly speaking, this translates into the fact that an index for a given strategy never encompasses all existing hedge funds following the strategy. The other is the existence of potential biases; roughly speaking, this translates into the fact that an index for a given strategy may encompass hedge funds deviating significantly from the given

¹See Hennessee group research report (2002).

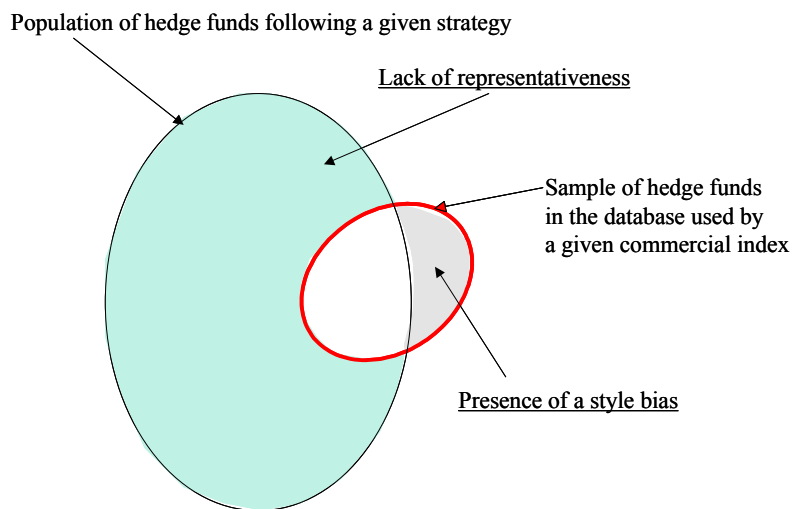


Figure 1: Problems with Style Indexes.

strategy.

Such shortcomings obviously also apply to traditional equity style indices and also, albeit to a lesser extent, to bond indices (see Reilly, Kao and Wright (1992)). In this paper, we choose to focus on the alternative investment universe where the problem is dramatically amplified due to the complex nature of the strategies involved, and the absence of regulation concerning performance disclosure (see details in section 1). In other words, we attempt to emphasize the need for a better understanding of investment style benchmarks where the problems are most visible, but the bulk of our message is more general and certainly extends to traditional equity investment styles.

In this paper, we provide a systematic investigation of how heterogeneous existing hedge fund indices are, and we also attempt to provide remedies to the problem through the construction of so-called “pure style indices”. We define a “pure style index” as being the true unobserved fully representative and unbiased index that would provide a fair representation of the return on a portfolio encompassing for a given strategy all hedge funds following that strategy and no other. We first explore a statistical approach to the problem of the construction of pure style indices, using Kalman filter techniques which are well-suited for the estimation of an unobservable factor from competing index return observations. Because, it is desirable that a pure index can be regarded as a portfolio of existing indices, we also suggest a portfolio approach to the problem of the construction of pure style indices. In particular, we suggest using principal component analysis to extract the “best possible one-dimensional summary” of a set of competing indices (addresses the problem of lack of representativeness), and using minimum variance analysis to extract the “least biased portfolio” from a set of competing indices (addresses the problem of biases). We also provide evidence of the ability of the pure

style indices to improve current techniques for factor analysis and benchmarking of hedge fund returns.

It should be noted that there is another type of problems in hedge fund benchmarks, which is the presence of measurement biases that hedge fund indices inherit from hedge fund databases they are built from. There are three main sources of difference between the performance of hedge funds in the database and the performance of hedge funds in the population, namely a *survivorship* bias, *selection* bias and *instant history* bias (see Fung and Hsieh (2001a)).² It is not our purpose to focus on the existence of these biases, a problem that has already been well addressed in the literature. In particular, Fung and Hsieh (2001a) offer a very detailed analysis of such biases, and also propose an alternative approach to the benchmarking of hedge fund returns. They suggest using returns on funds of funds, which provide a cleaner estimate of the investment experience of hedge fund investors. Our paper complements that paper in the following way. Assuming that an investor is willing to follow Fung and Hsieh (2001a) insightful suggestion to use the return on funds of funds as proxies for hedge fund indices, the problem remains to know which fund of funds return should be used to proxy a given style, as different funds of funds post very different returns.³ Insights into that question can be obtained by applying our methodology to competing fund of funds, just as we apply it to competing indices in this paper. The difference in approach between the two papers can be perhaps best summarized in the following way. Fung and Hsieh (2001a) focus on correcting for biases that can not be diversified away in a portfolio of indices: survivorship, selection and instant history biases are strongly correlated across hedge fund indices; all indices suffer from these biases, and yet they exhibit significantly different performances. We, on the other hand, focus on diversifiable biases. Competing hedge fund indices provide heterogeneous information because an index for a given strategy never encompasses all existing hedge funds following the strategy, while it may encompass hedge funds deviating significantly from the given strategy. Intuitively, the bulk of our message is that it is possible to significantly mitigate such biases by mixing portfolio of indices. The resulting “pure index”, however, still suffer from undiversifiable biases (survivorship, selection and instant history biases), the impact of which can and should be measured and accounted for as described in the literature (see Agarwal and Naik (2000b), Liang (1999), Park, Brown and Goetzmann (1999) and Fung and Hsieh (2000a)).

Also related to ours are recent papers by Brittain (2001) and Schneeweis et al. (2001), documenting measurement and interpretation problems with existing hedge fund indices. In the same vein, Brooks and Kat (2001) report some evidence of significant heterogeneity between

²It is believed that these biases account for a total approaching at least 4.5% annually (see Agarwal and Naik (2000b), Liang (1999), Park, Brown and Goetzmann (1999) and Fung and Hsieh (2000a)).

³Specialized fund of funds are not available for all hedge fund strategies. This makes impossible a complete benchmarking of the entire universe by fund of funds.

indices that aim to reflect the same type of strategy. Our paper differs from these in that we provide a systematic investigation of how heterogeneous existing hedge fund indices are, while the aforementioned authors have different focuses and merely report illustrations and examples related to the problem. Also, we attempt to provide remedies to the problem through the construction of so-called “pure style indices”.

The rest of the paper is organized as follows. In section 1, we review the main providers of hedge fund indices and discuss the database. In section 2, we provide strong evidence that competing indices offer a very contrasted view of hedge fund performance for a given style. In section 3, we offer three possible remedies and discuss the derivation of “pure style indices” from a set of competing indices. Section 4 is devoted to testing the performance of these pure style indices. In section 5, we present our conclusions and suggestions for further research. Some information on hedge fund strategies as well as detailed information on competing hedge fund indices are delegated to the Appendix.

1 The World of Hedge Fund Indices

We have listed almost a dozen hedge fund index providers.⁴ We report some key information about them in table 1, while more details on each of these index providers can be found in Appendix B.⁵

These indices have been set up to provide the rigorous data and analytics that both managers and investors increasingly demand for measuring performance and risk in this rapidly growing asset class. As argued in the introduction, one serious problem is that existing hedge fund style indices provide a somewhat confusing view of the alternative investment universe, because the collection of such indices is neither collectively exhaustive, nor mutually exclusive.

More specifically, hedge fund indices are built from databases of individual fund returns, and therefore inherit their shortcomings in terms of scope and quality of data, which vary a lot among various data vendors. There are three main competing databases (TASS, MAR, HFR) that are used by providers of hedge fund indices. While all three data bases are marred to some extent by the presence of the usual biases (survivorship, selection and instant history biases), they are far from being homogeneous in terms of population. For instance, HFR excludes managed futures from its databases while TASS and MAR take them into account. The majority of funds are present in one but not the other: of the 1,162 HFR funds and the

⁴Standard & Poor’s and MSCI have recently announced plans to create hedge fund indices (see Appendix B for more details).

⁵Magnum publishes the performance of funds of funds (since January 1997), as opposed to non-investible indices. Because they are well-established in the industry, we have chosen to include them in our analysis. This is consistent with Fung and Hsieh (2001a) who suggest using returns on funds of funds as proxies for hedge fund indices.

Providers	# of Indices	Launch Date	# of Funds	Website
EACM	13	1996	100	eacmalternative.com
HFR	15	1994	1,100	hfr.com
CSFB/Tremont	9	1999	340	hedgeindex.com
Zürich Capital	5	2001	60	zcmgroup.com
Van Hedge	12	1995	750	vanhedge.com
Hennessee Group	22	1992	450	hedgefund.com
Hedgefund.net	33	1979	1,800	hedgefund.net
LJH Global Investments	16	1992	800	ljh.com
MAR	15	1990	1,300	marhedge.com
Altvest	13	2000	1,400	altvest.com
Magnum	8	1994	NA	magnum.com

Table 1: Competing Indices in Hedge Fund Universe. This table provides a listing of competing hedge fund index providers, with information on the number of strategies, launch date, number of selected funds and web site.

1,627 TASS funds, only 465 are common to both databases. 59% of the funds that are still in activity and 68% of the funds that no longer report to HFR are not part of the TASS database. Out of the 465 funds in common between the HFR and TASS databases, only 154 (or 33.1%) have been included in both databases at the same time (cf. Liang (2001)).

As a result of the incompleteness and heterogeneity of hedge fund data, existing hedge fund indices, which are built from that data, potentially suffer from the following two major shortcomings (see figure 1).

- *Existing indices are not fully representative.* In the one-factor CAPM world of the sixties and seventies, the notion of a good index was one that was representative of the value-weighted portfolio of all traded assets, and the real challenge was to provide investors with the closest approximation of the true market portfolio (see Roll (1977)). The market cap logic, however, does not easily extend to the alternative investment universe. First, hedge funds are not submitted to reporting requirements so that information on the asset under management is very hard to gather with some degree of accuracy. This is the reason why all existing hedge fund indices, to the notable exception of CSFB/Tremont, use an equally-weighted, as opposed to value-weighted, scheme. Beside, because of the lack of regulation on hedge fund performance disclosure, existing data bases only cover a relatively small fraction of the hedge fund population. Probably only a little more than half of existing hedge funds choose to self-report their performance to one of the major hedge fund databases - see Appendix B for more details. As simple evidence of the fact that existing indices are not fully representative of the universe, it perhaps suffices to

note that one of the most popular hedge fund indices, the EACM 100, does not account for more than a tiny percentage of all existing hedge funds (100 among more than 7,000 funds).

- *Existing indices are biased.* Most hedge fund indices (actually all indices, except the Zürich indices - see Appendix B) are based upon managers' self-proclaimed styles. Given that hedge fund managers jealously protect the secret of their investment strategies (the so-called black-box problem), relying on managers' self-proclaimed style is actually almost a necessity. The problem is that this procedure only makes sense under the following two conditions: (1) a manager follows a unique investment style;⁶ (2) a manager's self-proclaimed style matches the manager's actual trading strategies. Of course, none of these assumptions can be taken for granted. In particular, it is well documented (see for example Lhabitant (2001)) that some significant *style drift* occurs; as opportunities eventually disappear in their original strategies, it is common practice for some hedge fund managers to start looking at other markets (e.g., managers who start pursuing fixed income arbitrage strategies and end up investing in emerging markets as arbitrage opportunities tend to disappear in liquid US markets).

2 Hedge Fund Indices are not Created Equal

Hedge fund indices come in very different shapes and forms. The existence of a profound heterogeneity in the set of assets under consideration, as well as some heterogeneity in the index construction method, result in some dramatic heterogeneity in the returns, which we document now.

2.1 The Data

There are some serious challenges one has to face when attempting to provide a detailed picture of the universe of hedge fund indices. We had to compile what we believe is an exhaustive database of all existing hedge fund indices. The data collection work involved was rather formidable as there is not, to the best of our knowledge, a single integrator of all these sources.⁷

⁶Similarly, building a growth index on the basis of growth stocks relies on the assumption that each stock can be labelled either 100% growth or 100% value. This, of course, is a somewhat heroic assumption (is AOL-Time Warner a growth or a value stock?). Some have proposed probabilistic classification techniques for style: 95% growth, 5% value (e.g., Russell, Salomon Brothers) (see Borger (1997)).

⁷PerTrac, a vendor of systems for asset management, offers integrated access to a series of databases including Hedgefund.net, Hedge Fund Research (HFR), Altvest, TASS, and MAR. PerTrac provides three types of manager search selections: information search, statistics search and style search. However, specific contracts are required with each of these prime data providers.

Fortunately, a significant number of index providers offer the possibility to download the data from their web site at no cost for registered users. When possible, we have set up an account with these data vendors, and collected data on every single hedge fund index they would carry. In other instances, we had to purchase the database from the vendor.⁸ Finally, we have also used some proprietary databases obtained from specific contacts we have developed in the hedge fund industry.⁹ As a result of this somewhat painful process of data collection, we have been able to obtain a database consisting of indices maintained by the dozen aforementioned hedge fund index providers.

A second step involves sorting the indices by strategy, i.e., listing all competing indices for a given strategy. One problem we had to face is that the terminology of hedge fund strategies is not entirely stabilized, as we are dealing with a relatively new industry. Therefore, the same strategy can be referred to under different names. For example, some hedge fund index providers use the name *convertible hedge*, while others use the name *convertible arbitrage*. In the same vein, EACM and HF Net use the label *risk arbitrage* to denote strategies that are otherwise referred to as *merger arbitrage* by Altvest, HFR, Zürich or Hennessee (see Appendix A for information on these strategies). We have used our knowledge of the hedge fund industry to try to generate the most consistent classification possible. While it is almost impossible to ascertain that a given classification does not pair together inconsistent strategies, or leave similar strategies apart, we feel confident that our classification scheme includes in a consistent way most of the available information. At the end of this process, we have finally listed as many as 25 different hedge fund strategies for which there are at least two competing index providers.

In the interest of brevity, we do not report the results for all 25 strategies in this paper, but only focus on the most popular ones. To that end, we have performed a selection based on the following rules: (1) we eliminate strategies for which not more than 3 competitors are available, (2) we eliminate strategies with narrow focus (e.g., *sectors - health care*). As a result of that selection, we are left with the following list of 12 styles, including the composite *fund of funds* style (see table 2), with 4 to 8 index providers for each style.

The strategies being left aside are listed in table 3. To that list should also be added composite global across-strategies indices.¹⁰ We do not imply of course that these strategies do not represent an important part of hedge fund investing. The results for these strategies are available from the authors upon request.

⁸Van Hedge in particular do not post the historical performance of their indices on the web, but such data can be purchased from them with a discount price charged when the data is used for academic purposes.

⁹In particular, we would like to thank Francisco Portillero from Zürich Capital Markets in London for providing us with access to Zürich index performance data.

¹⁰CSFB, Altvest, Van Hedge and Hennessee offer a global index; MAR, Zurich, Magnum, HFR and HF Net do not.

Sub-Universe	List of Competing Indices	Date
Convertible Arbitrage	CSFB, HFR, EACM, Zürich, Hennessee, HF Net	01/98
Emerging Markets	CSFB, Altvest, HFR, MAR, Van Hedge, Hennessee, HF Net	01/96
Equity Market Neutral	CSFB, Van Hedge, HFR, MAR, Hennessee, HF Net	01/96
Event Driven	CSFB, Altvest, MAR, EACM, HFR, Hennessee, HF Net, Zürich	01/98
Fixed Income Arbitrage	CSFB, HFR, Van Hedge, Hennessee, HF Net	01/96
Global Macro	CSFB, Altvest, Van Hedge, MAR, HFR, Hennessee, HF Net, Magnum	02/97
Long/Short	CSFB, Altvest, Zürich, EACM, HF Net	01/98
Merger Arbitrage	Altvest, HFR, Zürich, Hennessee, EACM, HF Net	01/98
Relative Value	Altvest, HFR, Van Hedge, EACM, HF Net	01/96
Short Selling	Altvest, HFR, Van Hedge, MAR, EACM	01/96
Distressed Securities	Van Hedge, Altvest, HFR, EACM, Zürich, HF Net, Hennessee	01/98
Fund of Funds	Van Hedge, Altvest, HFR, Zürich	01/98

Table 2: Competing Indices in Hedge Fund Universe. This table provides a listing of competing indices in the hedge fund universe, with details on the competing indices and the starting date. Van Hedge offer two market neutral indices, respectively labelled Arbitrage and Securities hedging. For the purpose of this study, we have combined these two sub-indices into a single one by taking a simple average return on the two strategies. EACM offers two long-short indices, respectively labelled Relative Value and Equity Hedge Fund. For the purpose of this study, we have combined these two sub-indices into a single one by taking a simple average return on the two strategies. EACM and HF Net use the label risk arbitrage for strategies otherwise known as merger arbitrage.

We have collected and merged monthly return data from competing indices for each strategy. As a result, we had to use the starting date of the database with the shortest history as the starting date for our study. The starting date turned out to be January 1998 for each sub-universe where Zürich has created an index, for example, as the starting date for Zürich indices was early 1998, and the ending date is December 2000.¹¹

¹¹It should be noted that the samples vary across sub-universes since the starting dates differ. We have chosen not to consider a unique sample history for our study because this would imply focusing on the period ranging from January 1998 to December 2000. Given that such a period is relatively short, and does not contain important events such as the LTCM crisis, we have decided to use as much data as possible, and consider the longest possible history for each sub-universe. This comes at the cost of a relative loss in comparability between different styles, which is acceptable for our purposes as we focus on heterogeneity within a given style.

Sub-Universe	List of Competing indices
Market Timing	HFR, Van Hedge
Aggressive Growth	Magnum, Van Hedge, HF Net
International	Magnum, MAR, EACM, Hennessee
Special Situations	Magnum, Van Hedge, HF Net
Opportunity	Magnum, Van Hedge, EACM, HF Net
Emerging Markets - Latin America	HFR, Hennessee
Fixed Income - High Yield	HFR, Hennessee
Regulation D	HFR, HF Net
Sectors - Energy	HFR, HF Net
Sectors - Financial	Van Hedge, Altvest, HFR, EACM, Zürich, HF Net
Sectors - Technology	Altvest, HFR, Magnum, HF Net
Sectors - Health Care	Altvest, HFR, HF Net, Hennessee
Statistical Arbitrage	HFR, HF Net
Capital growth	Magnum, Hennessee

Table 3: Strategies Not Covered. This table provides a listing of competing indices for hedge strategies that we do not explicitly provide results for in this paper. Results for these strategies can be obtained from the authors upon request.

2.2 Heterogeneity in Competing Hedge Fund Index Returns

We have computed various measures of homogeneity/heterogeneity for each given sub-universe. The first measure is the maximum difference in monthly returns in the sample period (from the starting date to December 2000). The results are reported in table 4.

As can be seen from table 4, differences in monthly returns are spectacular and can be greater than 20%! For example, Zürich reports a 20.48% return on *long/short* strategies in February 2000, while EACM reports a -1.56% return in the same month for the same strategy. Obviously, replicating or outperforming a *long/short* strategy index was considerably easier that month if the benchmark used was EACM as opposed to Zürich! *Short selling* also posts maximum differences in returns above 20%, while *emerging market* and *global macro* are very close to that number (19.45% and 17.80%, respectively). We also note that the maximum differences tend to be recorded in periods of crisis: for seven strategies, maximum differences are recorded in the period ranging from August to October 1998, which corresponds to the LTCM crisis. This suggests that index returns become less homogeneous in turbulent times. While not surprising, this is bothersome because hedge fund indices fail to agree precisely when reliable information is most needed. Such maximum differences become even more dramatic when computed at a quarterly frequency. For example, we find a maximum difference equal

Sub-Universe	Max Difference (with dates and indices)
Convertible Arbitrage	4.75% (Oct 98; CSFB (-4.67) / Hennessee (0.08))
Emerging Markets	19.45% (Aug 98; (MAR -26.65) / Altvest (-7.2))
Equity Market Neutral	5.00% (Dec 99; Hennessee (0.2) / Van Hedge (5.2))
Event Driven	5.06% (Aug 98; CSFB (-11.77%) / Altvest (-6.71))
Fixed Income Arbitrage	10.98% (Oct 98; HF Net (-10.78) / Van Hedge (0.2))
Global Macro	17.80% (May 00: Van Hedge (-5.80) / HF Net (12))
Long/Short	22.04% (Feb 00: EACM (-1.56) / Zürich (20.48))
Merger Arbitrage	1.85% (Sep 98: Altvest (-0.11) / HFR (1.74))
Relative Value	10.47% (Sep 98: EACM (-6.07) / Van Hedge (4.40))
Short Selling	21.20% (Feb 00: Van Hedge (-24.3) / EACM (-3.09))
Distressed Securities	7.38% (Aug 98: HF Net (-12.08) / Van Hedge (-4.70))
Fund of Funds	8.01% (Dec 99: MAR-Zürich (2.41) / Altvest (10.42))

Table 4: Measures of Heterogeneity in Hedge Fund Indices (1). This table provides the maximum monthly return difference between competing indices for the same style.

to 30.08% for *long/short* or 16.52% for *relative value*, as opposed to, respectively 22.04% or 10.47%, at the monthly level. This strongly suggests that differences in competing index returns do not smoothen out at lower frequencies.

We also compute the average and lowest correlation between various indices in each given universe. Table 5 summarizes that information.

We find again that there is evidence of strong heterogeneity in the information conveyed by competing indices. For example, the correlation between two competing indices for the same strategy can be as low as .16 (correlation between Hennessee and MAR *equity market neutral* indices). It can even reach a negative -0.1901, as in the case of EACM and Zürich *long/short* indices, thus reflecting a sharp difference in net exposure of these two competing indices. The mean correlation between competing indices within a particular style can also be low, as low as .4. In particular, *equity market neutral* strategies exhibit a low 0.4276 average correlation. Interestingly enough, we find that hedge fund strategies for which mean correlation is the lowest are the ones which are known to come closest to market neutrality, i.e., *equity market neutral*, *long/short*, or to a lesser extent *fixed-income arbitrage*. The intuitive explanation is that these fund managers attempt to follow pure alpha strategies with little, if any, systematic exposure to pervasive risk factors, while more directional strategies maintain a large exposure to standard asset classes in a way that makes them more similar. At the other extreme, we find for example *emerging markets*, *merger arbitrage* or *event driven*, for which there is a fair amount of homogeneity in the information provided by competing indices. For example, in

Sub-Universe	Average Correlation	Lowest Correlation
Convertible Arbitrage	0.8183	0.6350
Emerging Markets	0.9284	0.8301
Equity Market Neutral	0.4276	0.1258
Event Driven	0.9232	0.8458
Fixed Income Arbitrage	0.5407	0.2254
Global Macro	0.5598	0.2698
Long/Short	0.4575	-0.1901
Merger Arbitrage	0.9193	0.8797
Relative Value	0.6752	0.3042
Short Selling	0.8811	0.7796
Distressed Securities	0.8645	0.7218
Fund of Funds	0.8757	0.7985

Table 5: Measures of Heterogeneity in Hedge Fund Indices (2). This table provides two measures of heterogeneity in the hedge fund index universe (average correlation and lowest correlation).

the case of *merger arbitrage*, we find that the maximum difference in monthly returns is a low 1.85% (see table 4) with an average correlation greater than .9 (see table 5). It actually turns out that managers pursuing *merger arbitrage* strategies tend to behave in a consistent manner which explains why competing indices, based on different sets of managers, tend to agree more than for other hedge fund styles. This is consistent with Mitchell and Pulvino (2001) who show that most *risk arbitrage* returns are positively correlated with market returns in severely depreciating markets but uncorrelated with market returns in flat and appreciating markets.¹²

We have created a customized heterogeneity index aimed at representing the percentage heterogeneity in the sub-universe. For that we simply compute $HI = 1 - \text{average correlation}$. Assume that a universe is made of K perfectly homogeneous indices. In that case, the correlation matrix is entirely made of 1, and the heterogeneity index is equal to $1 - 1 = 0\%$ heterogeneity. In general, this yields a number between 0 and 1 that can be regarded as a percentage heterogeneity in the sub-universe under consideration.¹³ We confirm that existing hedge fund indices provide a very contrasted view of the return on *equity market neutral* and *long/short* strategies, as we obtain a heterogeneity index which rises as high as 57.24% and

¹²Based on this evidence, they argue that one may passively replicate a significant proportion of the returns generated by hedge funds pursuing *merger arbitrage* types of strategies using portfolios of uncovered index put options.

¹³If the average correlation is negative, HI gets greater than 1. A negative average correlation across indices for a given strategy, however, would be surprising and never happens in our sample.

54.25%, respectively.

2.3 Implications for Factor Analysis and Asset Allocation

It has been reported in the literature that hedge funds are exposed not only to market risks, but also to volatility, credit or liquidity risks (see for example Schneeweis and Spurgin (2000) or Amenc, Curtis and Martellini (2001)). We now test for the heterogeneity in hedge fund indices by documenting the differences in their exposure to a set of broad factors. The factors that we use are market risks, including equity risk, proxied by the return on the S&P 500 index, equity volatility risk, proxied by changes in the average of intra-month values of the VIX contract¹⁴, fixed income risks including level risk, proxied by the 3-month T-Bill rate, slope risk or term premium risk, proxied by monthly observations of the difference between the yield on 3-month Treasuries and 10-year Treasuries, currency risk, proxied by changes in the level of a volume-weighted exchange index of currencies versus US dollar, commodity risk, proxied by changes in the level of a volume-weighted index of commodity prices, credit risk, proxied by changes in the monthly observations of the difference between the yield on long term Baa bonds and the yield on long term AAA bonds, and liquidity risk, proxied by changes in the monthly market volume on the NYSE.

Table 6 summarizes the information on the correlation of competing indices with these factors for the *fixed-income arbitrage* universe. (Similar results on other strategies can be obtained from the authors upon request; they are not included in the paper in the interest of brevity.)

It can be seen from table 6 that competing indices vary significantly in the way they correlate with a set of broad-based factors. For example, the HFR *fixed-income arbitrage* index is negatively correlated with the return on the S&P 500 (-0.16), while the Van Hedge *fixed-income arbitrage* index exhibits a positive correlation greater than .5. Similarly, some *fixed-income arbitrage* strategies are positively correlated with volatility risk (CSFB, HFR, HF Net), while others are negatively correlated with it (Van Hedge, Hennessee).

Because different hedge fund indices correlate in a very different way with broad-based economic factors, the diversification benefits obtained from including them in a traditional stock and bond portfolio also vary dramatically. As an illustration, we generate efficient frontiers obtained from adding various *fixed-income arbitrage* indices to an equity and fixed-income portfolio, using the S&P 500 and Lehman Brothers Aggregate as proxies for equity and fixed-income markets, respectively.¹⁵ This is reported in figure 2, which is based on monthly

¹⁴VIX, introduced by CBOE in 1993, measures the volatility of the U.S. equity market. This index is calculated by taking a weighted average of the implied volatilities of eight OEX calls and puts. The chosen options have an average time to maturity of 30 days.

¹⁵Mean-variance analysis is a standard way to analyze and formalize the benefits of investing in hedge funds.

Fixed-Income Arbitrage	F1	F2	F3	F4	F5	F6	F7	F8
CSFB	0.00	0.12	0.15	0.23	0.42	0.05	-0.38	-0.10
HFR	-0.16	0.14	0.25	0.19	0.57	0.07	-0.24	-0.18
Van Hedge	0.53	-0.47	0.09	0.02	-0.13	0.14	-0.16	-0.05
Hennessee	0.37	-0.37	0.06	0.19	0.26	0.12	-0.22	-0.12
HF Net	-0.10	0.20	0.22	0.20	0.42	0.03	-0.37	-0.01

Table 6: Sensitivity to Broad-Based Factors - The Case of Fixed-Income Arbitrage. This table provides the correlation between competing convertible arbitrage indices and a set of broad-based factors. F1 stands for the return on the S&P 500 index, F2 stands for changes in VIX, F3 stands for the T-Bill rate, F4 stands for the difference between the yield on 3-month Treasuries and 10-year Treasuries, F5 stands for changes in the volume-weighted index of currencies versus the US dollar, F6 stands for changes in the level of a volume-weighted index of commodity prices, F7 stands for changes in the difference between the yield on long term Baa bonds and the yield on long term AAA bonds and F8 stands for changes in the monthly market volume on the NYSE.

data for the period extending from January 1996 to October 2001.

We first note that the introduction of alternative investment strategies in a fixed-income portfolio is likely to generate a dramatic enhancement of the efficient frontier. On the other hand, the diversification benefits vary significantly depending on which index is used as a proxy for the return on *fixed-income* arbitrage strategies. The asset allocation decision is also greatly affected. As an illustration, we find that the minimum variance portfolio corresponds to an allocation of almost 40% in fixed-income arbitrage hedge funds when CSFB index is used, while it corresponds to almost half that number (23.1%) when Hennessee index is used instead.

3 Desperately Seeking Pure Style Indices

In the presence of many different competing indices, one may be at a loss to decide which one to use for performance benchmarking or asset allocation decisions. The problem is that there is no clear and definitive judgement that one can make in terms of a qualitative assessment

It is however well-known that it requires suitable assumptions on investor preferences (quadratic preferences) or asset return distribution (normal returns). There is clear evidence, however, that hedge fund returns may not be normally distributed (see for example Amin and Kat (2001) or Lo (2001)). Hedge funds typically exhibit non-linear option-like exposures to standard asset classes (Fung and Hsieh (1997a, 2000), Agarwal and Naik (2000)) because they can use derivatives, follow all kinds of dynamic trading strategies, and also because of the explicit sharing of the upside profits (post-fee returns have option-like element even if pre-fee returns do not). As a result, hedge fund returns may not be normally distributed even if traditional asset returns were. In this paper, we mainly use mean-variance analysis for illustration purposes.

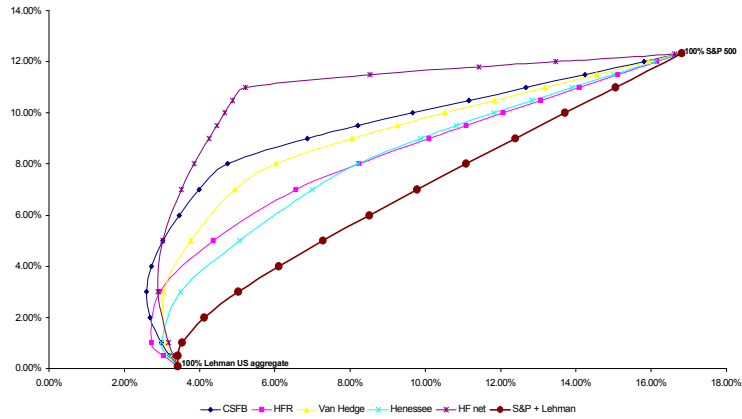


Figure 2: Heterogeneity in Diversification Benefits. In this figure, we generate efficient frontiers obtained from adding various fixed income arbitrage indexes (CSFB, HFR, Van Hedge, Hennessee, HF Net) to an equity and fixed income portfolio, using the S&P 500 and the Lehman Brothers Aggregate as proxies for equity and fixed income markets, respectively. This figure is based on monthly data for the period extending from January 1996 to October 2001.

of which index provider is doing the best job at representing a given market segment. All existing indices have both advantages and drawbacks. For example, Zürich Capital Markets (ZCM) have tried to design a process to ensure the style-purity of their indices by using an objective statistical cluster-based classification procedure for style classification, as opposed to managers' self-proclaimed styles. On the other hand, ZCM do not use as exhaustive a database as some of their competitors: their hedge fund indices are based upon a data set of 60 hedge funds, while HFR for example uses as many as 1,100 hedge funds (see Fung and Hsieh (2001b) for more institutional details). Therefore, while Zürich indices may be less biased than some of their competitors, they fail to properly represent a significant fraction of the entire universe and are therefore less representative than some of their competitors.

In this context, it seems desirable to try and use all available information, as opposed to focus on a single specific index. In what follows, we attempt to provide remedies to both the lack of representativeness and existence of bias problems and suggest various methodologies designed to help extract a “pure style index”, or “index of the indices” from the information conveyed by competing index providers.

3.1 Statistical Approach to the Construction of Pure Style Indices

There is a whole body of the statistical literature that deals with (linear and non linear) factor models with unobservable factors. For example, at the cost of specific assumptions (e.g., assumption that the return on the indices are normally distributed), one may use filtering

theory (Kalman filter) to draw inferences on the unobservable variable, based on the observed returns.¹⁶

Many time-series models, including the classical linear regression model and ARIMA models, can be represented in a general form known as the state space form. There are two main benefits to representing a dynamic system in state space form. First, the state space allows unobserved variables (known as the state variables) to be incorporated into, and estimated along with, the observable model. In the present context, the observed variables are the competing indices, and the unobserved variable is the “true” pure index. Second, state space models can be estimated using a powerful recursive algorithm known as the Kalman filter. The Kalman filter is used both to evaluate the likelihood function and to forecast and smooth the unobserved state variables. In this paper, our motivation is to estimate (smooth) the unobserved state variable, based on the observed returns. State space models have been applied in the econometrics literature to model unobserved variables such as (rational) expectations, measurement errors, missing observations, permanent income, unobserved components (cycles and trends), and the natural rate of unemployment. Extensive surveys of applications of state space models in econometrics can be found in Hamilton (1994a, 1994b) and Harvey (1989).

We present here a brief discussion of the specification and estimation of a state space model in the context of pure hedge fund indices. Those desiring greater detail are directed to Hamilton (1994a) and Harvey (1989). The basic intuition is that we want to model the return on these indices as the sum of a true unobservable pure style index I and a white noise capturing the existence of biases and lack of full representativeness. The state space representation of the dynamics of a vector $R_t = (R_{it})_{i=1,\dots,n}$ of the return on competing indices is then given by the following system of equations

$$R_t = \mathbf{1} \cdot I_t + \varepsilon_t \quad (1)$$

$$I_{t+1} = \bar{I} + \nu_{t+1} \quad (2)$$

where $\mathbf{1}$ is a $(n \times 1)$ vector of ones, I_t is the scalar return on the unobservable pure index, \bar{I} the mean of the unobserved pure index, and where ε_t and ν_t are assumed to be independent white noise with $\text{Var}[\varepsilon_{it} | \mathcal{F}_t] = \sigma_\varepsilon^2 = (\sigma_{\varepsilon_i}^2)_{i=1,\dots,n}$, $\text{Var}[\nu_t | \mathcal{F}_t] = \sigma_I^2$ and $\text{Cov}[\varepsilon_{it}, \nu_t | \mathcal{F}_t] = 0$ for all $i = 1, \dots, n$.¹⁷

¹⁶The assumption that hedge fund returns are normally distributed is at odd with empirical evidence. It can, however, be shown, that, while the Kalman filter forecasts need no longer be optimal for systems that are not normal, no other forecast based on a linear function of I_t will have a smaller mean squared error (see Hamilton (1994b)). These results parallel the Gauss-Markov theorem for ordinary least squares regression.

¹⁷Note that because of the presence of biases such as self-selection and instant history biases, the noise term ε may have a positive mean. This unambiguous effect can be accounted for separately from the analysis we present in this section by subtracting an estimate of the impact of the combined biases from the return of competing indices.

Equation (1) is known as the observation (or measurement) equation while equation (2) is known as the state (or transition) equation, which specifies the dynamics of the return on the index. It should be noted that one may consider the more general expression $R_t = A'X + H \cdot I_t + \varepsilon_t$, where A and H are $n \times 1$ vectors of parameters, and X is a $n \times 1$ vector of observable factors. We have also tested the latter, more general, version of the model.¹⁸ We do not report the results here in the interest of brevity; they can be obtained from the authors upon request.

Given observations (R_t) for $t = 1, 2, \dots, T$, our goal is to estimate the parameters σ_ε^2 and σ_I^2 , and make inferences about the state vector I_t . The Kalman filter is a recursive algorithm for sequentially updating the state vector given past information. While we use the more familiar terminology of Kalman *filter*, we actually use in this paper the Kalman *smoother*. The difference between the two is the conditioning information set. The filter is conditional upon information up to time t , and therefore is well-suited for prediction. In the context of prediction, the problem is to provide an optimal forecast of the value of the state vector at date $t+1$ based on information available at date t , denoted $\widehat{I}_{t+1|t}$. In our application, the value of the state vector is of interest in its own right. In such cases it is desirable to use information through the end of the sample (date T) to help improve the inference about the historical value that the state vector took on at any particular date t in the middle of the sample. Such an inference is known as a smoothed estimate, denoted $\widehat{I}_{t|T} = \mathbb{E}[I_t | I_T]$. The mean squared error of this estimate is denoted $P_{t|T} = \mathbb{E}\left[\left(\widehat{I}_{t|T} - I_t\right)^2\right]$.

The smoothed series, which is the best estimate (in terms of mean squared error) of the state series using the full sample T , is obtained as follows. We first compute $\widehat{I}_{t|t}$ and $P_{t|t}$, for $t = 1, 2, \dots, T$, using the recursive Kalman filter procedure, which reads in the general case

$$\begin{aligned}\widehat{I}_{t|t} &= \widehat{I}_{t-1|t-1} + S_{t-1}H (H'S_{t-1}H + \sigma_\varepsilon^2)^{-1} (R_t - A'X - H\widehat{I}_{t-1|t-1}) \\ P_{t|t} &= S_{t-1} - S_{t-1}H (H'S_{t-1}H + \sigma_\varepsilon^2)^{-1} HS_{t-1}\end{aligned}$$

where $S_{t-1} = P_{t|t} + \sigma_I^2$. Note that the recursion for P does not depend on the forecasted state vector $\widehat{I}_{t-1|t-1}$, or on the observed data (R, X) . The smoothed series and its mean squared errors are then computed by the backward recursion

$$\begin{aligned}\widehat{I}_{t|T} &= \widehat{I}_{t|t} + J_t \left(\widehat{I}_{t+1|T} - \widehat{I}_{t+1|t} \right) \\ P_{t|T} &= P_{t|t} + J_t^2 (P_{t+1|T} - P_{t+1|t})\end{aligned}$$

¹⁸We have actually tested 2 versions of the model, one with $R_t = A + H \cdot I_t + \varepsilon_t$, where A and H are $n \times 1$ vectors of parameters, and another with $R_t = A'X + H \cdot I_t + \varepsilon_t$, where A and H are $n \times 1$ vectors of parameters, and X is a $n \times 1$ vector of observable factors. For the latter formulation of the model, we have used factors that have been found to help explain equity returns (proxies for market, credit and liquidity risks) and subsequently performed Kalman filtering.

for $t = T - 1, T - 2, \dots, 1$, where $J_t = P_{t|t}P_{t+1|t}^{-1}$.

In table 7, we provide information on the mean, volatility, estimation error, kurtosis, skewness as well as average correlation with competing indices of pure indices generated using the Kalman filter smoothing techniques. We also provide (in parenthesis) average values for these parameters for competing indices.

Sub-Universe	Mean	Volatility	Est. Error	Average Correlation
Conv. Arbitrage	0.75% (0.88%)	1.04% (1.35%)	0.01%	0.905
Emerging Markets	0.49% (0.56%)	4.32% (5.08%)	0.01%	0.966
Eq. Market Neutr.	0.71% (0.83%)	0.55% (0.90%)	0.01%	0.707
Event Driven	0.72% (0.82%)	1.70% (2.01%)	0.01%	0.954
Fixed Income Arb.	0.34% (0.41%)	1.09% (1.67%)	0.02%	0.789
Global Macro	0.60% (0.68%)	1.77% (2.44%)	0.01%	0.822
Long/Short	0.84% (1.01%)	2.22% (3.00%)	0.02%	0.704
Merger Arb.	0.71% (0.83%)	1.07% (1.29%)	0.01%	0.969
Relative Value	0.75% (0.88%)	1.39% (1.90%)	0.01%	0.807
Short Selling	0.53% (0.66%)	5.68% (7.42%)	0.02%	0.954
Distressed	0.65% (0.75%)	1.73% (2.14%)	0.01%	0.923
Fund of Funds	0.69% (0.86%)	1.83% (2.40%)	0.02%	0.948

Table 7: Pure Indices from Kalman Filter Analysis. This table provides information (mean, volatility, estimation error as well as average correlation with competing indices) on the monthly returns of pure indexes generated using the Kalman filter smoothing techniques. Numbers in paranthesis are average values for these parameters for competing indices.

As an illustration, we plot in figure 3 the return for the pure index, with the corresponding 95% confidence interval, as well as the return on competing indices for the strategy *convertible arbitrage* on the period 1998-2000.

From table 7 and figure 3, it appears pure indices generated from Kalman filter analysis have relatively high correlations with corresponding competing indices. On the other hand, they have smoother paths than that of the competing indices, suggesting that noise reduction has actually occurred. On the other hand, one key problem with this approach is that the estimated pure index can not be expressed as a linear combination of competing indices. As a result, a pure index generated from Kalman smoothing analysis can not be regarded as an index in its own right, since it can not be expressed as a portfolio of individual hedge funds. In what follows, we discuss a portfolio approach to the construction of pure style indices, which allows us to alleviate this black-box problem.

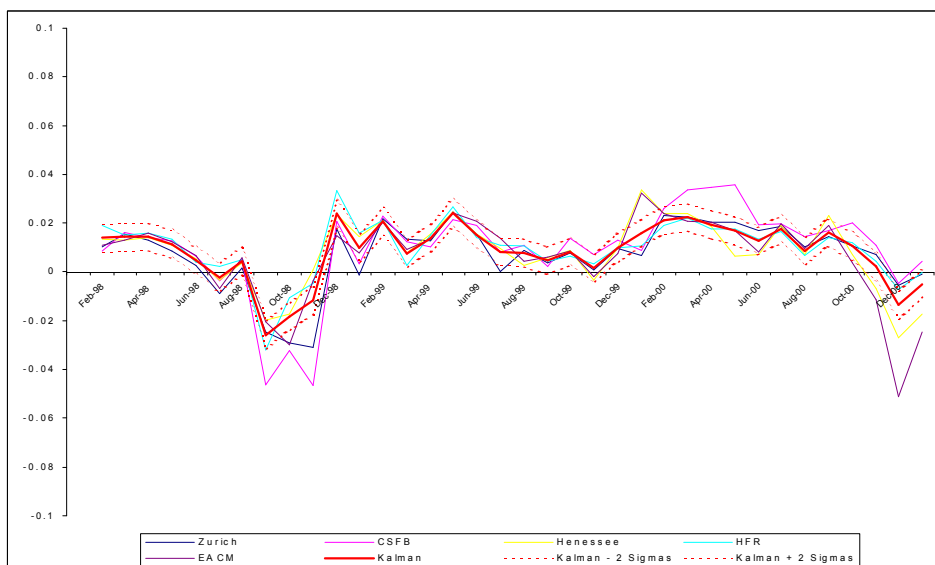


Figure 3: Convertible Arbitrage Pure Index from Kalman Analysis. This figure displays the return for the pure index, with the corresponding 95% confidence interval, as well as the return on competing indexes for the strategy *convertible arbitrage* on the period 1998-2000.

3.2 Portfolio Approach to the Construction of Pure Style Indices

Given that it is impossible to come up with an objective judgement on what is the best existing index, a natural idea consists of using some combination of competing indices to reach a better understanding of what the common information about a given investment style would be.

One straightforward method for obtaining a composite index based on various competing indices would involve computing an equally-weighted portfolio of all competing indices.¹⁹ This would obviously provide investors with a convenient one-dimensional summary of the contrasted information contained in competing indices. In particular, because competing hedge fund indices are based on different sets of hedge funds, the resulting portfolio of indices would be more exhaustive than any of the competing indices it is extracted from (see theorem 2 section 4.1). In this paper, we wish to push the logic one step further and suggest using factor analysis techniques to extract the *best possible one-dimensional summary* of a set of competing indices, and design what can be called “pure style” indices. Our method is a natural generalization of the idea of taking a portfolio of competing indices. The refinement involves relaxing the assumption of an equally-weighted portfolio.

¹⁹Value-weighting is not a viable alternative, as information on the assets under management is not usually part of the information made available to the public by data vendors. Besides, funds frequently report assets under management with a considerable lag, which implies that such information could not be made available on a timely basis anyway.

3.2.1 Maximization of Representativeness

We first suggest to use factor analysis techniques to generate a set of pure indices that can be thought of as the best possible one-dimensional summaries of information conveyed by competing indices for a given style, in the sense of the larger fraction of the variance explained. Here, we are looking for the portfolio weights that make the combination of competing indices capture the largest possible fraction of the information contained in the data from the various competing indices. Technically speaking, this amounts to using the first component of a PCA of competing indices as a candidate for a pure style index. Note that the first component typically captures a large proportion of cross-sectional variations because competing styles tend to be at least somewhat positively correlated. This is confirmed by the numbers in table 8 below.

The PCA of a time-series involves studying the correlation matrix of successive shocks. Its purpose is to explain the behavior of observed variables using a smaller set of unobserved implied variables. From a mathematical standpoint, it involves transforming a set of K correlated variables into a set of orthogonal variables, or implicit factors, which reproduces the original information present in the correlation structure. Each implicit factor is defined as a linear combination of original variables. Define R as the following matrix

$$R = (R_{tk})_{\substack{1 \leq t \leq T \\ 1 \leq k \leq n}}$$

We have n variables, i.e., monthly returns for n different competing indices, and T observations of these variables.²⁰ PCA enables us to decompose R_{tk} as follows²¹

$$R_{tk} = \sum_{i=1}^n \sqrt{\lambda_i} U_{ik} V_{ti} \quad (3)$$

where

$(U) = (U_{ik})_{1 \leq i, k \leq n}$ is the matrix of the n eigenvectors of $R'R$.

$(U^\top) = (U_{ki})_{1 \leq k, i \leq n}$ is U transposed.

$(V) = (V_{ti})_{\substack{1 \leq t \leq T \\ 1 \leq i \leq n}}$ is the matrix of the n eigenvectors of RR' .

Note that these n eigenvectors are orthonormal. λ_i is the eigenvalue (ordered by degree of magnitude) corresponding to the eigenvector U_i . Denoting $s_{ik} = \sqrt{\lambda_i} U_{ik}$ the principal component sensitivity of the k^{th} variable to the i^{th} factor, and $V_{ti} = F_{ti}$, one can equivalently write equation (3)

$$R_{tk} = \sum_{i=1}^n s_{ik} F_{ti}$$

²⁰The asset returns have first been normalized to have zero mean and unit variance.

²¹For an explanation of this decomposition in a financial context, see for example Barber and Copper (1996).

where the n factors F_i are a set of orthogonal variables. One may use the method to describe each variable as a linear function of a reduced number of factors. To that end, one needs to select a number of factors I such that the first I factors capture a large fraction of asset return variance, while the remaining part can be regarded as statistical noise

$$R_{tk} = \sum_{i=1}^I \sqrt{\lambda_i} U_{ik} V_{ti} + \varepsilon_{tk} = \sum_{i=1}^I s_{ik} F_{ti} + \varepsilon_{tk} \quad (4)$$

where some structure is imposed by assuming that the residuals ε_{tk} are uncorrelated one to another. The percentage of variance explained by the first I factors is given by $\sum_{i=1}^I \lambda_i / \sum_{i=1}^N \lambda_i$. By taking $I = 1$ in equation (4) this method can be used to generate “the best one dimensional”

summary of a set of competing indices. Furthermore, a simple normalization

$$R_{tk} = \sum_{i=1}^K \frac{s_{ik}}{\sum_{k'=1}^K s_{ik'}} F_{ti}$$

allows one to obtain an index which can be regarded as a portfolio of competing indices, so that an actual decomposition in terms of actual funds in the index can easily be obtained as long as information is available in each competing index composition.

Table 8 displays the ratio of the eigenvalue associated with the first component to the sum of all eigenvalues. That number can be regarded as the percentage of the information contained in the time-series of competing indices that is captured by the pure index (column 3). An information loss ratio can be computed by simply taking 100% minus the percentage of variance explained by the first factor. Table 8 also provides the number of competing indices for each category in column 2.

We find that pure style indices are able to capture a very large fraction of the information. The average (resp. median) percentage of variance explained by the pure style indices is 79.12% (resp. 81.12%) across all sub-universes. The percentage of variance explained by the pure index is, of course, all the more significant in that the correlation between competing indices was high. For example, *emerging market* style indices have a percentage of variance explained greater than 90% while it originates from a population of 7 competing indices. From table 4, we see that the mean correlation was almost 0.93 for *emerging market* indices. In the same vein, *event driven* and *merger arbitrage* PCA indices capture more than 80% of the information originally available in a set of 8 and 4 competing indices, respectively. The *fund of funds* PCA index also enjoys very low information loss as more than 91% of the information is captured by the one-dimensional summary. On the other hand, the percentage of information loss is higher in the case of *equity market neutral* (41.09% = 100% – 58.91% information loss) and *fixed-income arbitrage* (35% = 100% – 65% information loss). This is because these

Sub-Universe	# of Indices	% of Variance Explained
Convertible Arbitrage	6	84.91
Emerging Markets	7	91.97
Equity Market Neutral	6	58.91
Event Driven	8	85.41
Fixed Income Arbitrage	5	65
Global Macro	8	74.13
Long/Short	6	86.8
Merger Arbitrage	4	83.81
Relative Value	5	71.26
Short Selling	5	78.42
Distressed Securities	7	77.6
Fund of Funds	5	91.19

Table 8: Pure Style Indices. This table provides the ratio of the eigenvalue associated with the first component to the sum of all eigenvalues. That number can be regarded as the percentage of the information contained in the time-series of competing indices that is captured by the pure index (column 3). It also provides the number of competing indices for each category in column 2.

strategies were the ones for which the heterogeneity of information provided by competing index providers was the most extreme (see tables 4 and 5).

Pure hedge fund indices generated as the first component in a factor analysis have an appealing built-in element of optimality, since there is no other linear combination of competing indices that implies a lower information loss. Another approach consists in focusing on minimization of the bias.

3.2.2 Minimization of Bias

In this approach, we still model the return on these indices as the sum of a true unobservable pure style index I and a white noise (the bias)

$$R_{it} = I_t + \varepsilon_{it}$$

We denote as $\mathcal{F}_t = (R_{is})_{s \leq t, i=1, \dots, n}$ the agents' information set. Note that investors do not observe the return on the true index but only the return on the competing indices. The following assumptions will be made for all $i, j = 1, \dots, n$

- Assumption 1: $\mathbb{E}[\varepsilon_{it} | \mathcal{F}_t] = 0$
- Assumption 2: $\text{Var}[\varepsilon_{it} | \mathcal{F}_t] = \sigma_{\varepsilon_i}^2$

- Assumption 3: $\text{Cov}[\varepsilon_{it}, I_t | \mathcal{F}_t] = 0$

Assumption 1 simply states that the bias is not systematic: sometimes the return on the commercial index i exceeds that of the true unobservable index, sometimes it is lower, on average the difference is zero. Assumption 2 is a homoscedasticity assumption which states that the variance of the bias is constant in time (note that $\sigma_{\varepsilon_i}^2$ is a measure of the size of the bias). Assumption 3 is common to linear models; it states the bias is not affected by the level of the true index.

Our goal is to define a portfolio of competing indices that would be as close as possible to the return on the true unobservable factor. We denote as

$$\begin{aligned} R_{pt} &= w' \cdot R_t \\ &= I_t + w' \cdot \varepsilon_t \end{aligned}$$

where w is a n -dimensional vector of weights, and R_t (respectively ε_t) is a n -dimensional vector of index returns (respectively biases). Finally $'$ denotes transpose. We obtain

$$\begin{aligned} \mathbb{E}[R_{pt} | \mathcal{F}_t] &= \mathbb{E}[I_t | \mathcal{F}_t] \\ \text{Var}[R_{pt} | \mathcal{F}_t] &= \sigma_I^2 + w' \cdot \Sigma_\varepsilon \cdot w \end{aligned}$$

where we are using assumption 1 and 3 in the first and second line, respectively, and where σ_I is the volatility of the true unobservable index, and Σ_ε is the variance-covariance matrix of the bias terms.

Our objective is to minimize the bias of the composite index R_p , that is $\underset{w}{\text{Min}} w' \cdot \Sigma_\varepsilon \cdot w$ such that $w' \cdot \mathbf{1} = 1$, where $\mathbf{1}$ is a n -dimensional vector of ones. The solution to this problem is well-known to be (see for example Cochrane (2001))

$$w^* = \frac{\Sigma_\varepsilon^{-1} \cdot \mathbf{1}}{\mathbf{1}' \cdot \Sigma_\varepsilon^{-1} \cdot \mathbf{1}} \quad (5)$$

This is formally similar to a variance minimization problem. Since $\text{Var}[R_{pt} | \mathcal{F}_t] = \sigma_I^2 + w' \cdot \Sigma_\varepsilon \cdot w$, it actually appears that minimization of the bias is equivalent to minimization of $\text{Var}[R_{pt} | \mathcal{F}_t]$. This can be achieved in the usual way, using an efficient estimate of the sample covariance matrix of hedge fund returns (see Amenc and Martellini (2002)). Note also that portfolio constraints may further be imposed to ensure that all the weights are positive. This may actually be a desired property since it is natural that the index of the indices thus obtained be eventually expressed as a portfolio with positive investments in individual funds. In principle, imposing positivity constraints obviously induces a deviation from optimal bias reduction. On the other hand, we find evidence that minimum bias portfolios in the presence of positivity constraints are sometimes more representative than in the absence of such constraint (see table 9 in section 4.2).

If we make one additional assumption (assumption 4 below), we can show that the problem of bias minimization actually admits a very simple and elegant closed-form solution (see equation 6).

- Assumption 4: $\mathbb{C}ov[\varepsilon_{it}, \varepsilon_{jt} | \mathcal{F}_t] = 0$ for $i \neq j$

Assumption 4 states that the bias on a given index is independent from that on another competing index, so that Σ_ε is diagonal. Because biases are meant to capture some idiosyncratic component in competing index returns, this can be regarded as a reasonable assumption. On the other hand, that assumption may not be taken for granted, as there might exist some common factors in the biases of various competing index providers for a given style.²² As often the case, a standard trade-off exists between model risk and estimation risk.

Using assumptions 4 and 2, we obtain that Σ_ε is a diagonal matrix with $\sigma_{\varepsilon_i}^2$ terms on the diagonal. Now

$$\sigma_{\varepsilon_i}^2 = \mathbb{V}ar[\varepsilon_{it} | \mathcal{F}_t] = \mathbb{V}ar[I_t - R_{it} | \mathcal{F}_t] = \sigma_I^2 + \sigma_i^2 - 2\mathbb{C}ov[I_t, R_{it} | \mathcal{F}_t]$$

where σ_i^2 is the variance on the return on the i^{th} competing index. Noting that (using assumption 3)

$$\mathbb{C}ov[I_t, R_{it} | \mathcal{F}_t] = \mathbb{C}ov[I_t, I_t + \varepsilon_{it} | \mathcal{F}_t] = \sigma_I^2$$

we finally obtain

$$\sigma_{\varepsilon_i}^2 = \sigma_i^2 - \sigma_I^2$$

From equation (5), we finally get

$$w_i^* = \frac{\frac{1}{\sigma_i^2 - \sigma_I^2}}{\sum_{j=1}^n \frac{1}{\sigma_j^2 - \sigma_I^2}} \quad (6)$$

a very simple and elegant expression.

In practice, the volatility σ_i of the return on the competing index i is replaced by its unbiased estimator $\hat{\sigma}_i$. The problem is that the optimal weight w_i^* is also a function of the volatility of the unobservable true index σ_I . That term can be estimated from the Kalman filter approach described above. Alternatively, one may elect to estimate that term in the following way. Just note that

$$\begin{aligned} \mathbb{C}ov[R_{it}, R_{jt} | \mathcal{F}_t] &= \mathbb{C}ov[I_t + \varepsilon_{it}, I_t + \varepsilon_{jt} | \mathcal{F}_t] \\ &= \sigma_I^2 + \mathbb{C}ov[I_t, \varepsilon_{it} | \mathcal{F}_t] + \mathbb{C}ov[I_t, \varepsilon_{jt} | \mathcal{F}_t] + \mathbb{C}ov[\varepsilon_{it}, \varepsilon_{jt} | \mathcal{F}_t] = \sigma_I^2 \end{aligned}$$

²²To check whether assumption 4 is consistent with the data, we have computed the correlation between $R_{it} - I_t$ where the return I_t on the pure index is obtained from Kalman analysis. From that analysis, we find evidence of non trivial correlations between the noise terms for competing indices.

where we use assumptions 3 and 4 in the last line. This suggests that one may use the average covariance between competing indices as an estimator for the variance on the return on the unobservable true factor

$$\hat{\sigma}_I^2 = \frac{1}{n^2} \sum_{i,j=1}^n \hat{\sigma}_{ij}$$

where $\hat{\sigma}_{ij} = \frac{1}{T} \sum_{t=1}^T (R_{it} - \bar{R}_i) (R_{jt} - \bar{R}_j)$ is the estimator of the covariance between the return on index i and index j .

In table 9, we provide information on the mean, volatility, as well as average correlation with competing indices of pure indices generated using equation (6), where the variance of the pure index is estimated from the Kalman filter approach. We also provide (in parenthesis) average values for these parameters for competing indices.

Sub-Universe	Mean	Volatility	Average Correlation
Convertible Arbitrage	1.08% (0.94%)	1.13% (1.34%)	0.90
Emerging Markets	0.80% (0.43%)	3.33% (5.34%)	0.95
Equity Market Neutral	0.85% (0.83%)	0.44% (1.03%)	0.58
Event Driven	0.95% (0.86%)	3.78% (2.09%)	0.87
Fixed Income Arbitrage	0.37% (0.36%)	1.33% (1.70%)	0.78
Global Macro	0.09% (0.69%)	3.73% (2.55%)	0.70
Long/Short	0.78% (1.11%)	0.76% (3.13%)	0.56
Merger Arb.	0.99% (0.89%)	1.15% (1.31%)	0.92
Relative Value	0.82% (0.90%)	1.00% (1.97%)	0.68
Short Selling	0.82% (0.49%)	6.95% (7.70%)	0.47
Distressed	0.46% (0.73%)	2.55% (2.26%)	0.80
Fund of Funds	0.94% (0.91%)	2.45% (2.52%)	0.95

Table 9: Pure Indices from Minimum Bias Analysis. This table provides information (mean, volatility, as well as average correlation with competing indices) on monthly returns of pure indices generated using minimum bias analysis where the variance of the pure index is estimated from the Kalman filter approach. Number in paranthesis are average values for these parameters for competing indices.

From table 9, we see that the volatility of all but one pure indices is lower than the average of competing indices, hence suggesting that a noise reduction has actually occurred. Pure hedge fund indices generated from minimum variance analysis have an appealing built-in element of optimality, since there is no other linear combination of competing indices that implies a lower bias, granted that assumptions 1 to 4 hold (see discussion in section 4).

It should be noted that the sample covariance matrix of historical returns is likely to generate high sampling error when the number of observations is relatively small compared to the number of asset classes.²³ Several methods that have been introduced to improve asset return covariance matrix estimation could be applied in the present context. One solution is to impose some structure on the covariance matrix to reduce the number of parameters to be estimated. Several models fall within that category, including the constant correlation approach (Elton and Gruber (1973)), the single factor forecast (Sharpe (1963)) and the multi-factor forecast (e.g., Chan, Karceski and Lakonishok (1999)). In these approaches, sampling error is reduced at the cost of some specification error. Several authors have studied the optimal trade-off between sampling risk and model risk in the context of optimal shrinkage theory. This includes optimal shrinkage towards the grand mean (Jorion (1985, 1986)), optimal shrinkage towards the single-factor model (Ledoit (1999)). Also related is a recent paper by Jagannathan and Ma (2000) who show that imposing weight constraints is actually equivalent to shrinking the extreme covariance estimates to the average estimates.

Since the focus of the paper is not on estimation of variance-covariance matrices, we use the sample estimate for simplicity. We, however, also try and impose positivity constraints, and we argue in section 5 below that it might help mitigate the estimation error risk.

4 How Pure is Pure?

One may wonder how better off an investor would be by focusing on these pure style indices, as opposed to any of the competing indices from which they are extracted. Backtesting the pure style indices is no easy task because of the definite chicken-and-egg flavor associated to it. If we knew what a good style index should be in the first place, we would have all competing index providers agreeing to a larger extent, and we would have no need for composite pure style indices!

In what follows, we first provide two simple results that show the superiority of any portfolio of indices over any single competing index for a given strategy. Obviously, PCA-based and minimum-variance-based pure style indices, which have been specifically designed to address, respectively, the information loss and minimization of the bias problems, should be better than an arbitrarily selected portfolio of competing indices. We also illustrate the performance of the pure style indices in terms of their ability to improve benchmarking of hedge fund returns.

²³In the present context, the number of assets is not very large, as we deal with portfolio of competing hedge fund indices. The data, however, is scarce, as hedge fund returns are typically available at a monthly frequency only.

4.1 The two Basic Theorems of Pure Indexing

The mere fact of composing a pure index as an index of the indices, i.e., as a portfolio of existing indices, always leads to a reduction in the size of the bias and an improvement of representativeness, even if the portfolio weights are not optimized. This is the content of the following two straightforward propositions.

Theorem 1 *An index of the indices is always less biased than the average of the set of indices it is extracted from.*

Proof. Straightforward application of the properties of portfolio diversification. The proof is expressed under the additional assumption 4. Let us consider a portfolio of competing indices $R_{pt} = \sum_{j=1}^n w_j R_{jt}$. Theorem 1 says that the variance of the portfolio bias is always smaller than the average variance of the competing index biases, whatever the portfolio weights, that is

$$\sigma_{\varepsilon_p}^2 \leq \sum_{j=1}^n w_j \sigma_{\varepsilon_j}^2$$

To see why, note that the variance of the portfolio $\sigma_{\varepsilon_p}^2$ is given by

$$\sigma_{\varepsilon_p}^2 = w' \cdot \Sigma_{\varepsilon} \cdot w = \sum_{j=1}^n w_j^2 \sigma_{\varepsilon_j}^2$$

where the portfolio weight vector w is defined as $w = (w_1, \dots, w_n)$. The announced result is obtained from

$$\sum_{j=1}^n w_j^2 \sigma_{\varepsilon_j}^2 \leq \sum_{j=1}^n w_j \sigma_{\varepsilon_j}^2$$

■

Obviously, the noise reduction is the greater, by construction, for the minimum biased index. In the same vein, considering a portfolio of hedge fund indices always enhances the representativeness of the index, because more funds are accounted for.

Theorem 2 *An index of the indices is always more presentative than any competing index.*

Proof. Obvious from figure 4 in the case of 3 indices.

We denote by $N_j = \text{card}(I_j)$ (respectively, $N_p = \text{card}(I_p)$) the number of funds used by index j (respectively, by the portfolio of index $R_{pt} = \sum_{j=1}^n w_j R_{jt}$), i.e., $R_{jt} = \sum_{k=1}^{N_j} p_k^{(j)} R_{kt}$, (respectively, $R_{pt} = \sum_{k=1}^{N_p} p_k^{(p)} R_{kt}$), where R_{kt} is the return at date t on individual fund k and $p_k^{(j)}$ (respectively, $p_k^{(p)}$) the weighting of fund k in the index j (respectively, in the portfolio of

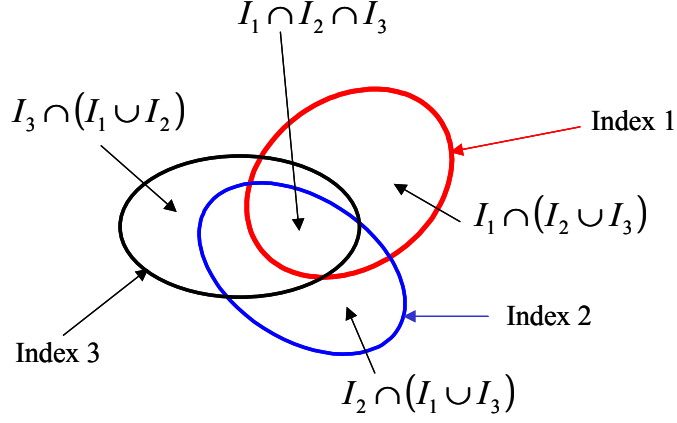


Figure 4: Representativeness of a Portfolio of Competing Indexes. A portfolio of competing indexes encompasses more individual funds than any of the competing indexes.

indices). In most cases, we have that $p_k^{(j)} = 1/N_j$, since all but one existing hedge fund indices are equally-weighted. We have that

$$I_p = \left(\bigcap_{j=1, \dots, n} I_j \right) \cup \left(\bigcup_{j=1, \dots, n} \left(I_j \cap \bigcap_{j' \neq j} I_{j'} \right) \right)$$

from which we obtain

$$N_p = \text{card}(I_p) = \text{card} \left(\bigcap_{j=1, \dots, n} I_j \right) + \text{card} \left(\bigcup_{j=1, \dots, n} \left(I_j \cap \bigcap_{j' \neq j} I_{j'} \right) \right) \geq \max_{j=1, \dots, n} (N_j)$$

Note that $N_p \neq \sum_{j=1}^n N_j$ because funds may report their performance to more than one database (i.e., $I_j \cap I_{j'} \neq \emptyset$). Note also that the index of the indices will not be in general an equally-weighted portfolio of individual funds. ■

In other words, even if PCA based portfolios are meant to focus on the representativeness dimension while minimum variance portfolios are meant to focus on the bias dimension, any portfolio of hedge fund indices (e.g., an equally-weighted portfolio of competing indices) should do better than any given index on both dimensions. When the correlation between competing indices is high, we actually expect the first component of the PCA analysis to generate an almost equally-weighted portfolio. On the other hand, when correlation is relatively low, i.e., when we really need a pure index, PCA guarantees the best possible one-dimensional representation.

4.2 Testing the Representativeness of Pure Hedge Fund Indices

To assess the superior representativeness of pure style indices with respect to existing indices, we have performed the following test.²⁴ We have first merged the three major databases providing information on individual fund return, MAR, TASS and HFR. We have also added data on funds which do not report to any data base, that had been directly obtained from administrators. As a result of that data collection process, we have gathered monthly returns on a total of 7,422 hedge funds, including 2,317 funds that do not report their returns to the major data bases.²⁵ As a result, our sample represents a very significant fraction of the hedge fund population.²⁶ The following table (table 10) provides information on the breakdown of that number into strategies.

Sub-Universe	Number of Funds
Convertible Arbitrage	657
Emerging Markets	198
Equity Market Neutral	206
Event Driven	1083
Fixed Income Arbitrage	438
Global Macro	263
Long/Short	2533
Merger Arb.	638
Relative Value	198
Short Selling	302
Distressed	554
Fund of Funds	352
Total	7,422

Table 10: Number of Funds in the Sample. This table provides information on the number of funds in the sample for each strategy.

Then, for each strategy covered in this paper, we have collected all funds with self-proclaimed style matching the strategy under consideration and form equally-weighted portfolios. Because we are following the funds' self-proclaimed style, these equally-weighted portfolios are undoubtedly biased. On the other hand, because they account for the return on all funds

²⁴We would like to express our deepest gratitude to Francois-Serge Lhabitant who has provided us with the data needed for this test.

²⁵Dead funds are kept in the sample. This alleviates the survivorship bias problem.

²⁶Even though it is impossible to have a precise estimate of that number, we believe that the sample represents approximately 70-80% of the population of hedge funds with more than \$20 million of assets under management.

for which there is public and some private information, they are as representative as possible.

In table 11, we report for each pure index (PCA, minimum bias with and without positivity constraints, and also Kalman) the correlation with the corresponding equally-weighted portfolio, as well as the percentage of time this correlation is higher than that of competing indices.²⁷

Sub-Universe	Kalman (%)	PCA (%)	Min. Bias (%)	Min. Bias Positive Weights
Convertible Arbitrage	0.84 (67%)	0.84 (67%)	0.76 (33%)	0.78 (50%)
Emerging Markets	0.95 (29%)	0.98 (86%)	0.95 (29%)	0.86 (14%)
Equity Market Neutral	0.38 (83%)	0.41 (83%)	0.38 (67%)	0.38 (67%)
Event Driven	0.95 (75%)	0.96 (100%)	0.92 (37%)	0.92 (37%)
Fixed Income Arbitrage	0.93 (80%)	0.81 (60%)	0.88 (60%)	0.76 (60%)
Global Macro	0.71 (86%)	0.77 (86%)	0.55 (43%)	0.78 (86%)
Long/Short	0.63 (40%)	0.98 (100%)	0.41 (20%)	0.24 (20%)
Merger Arbitrage	0.85 (67%)	0.86 (67%)	0.86 (67%)	0.84 (67%)
Relative Value	0.88 (83%)	0.89 (83%)	0.61 (17%)	0.88 (83%)
Short Selling	0.69 (50%)	0.73 (50%)	0.44 (0%)	0.75 (100%)
Distressed	0.95 (100%)	0.94 (86%)	0.82 (14%)	0.91 (43%)
Fund of Funds	0.93 (75%)	0.94 (100%)	0.93 (75%)	0.83 (25%)

Table 11: Testing the Representativeness of Pure Indices. In this table we report for each pure index (Kalman, PCA, minimum bias with and without positivity constraints) the correlation with the corresponding equally-weighted portfolio, as well as the percentage of time this correlation is higher than that of competing indices.

A number of conclusions can be inferred from these results. First it appears that the PCA-based pure indices are always more correlated with the equally-weighted as-representative-as-possible portfolios than more than 50% of the competing indices they are extracted from. Overall, these results strongly suggest that PCA-based indices do achieve the improvement of representativeness they were designed for. Also PCA-based indices are more correlated with these portfolios than minimum bias and Kalman indices for all but very few strategies (fixed-income arbitrage and distressed securities). In all cases but one (emerging markets), Kalman-based pure indices are always more correlated with the equally-weighted as-representative-as-possible portfolios than more than 50% of the competing indices they are extracted from. On the other hand, indices generated from minimum bias analysis do not always outperform more

²⁷The minimum bias indices without positivity constraints have been generated using equation (6). Minimum bias indices with positivity constraints have been generated from minimum variance analysis based on the sample covariance matrix of competing indices.

than half of the competing indices they were extracted from when no further constraints are imposed on the weights. Under the condition of positive weights, the correlation with the equally-weighted portfolios increases in some cases, suggesting that a better representativeness may then be achieved.

4.3 Testing the Level of Bias in Pure Hedge Fund Indices

To measure the improvement of the pure style indices with respect to existing indices along the bias dimension, we test for their correlation with Kalman indices, which are built to be unbiased but do not allow for a portfolio interpretation.

In table 12, we report for each pure index (PCA, minimum bias with and without positivity constraints, and also Kalman) the correlation with the corresponding equally-weighted portfolio, as well as the percentage of time this correlation is higher than that of competing indices.²⁸

Sub-Universe	PCA (%)	Min. Bias (%)	Min. Bias Positive Weights
Convertible Arbitrage	1.000 (100.00%)	0.970 (83.33%)	0.934 (83.33%)
Emerging Markets	1.000 (100.00%)	0.973 (42.86%)	0.891 (0.00%)
Equity Market Neutral	0.975 (100.00%)	0.996 (100.00%)	0.975 (100.00%)
Event Driven	1.000 (100.00%)	0.949 (16.67%)	0.958 (16.67%)
Fixed Income Arbitrage	0.993 (100.00%)	0.968 (100.00%)	0.965 (100.00%)
Global Macro	0.995 (100.00%)	0.871 (71.43%)	0.872 (71.43%)
Long/Short	0.996 (100.00%)	0.565 (20.00%)	0.404 (20.00%)
Merger Arbitrage	0.999 (100.00%)	0.939 (0.00%)	0.978 (100.00%)
Relative Value	1.000 (100.00%)	0.655 (16.67%)	0.891 (50.00%)
Short Selling	1.000 (100.00%)	0.357 (0.00%)	0.994 (100.00%)
Distressed	1.000 (100.00%)	0.896 (14.29%)	0.973 (85.71%)
Fund of Funds	1.000 (100.00%)	1.000 (100.00%)	0.917 (0.00%)

Table 12: Testing the Level of Bias of Pure Indices. In this table we report for each portfolio-like pure index (PCA, minimum bias with and without positivity constraints) the correlation with the corresponding Kalman pure index, as well as the percentage of time this correlation is higher than that of competing indices.

We find that PCA-based pure indices exhibit a very high correlation with Kalman filter indices, which are supposed to be as unbiased as possible. In all cases, they are more correlated

²⁸The minimum bias indices without positivity constraints have been generated using equation (6). Minimum bias indices with positivity constraints have been generated from minimum variance analysis based on the sample covariance matrix of competing indices.

with Kalman indices than any of the competing indices. Overall, this suggests that PCA indices do not only enjoy good properties in terms of representativeness, but also in terms of lack of bias.

It is somewhat surprising to note that, while they have been designed to exhibit as little bias as possible, minimum bias indices are however less correlated than PCA indices with Kalman indices. One possible explanation is related to the well-documented sensitivity of minimum variance analysis: small changes in the variance-covariance estimates lead to significant changes in the minimum variance portfolio weights, which in turn greatly affect the performance of such portfolio. Under the condition of positive weights, the correlation with the Kalman filter index increases in some cases, suggesting that a better representativeness may then be achieved. This is in accordance with the intuition that portfolio constraints tend to reduce estimation risk. On the other hand, a fairly high level of noise is probably still present in the variance-covariance estimate, and this is probably part of the reason why the minimum bias indices are not as close to Kalman indices as we would have expected.

5 Conclusion

In this paper, we attempt to emphasize the need for a better understanding of investment style benchmarks by focusing on the alternative investment universe, where the problems are most visible. Our contribution is two-fold. First, we provide detailed evidence of strong heterogeneity in the information conveyed by competing indices. Second, we attempt to provide remedies to the problem and suggest a methodology designed to help build a “pure style index” or “index of the indices” for a given style. In particular, we explore a statistical approach to the problem of the construction of pure style indices, using Kalman filter techniques for the estimation of an unobservable factor from competing index return observations. Because it is desirable that a pure index can be regarded as a portfolio of existing indices, we also suggest a portfolio approach to the problem. In particular, we suggest using principal component analysis to extract the “best possible one-dimensional summary” of a set of competing indices (addresses the problem of lack of representativeness), and using minimum variance analysis to extract the “least biased portfolio” from a set of competing indices (addresses the problem of biases).

The relevance of this work is underlined by the recent recognition that asset allocation models and modern portfolio theory can be applied to hedge funds as well as to traditional investment vehicles (Amenc and Martellini (2001, 2002), Amenc, El Bied and Martellini (2002), Cvitanic et al. (2001)). In particular, passive strategies start to emerge in the alternative investment universe, as they have developed in the traditional stock and bond markets over the past 10 years. Very recently, a series of investment products designed to track the performance

of hedge-fund indices have actually been launched and various institutions (Credit Swiss First Boston and Zürich Capital Markets, among others) now offer index funds aiming at capturing the average return of a specific hedge fund universe.²⁹

Because we believe that there is a real value in providing investors with a more objective, or more global, assessment of the performance of alternative investment styles, we have suggested that a database of pure style indices be maintained at the EDHEC-MISYS Risk and Asset Management Research Center, and posted on a dedicated web site (www.edhec-risk.com), starting as of December 2002. We have chosen to focus on the PCA approach, which seems to yield the best representativeness/unbiasness trade-off, that we have applied to the production of pure indices for the following list of 11 styles: *convertible arbitrage*, *emerging markets*, *equity market neutral*, *event driven*, *fixed-income arbitrage*, *global macro*, *long/short*, *merger arbitrage*, *relative value*, *short selling* and *distressed securities*. The index construction methodology involves the following 3 steps.

- Step 1: Use the first 3 years of monthly returns to calibrate the model.
- Step 2: Perform a PCA analysis of competing indices for each strategy on the data used for calibration purposes, and obtain “pure style indices”.
- Step 3: These portfolios are held for 3 months, their monthly returns are recorded, and the same process is repeated.

Our results can easily be extended to traditional investment styles such as growth/value, small-cap/large-cap and a series of returns on pure indices can be constructed on the basis of the S&P/BARRA, MSCI, Dow Jones, Wilshire, Russell, etc., equity style indices.

6 References

Agarwal, V., and N. Naik, 1999, On taking the alternative route: risks, rewards and performance persistence of hedge funds, forthcoming, *Journal of Alternative Investments*.

Agarwal, V., and N. Naik, 2000, Performance evaluation of hedge funds with option-based and buy-and-hold strategies, working paper, London Business School.

Amenc, N., S. Curtis, and L. Martellini, 2001, The alpha and Omega of hedge fund performance measurement, working paper, EDHEC-MISYS Risk and Asset Management Research Center.

²⁹See the article “Critics spotlight shortcomings in index funds” in the weekly newsletter “Hedge Fund Alert”, March 6, 2002.

Amenc, N., S. El Bied, and L. Martellini, 2002, Evidence of predictability in hedge fund returns and multi-style multi-class tactical style allocation decisions, second round of submission, *Financial Analysts Journal*.

Amenc, N., and L. Martellini, 2001, It's time for asset allocation!, *Journal of Financial Transformation*, 3, 77-88, 2001.

Amenc, N., and L. Martellini, 2002, Portfolio optimization and hedge fund style allocation decisions, *Journal of Alternative Investments*, forthcoming.

Brittain, Bruce, 2001, Hedge funds and the institutional investor, *Journal of International Financial Management and Accounting*, forthcoming.

Brown J., W., Goetzmann W., and R., Ibbotson, 1999, Offshore hedge funds, survival and performance, 1989-1995, *Journal of Business*, 72.

Borger, D., 1997, Fundamental factors in equity style classification, in *The Handbook of Equity Style Management*, 2nd edition, 73-92.

Brooks, C., and H. Kat, 2001, The statistical properties of hedge fund index returns and their implications for investors, working paper, The University of Reading, ISMA Centre.

Chan, L., J. Karceski, and J. Lakonishok, 1999, On portfolio optimization: forecasting covariances and choosing the risk model, *Review of Financial Studies*, 12, 937-74.

Cvitanic, J., A. Lazrak, L. Martellini, and F. Zapatero, 2001, Optimal Investment in Alternative Portfolio Strategies, working paper, USC.

Elton, E., and M. Gruber, 1973, Estimating the dependence structure of share prices - implications for portfolio selection, *Journal of Finance*, 28, 1203-1232.

Fung, W., and D. Hsieh, 1997, Empirical characteristics of dynamic trading strategies, *Review of Financial Studies*, 10, 275-302.

Fung, W., and D. Hsieh, 2000a, The risk in hedge fund strategies: theory and evidence from trend followers, *Review of Financial Studies*, forthcoming.

Fung, W., and D. Hsieh, 2000b, Performance characteristics of hedge funds and commodity funds: natural versus spurious biases, working paper, Fuqua School of Business, Duke University.

Fung, W., and D. Hsieh, 2001a, Benchmarks of hedge fund performance: information content and measurement biases, *Financial Analyst Journal*, forthcoming.

Fung, W., and D. Hsieh, 2001b, Asset-based hedge fund styles and portfolio diversification, working paper, Fuqua School of Business, Duke University.

Gatev, E., W. Goetzmann and K. Rouwenhorst, 1999, Pairs Trading: performance of a relative value arbitrage rule, Working Paper, Yale School of Management.

Hamilton, J., 1994a, Time series analysis, Princeton University Press.

Hamilton, J., 1994b, State space models, in Chapter 50 of Handbook of Econometrics, Volume 4, North-Holland.

Harvey, A., 1989, Forecasting, structural time series models and the Kalman filter, Cambridge University Press.

Jagannathan R., and T. Ma, 2000, Covariance matrix estimation: myth and reality, working paper, Northwestern University.

Jorion, P., 1985, International portfolio diversification with estimation risk, *Journal of Business*, 58, 259-278.

Jorion, P., 1986, Bayes-Stein estimation for portfolio analysis, *Journal of Financial and Quantitative Analysis*, 21, 3, 279-292.

Ledoit, O., 1999, Improved estimation of the covariance matrix of stock returns with an application to portfolio selection, working paper, UCLA.

Litterman, R., and K. Winkelmann, 1998, Estimating covariance matrices, Goldman Sachs Risk Management Series.

Lhabitant, F., 2001, Hedge fund investing: a quantitative look inside the black box, working paper, Union Bancaire Privée.

Lobosco, A., and D. DiBartolomeo, 1997, Approximating the confidence intervals for Sharpe style weights, *Financial Analysts Journal*, July/August, 80-85.

Malkiel, B., 1995, Returns from investing in equity mutual funds: 1971 to 1991, *Journal of Finance*, 50, 2, 549-572.

Merton, R. C., 1973, An intertemporal capital asset pricing model, *Econometrica*, 41, 867-888.

Mitchell, M., and T. Pulvino, 2001, Characteristics of risk in risk arbitrage, forthcoming, *Journal of Finance*.

Park J., S. Brown, and W. Goetzmann, 1999, Performance benchmarks and survivorship bias for hedge funds and commodity trading advisors, *Hedge Fund News*, August 1999.

Reilly, F., G. Kao, and D. Wright, 1992, Alternative bond market indices, *Financial Analysts Journal*, 44-58.

Roll, R., 1977, A critique of the asset pricing theory's tests, *Journal of Financial Economics*, 4, 129-176.

Ross, S., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory*, 13, 341-475.

Sharpe, W., 1963, A Simplified model for portfolio analysis, *Management Science*, 277-293.

Sharpe, W., 1964, Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance*, 19, 425-442.

Sharpe, W., 1988, Determining a fund's effective asset mix, *Investment Management Review*, 2, 6, 56-69.

Sharpe, W., 1992, Asset allocation: management style and performance measurement, *Journal of Portfolio Management*, 18, 7-19.

Schneeweis, T., and R. Spurgin, 2000, Quantitative analysis of hedge funds and managed futures return and risk characteristics, working paper CISDM, University of Massachusetts, Amherst.

Schneeweis, T., R. Spurgin and V. Karavas, 2000, Alternative investments in the institutional portfolio, research paper, Alternative Investment Management Association (AIMA).

Schneeweis, T., H. Kazemi and G. Martin, 2001, Understanding hedge fund performance, research paper, Lehman Brothers.

A Information on Hedge Fund Strategies

- **Convertible Arbitrage.** Attempts to exploit anomalies in prices of corporate securities that are convertible into common stocks (convertible bonds, warrants and convertible preferred stocks). Convertible bonds tends to be under-priced because of market segmentation; investors discount securities that are likely to change types: if the issuer does well, the convertible bond behaves like a stock; if the issuer does poorly, the convertible bond behaves like distressed debt. Managers typically buy (or sometimes sell) these securities and then hedge part or all of the associated risks by shorting the stock. Delta neutrality is often targeted. Over-hedging is appropriate when there is concern about default as the excess short position may partially hedge against a reduction in credit quality.
- **Dedicated Short Bias.** Sells securities short in anticipation of being able to re-buy them at a future date at a lower price due to the manager's assessment of the overvaluation of the securities, or the market, or in anticipation of earnings disappointments often due to accounting irregularities, new competition, change of management, etc. Often used as a hedge to offset long-only portfolios and by those who feel the market is approaching a bearish cycle.
- **Emerging Markets.** Invests in equity or debt of emerging (less mature) markets that tend to have higher inflation and volatile growth. Short selling is not permitted in many emerging markets, and, therefore, effective hedging is often not available, although Brady debt can be partially hedged via U.S. Treasury futures and currency markets.
- **Long/Short Equity.** Invests both in long and short equity portfolios generally in the same sectors of the market. Market risk is greatly reduced, but effective stock analysis and stock picking is essential to obtaining meaningful results. Leverage may be used to enhance returns. Usually low or no correlation to the market. Sometimes uses market

index futures to hedge out systematic (market) risk. Relative benchmark index is usually T-bills.

- Equity Market Neutral. Hedge strategies that take long and short positions in such a way that the impact of the overall market is minimized. Market neutral can imply dollar neutral, beta neutral or both.
 - Dollar neutral strategy has zero net investment (i.e., equal dollar amounts in long and short positions).
 - Beta neutral strategy targets a zero total portfolio beta (i.e., the beta of the long side equals the beta of the short side). While dollar neutrality has the virtue of simplicity, beta neutrality better defines a strategy uncorrelated with the market return.

Many practitioners of market-neutral long/short equity trading balance their longs and shorts in the same sector or industry. By being sector neutral, they avoid the risk of market swings affecting some industries or sectors differently than others.

- Event Driven : corporate transactions and special situations
 - Deal Arbitrage (long/short equity securities of companies involved in corporate transactions)
 - Bankruptcy/Distressed (long undervalued securities of companies usually in financial distress)
 - Multi-strategy (deals in both deal arbitrage and bankruptcy)
- Fixed Income Arbitrage. Attempts to hedge out most interest rate risk by taking offsetting positions. May also use futures to hedge out interest rate risk.
- Global Macro. Aims to profit from changes in global economies, typically brought about by shifts in government policy that impact interest rates, in turn affecting currency, stock, and bond markets. Participates in all major markets – equities, bonds, currencies and commodities – though not always at the same time. Uses leverage and derivatives to accentuate the impact of market moves. Utilizes hedging, but the leveraged directional investments tend to have the largest impact on performance.
- Managed Futures. Opportunistically long and short multiple financial and/or non financial assets. Sub-indices include Systematic (long or short markets based on trend-following or other quantitative analysis) and Discretionary (long or short markets based on qualitative/fundamental analysis often with technical input).

B Information on Hedge Fund Indices

B.1 Main indices

There are three main providers of hedge fund indices.

B.1.1 Evaluation Associates Capital Markets (EACM)

Evaluation Associates Capital Markets offer one aggregate index, the EACM 100. This index is an equally-weighted composite of unaudited performance information provided by 100 private investment funds chosen by EACM. There are five broad strategies and 13 underlying sub-strategy styles: Relative Value (long/short equity specialists, convertible hedgers, bond hedgers, multi-strategy), Event Driven (deal arbitrageurs, bankruptcy/distressed debt specialists, multi-strategy managers), Equity Hedge Funds (domestic long biased, domestic opportunistic, global/international), Global Asset Allocators (systematic traders, discretionary managers) and Short Selling. Funds are assigned categories on the basis of how closely they match the strategy definitions. Names in the funds are not disclosed. Investment managers in the index are selected based on guidelines established by EACM. Investment manager allocations are rebalanced at the beginning of each calendar year. It was launched in 1996 with data going back to 1990.

B.1.2 Hedge Fund Research (HFR)

Hedge Fund Research provides indices for seven strategies (convertible arbitrage, equity hedge, event-driven, merger arbitrage, distressed securities), as well as an equally-weighted aggregate index based on 1,100 funds drawn from a database of 1,700 funds. Funds of funds are not included in the composite index. Funds are assigned to categories based on the descriptions in their offering memorandums. One advantage is that the indices eliminate the survivor bias problem by incorporating funds that have ceased to exist. The index was launched in 1994 with data going back to 1990. Hedge Fund Research offers a daily “investible” index to its institutional investors.

B.1.3 Credit Swiss First Boston/Tremont (CSFB/Tremont)

The CSFB/Tremont index is an index that weights component hedge funds according to the relative size of their assets. It is currently the industry’s only asset-weighted hedge fund index. In principle, asset-weighting, as opposed to equal-weighting, provides a more accurate depiction of an investment in the asset class.³⁰ The CSFB/Tremont indices cover nine strategies

³⁰However, for indices such as the Zurich Capital and EACM indices, which select from a small pool of large, established managers, equal weighting will provide the most efficient estimate of the performance of a

(convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, fixed income arbitrage, global macro, long-short equity and managed futures), and is based on 340 funds representing \$100 billion in invested capital, selected from a database, the TASS database, which tracks over 2,600 funds. A fund must have US \$10 million in assets to be included. Only funds with audited financials are included. The index is calculated on a monthly basis, and funds are re-selected on a quarterly basis as necessary. Funds are not removed from the index until they are liquidated or fail to meet the financial reporting requirements. The index was launched in 1999 with data going back to 1994.

B.2 Other Hedge Fund indices

There is also a variety of other hedge fund index providers.

B.2.1 Zürich Capital Markets

Zürich Capital Markets hedge fund indices consist of equally weighted portfolios of funds that satisfy a number of qualitative criteria for institutional investment as well as a statistical classification procedure for style classification. The indices are based on 60 funds selected from a universe of several thousand. Funds within each category must meet asset, years in existence, and statistically-based style purity constraints. Funds that meet these restrictions are asked to participate in the index; however, only those managers who agree to meet reporting constraints are included. Five strategies are available: convertible arbitrage, merger arbitrage, distressed securities, event driven and hedged equity. The indices were launched in 2001 with data going back to 1998. They are equally weighted and are rebalanced quarterly. The Zürich Hedge Fund indices are the only ones to have an independent advisory board. Investible portfolios, i.e., replicating portfolios with an approximate 2.5% tracking error, are available for each of these 5 indices with monthly liquidity ensured by Zürich Capital Markets. These indices differ from existing hedge fund indices by focusing only on those funds/managers that are 1) strategy pure in their style 2) have a two-year minimum performance track record and 3) sufficient assets under management to demonstrate organizational and managerial infrastructure, scalable strategies and the ability to raise funds from sophisticated investors.

B.2.2 Van Hedge

Van Hedge fund indices cover 12 strategies: aggressive growth, distressed securities, emerging markets, fund of funds, income, macro, market timing, market-neutral securities hedging, particular style. Using equal weights also simplifies independent confirmation of index performance, as funds frequently report assets under management with a considerable lag.

market-neutral arbitrage, opportunistic, short selling, special situations and value. The company's database, which is used in the construction of the indices, contains detailed information on over 3,400 hedge funds (2,000 U.S. and 1,400 offshore). There are no performance or size criteria and funds are assigned to categories based on their offering memorandums and interviews with the individual managers. Van Hedge Fund Advisors International provides research and advisory services to individual and institutional investors.

B.2.3 Hennessee Group

Twenty-two strategies are available: convertible arbitrage, distressed, event driven, financial equities, fixed income, growth, healthcare and bio tech, high yield, macro, market neutral, merger arbitrage, multiple arbitrage, opportunistic, regulation D, short biased, value, emerging markets, Europe, Pacific rim, Latin America, technology and telecom and media. Results are based on 450 funds including 150 in which Hennessee clients invest, from a database of 3,000 funds. Assets of \$160 billion are represented in the index. The indices were created in 1987 and first published in 1992. Hennessee Group LLC provides research and consulting to hedge fund advisors.

B.2.4 Hedgefund.net

Hedgefund.net's so-called tuna indices are an equally-weighted average of all fund returns. They cover 33 strategies: aggressive growth, convertible arbitrage, country specific, commodity trading advisor, distressed, emerging markets, energy sector, event driven, finance sector, fixed income arbitrage, fixed income, fund of funds, healthcare sector, long only, long/short hedged, macro, market neutral, market timer, opportunistic, options arbitrage, options strategies, other, other relative value, regulation D, risk arbitrage, short bias, short-term trading, small/micro-cap, special situations, statistical arbitrage, technology sector, value and VC/private equity. They are updated from a database of 1,800 hedge funds and funds of funds. The data goes back to 1979 and managers select their own categories. They are among the first to report performance results each month. Hedgefund.net is operated by Links Securities LLC, a NASD registered broker-dealer, and is owned by Links Holding and Capital Z Investments.

B.2.5 LJH Global Investments

LJH hedge fund indices are equally weighted and are calculated as the average performance of all managers for each style. They cover 16 strategies, each composed of 25 to 50 funds: Asian hedge, convertible arbitrage, distressed securities, domestic hedge, emerging markets, emerging markets fixed income, event driven, fixed income arbitrage, European hedge, global hedge,

global macro, hedge, market neutral equity, risk arbitrage, short only and technology fund. These indices are rebalanced quarterly or semiannually, depending upon the strategy. Funds must have audited statements and have passed some level of LJH due diligence. Funds are assigned categories by LJH. LJH Global Investments is a consulting and investment advisory firm.

B.2.6 Managed Account Reports (MAR)

The MAR database contains 1,300 funds and managers usually select their own categories. A composite index is not available. There are 9 categories (“medians”), some of which are combined into sub-categories (“sub-mediands”): Zürich Event-Driven Median (Distressed securities and Risk arbitrage sub-mediands), Zürich Global Emerging Median, Zürich Global International Median, Zürich Global Established Median (Global Established growth, Global Established small-cap and Global Established value sub-mediands), Zürich Global Macro Median, Zürich Market Neutral Median (Market Neutral arbitrage, Market Neutral long/short and Market Neutral mortgage-backed sub-mediands), Zürich Sector Median, Zürich Short-Sellers Median, Zürich Fund of Funds Median (Fund of Funds diversified and Fund of Funds niche sub-mediands). MAR was recently acquired by Zürich Capital Markets.

B.2.7 Altvest

Altvest hedge fund indices cover 13 strategies: capital structure arbitrage, currency trading, distressed securities, emerging markets, event driven, fund of funds, health care, long/short equity, macro, merger arbitrage, relative value, short selling and technology. Each fund is assigned to the category in which the largest percentage of its assets is invested. Index results are based on reports from more than 1,400 hedge funds in a database of 1,800 funds. The index was launched in 2000 with data going back to 1993. Altvest is owned by InvestorForce.

B.2.8 Magnum

Founded in April 1994, Magnum focuses on identifying hedge funds likely to generate superior returns and combines them into funds of hedge funds designed to deliver targeted levels of return for given levels of risk. Magnum offers 17 offshore funds of hedge funds, five feeder funds (with lower minimum investment levels) and 2 funds of hedge funds. The various indices they publish are the Magnum Aggressive Growth Fund, Magnum Bull & Bear Fund, Magnum Capital Growth Fund, Magnum e-Com Fund, Magnum Edge Fund, Magnum Europe Equity Fund, Magnum Fund, Magnum Global Equity Fund, Magnum International Equity Fund, Magnum Macro Fund, Magnum Multi Fund, Magnum Opportunity Fund, Magnum Special

Situations Fund, Magnum Tech Fund, Magnum Turbo Growth Fund and Magnum U.S. Equity Fund.

B.3 Forthcoming Indices

Two major providers of traditional indices have recently announced their intention to launch hedge fund indices.

B.3.1 S&P

Standard & Poor's has recently announced plans to create the S&P Hedge Fund Index, aimed at providing a transparent benchmark of the hedge fund asset class. The S&P Hedge Fund Index will offer investors an investable benchmark that is broadly representative of the range of major strategies that hedge funds employ. The Index will contain 40 funds divided into three sub-indices: Arbitrage, Event Driven and Tactical, which in turn represent a total of nine specific strategies. These strategies include: Macro, Equity Long/Short, Managed Futures, Special Situations, Merger Arbitrage, Distressed, Fixed Income Arbitrage, Convertible Arbitrage and Equity Market Neutral. The strategies will be equal weighted to ensure well-rounded representation of hedge fund investment approaches and to avoid over-representation of currently popular strategies. The index is expected to be launched during the third quarter of 2002.

Potential index constituents must pass a series of quantitative screening criteria to ensure that they conform to their stated strategy's return and risk characteristics. After this screening, eligible funds must agree to offer daily transparency so that their valuations can be verified by a third party, in this case Derivatives Portfolio Management (DPM), and so that the index may be computed on a daily basis. Finally, the candidate funds must pass a rigorous due diligence process conducted by Albourne Partners Ltd., a hedge fund consultant to Standard & Poor's, to ensure that they are appropriately managed, adhere to their stated strategy or style, and maintain all necessary risk controls and operational infrastructure. Index values will be posted daily to the www.spglobal.com website.

The index will be maintained by an Index Committee managed by Standard & Poor's that will meet regularly to ensure that inclusion criteria are being met and to implement necessary rebalancing.

B.3.2 Morgan Stanley Capital International (MSCI)

MSCI Hedge Fund Indices are classified according to four basic categories: directional trading, relative value, specialist credit and stock selection. Within each category, indices will be segregated based on asset class (fixed income, commodities, currencies and stocks) and geographical

region. The indices will be supported by a platform that allows subscribers to look at the data at a more detailed level (industry focus, fund size, open vs. closed, etc.). Morgan Stanley Capital International, Geneva, has formed a partnership with Financial Risk Management, New York, to produce the hedge fund indices. Financial Risk Management provides a large private hedge fund database that tracks 3,000 funds. This database will serve as the initial core of the indices.