

An EDHEC Risk and Asset Management Research Centre Publication

Asset-Liability Management Decisions in Private Banking

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Foreword

High net worth individuals (HNWIs) have numerous characteristics, in terms of assets under management and the sophistication of their requirements, that they share with institutional investors. This is a fact that has long been recognised by the marketing departments of asset management companies and private banks, who typically have special consideration for these profiles in their marketing and sales segmentation. We can therefore consider, with strong justification, that a similar approach would be appropriate for the investment management techniques employed for HNWIs and institutional investors. This is the logic that we have applied in the present research paper, which is drawn from EDHEC's ALM and Asset Management research programme.

This programme aims to apply recent research in asset-liability management for institutional investors and to improve asset management techniques, and in particular strategic allocation tools, to positively impact the performance of ALM programmes. Recent EDHEC publications in this field include 'Assessing the Impacts of IFRS and Solvency II Rules on the Financial Management of European Insurance Companies,' a major study which was jointly produced by the EDHEC Financial Analysis and Accounting Research Centre and the EDHEC Risk and Asset Management Research Centre; an academic analysis of Liability-Driven Investing by Lionel Martellini, 'The Theory of LDI,' which was published in the May 2006 issue of *Life and Pensions*; and a paper entitled 'The Benefits of Hedge Funds in Asset Liability Management', by Lionel Martellini and Volker Ziemann, which appeared in the *Alternative Investment Quarterly* in 2005.

The current paper discusses the sources of added-value in private wealth management, and argues through a series of illustrations that asset-liability management is the natural approach for the design of truly client-driven services in private banking.

We would like to extend our sincere thanks to our partners at Pictet & Cie, who lent considerable support to this project. We hope you will find the study both interesting and informative.



Noël Amenc, PhD,
Director of the EDHEC Risk
and Asset Management Research Centre

Executive Summary



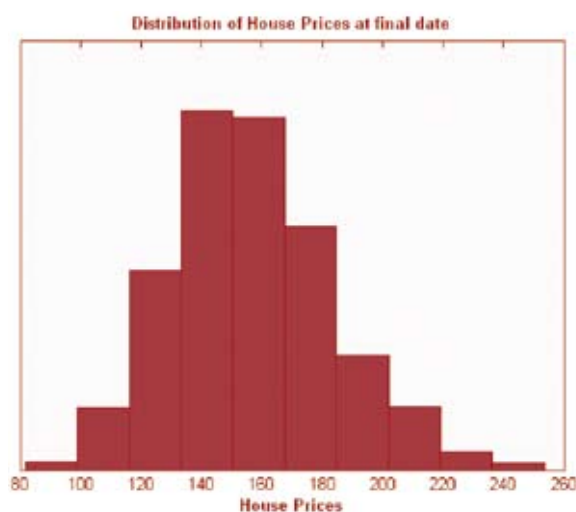
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The private wealth management industry has now become a very significant industry due to continuing strong economic growth in specific regions of the world. This increase is currently driving a larger wealth management market creating greater opportunities for wealth advisors to leverage new technology with a view to acquiring new clients and boosting profits. As a result, competition among wealth advisory firms is increasing to find ways to improve existing client relationships and provide new tools to improve advisor efficiency. Current private banking tools are typically tax and estate planning geared towards one specific country and financial simulation software, relying on single period mean-variance optimization of the asset portfolio. These tools suffer from significant limitations and cannot satisfy the needs of a sophisticated clientele.

While some industry players have recently developed planning tools that model assets in a multi-period stochastic framework, asset-liability matching for individuals remains an area for exploration. This paper adapts Asset-Liability Management (ALM) techniques developed for institutional investors to the context of private banking customers. Asset-Liability Management (ALM) denotes the adaptation of the portfolio management process in order to handle the presence of various constraints relating to the commitments of an investor's liabilities. We argue that portfolio optimization techniques used by institutional investors, e.g., pension funds, could usefully be transposed to the context of private wealth management because they have been engineered precisely to allow for the incorporation of an investor's specific constraints, objectives and horizon in the portfolio construction process. Taking investor's liability constraints and specific objectives into account actually has a dramatic impact on asset allocation decisions. For example, clients who wish to maintain a given level of expenses for their retirement years will expect the investment process performed on their current wealth to be able to generate cash-flows sufficient to meet their consumption needs, which justifies a focus

on inflation hedging that is not typically involved in a standard asset management solution.

As an illustration, we consider the situation of an investor who wishes to invest fixed annual contributions (€x) for a future expenditure, e.g., the purchase of a house in 5 years, for which the current value is normalized at €100. We introduce an explicit model for the dynamics of real estate prices and the exhibit below shows the impact of real estate price uncertainty on the value of the €100 payment scheduled to be paid in 5 years from now. As we can see, real estate price risk is significant, with a nominal amount to be secured equal to €156.59 on average and a €27.18 standard deviation.

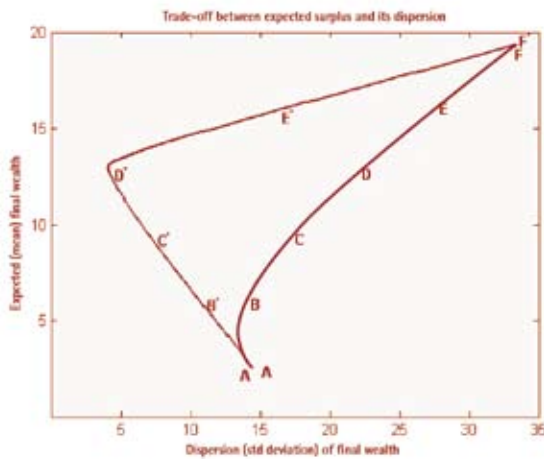


Distribution of house prices at final date; mean value = 156.59; standard deviation = 27.18.

In practical terms, the goal is to generate a lump sum payment at horizon date (5 years). It is not possible in general to find a perfect liability-matching portfolio. The existence of a perfect liability-matching portfolio is actually only ensured on the following two conditions: the investor must be able to borrow against future income and invest the present value of the future contributions at the initial date; and there must be an investment vehicle (e.g., REITS) with a payoff which is directly related to real estate price uncertainty. We test two different situations: an opportunity set containing stocks, bonds and TIPS and an opportunity set containing stocks, bonds, TIPS and real estate (modelled as

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an investment that will pay the compounded return on real estate). To generate comparable portfolios, we looked at the improvement in surplus volatility for a given level of expected surplus.



ALM Efficient Frontiers without Real Estate (A, B, C, D, E, F) and with Real Estate (A', B', C', D', E', F)

cases), the surplus volatility at the optimal level reaches 21.95 when the opportunity set does not contain a real estate asset, while it merely amounts to 4.25, a dramatic risk reduction, when the real estate asset is included. Again this signals the relevance of an ALM approach to private wealth management: it is only by trying to fit the client liability constraints that truly optimal solutions can be proposed.

In the same vein, we also consider a number of other illustrations that are typical of standard private wealth management problems and show that optimal solutions are strongly affected by the presence of liability constraints. In particular, we focus on various pension-related objectives and consider an individual who is either already retired or still employed, and who seeks to ensure a stream of inflation-protected fixed payments,

Portfolio	Weights				Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
	Stocks	Bonds	TIPS	Real Estate							
A	0%	0%	100%	-	2.46	14.64	41.5%	11.40	(12.5%)	19.07	4.7%
B	10%	90%	0%	-	5.84	13.96	31.5%	10.32	(11.3%)	17.85	10.8%
C	33%	67%	0%	-	9.22	17.16	28.9%	11.03	(12.1%)	16.70	16.5%
D	55%	45%	0%	-	12.60	21.95	28.5%	12.93	(14.2%)	15.62	21.9%
E	78%	22%	0%	-	15.98	27.51	28.0%	15.92	(17.5%)	14.60	27.0%
F	100%	0%	0%	-	19.36	33.46	27.9%	19.09	(21.0%)	13.63	31.8%
A'	0%	0%	100%	0%	2.46	14.64	41.5%	11.40	(12.5%)	19.07	4.7%
B'	0%	0%	68%	32%	5.84	10.87	28.6%	7.44	(8.2%)	17.85	10.7%
C'	0%	0%	37%	63%	9.22	7.27	10.6%	4.57	(5.0%)	16.70	16.5%
D'	0%	0%	5%	95%	12.60	4.25	0.6%	2.07	(2.3%)	15.61	21.9%
E'	46%	0%	0%	54%	15.98	16.43	16.7%	7.36	(8.1%)	14.59	27.0%
F'	100%	0%	0%	0%	19.36	33.46	27.9%	19.09	(21.0%)	13.63	31.8%

Allocation strategies and risk-return indicators; all values are given as present values at initial date based on a €20 annual contribution for 5 years, the present value of which amounts to €90.91 given our choice of parameter values. Expected shortfall is expressed as a percentage of this value. The relative contribution saving corresponds to the increase (in percentage) in initial investment that should have taken place with a given strategy so as to generate an expected surplus equal to zero.

The graph shows the efficient frontier in both cases, while risk-return indicators are reported in the table. As expected, the presence of assets allowing investors to span real estate price uncertainty proves to be a key element in improving the efficient frontiers obtained from an ALM perspective. Looking for example at portfolio D and D' in the table, we see that for the same level of expected surplus (12.60 in both

based either on a lump-sum contribution or a series of annual contributions. We also introduce a variety of bequest-related objectives.

In conclusion, we argue that it is not the performance of a particular fund nor that of a given asset class (including commodities or hedge funds) that will be the determining factor in the ability of private wealth management to meet

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investors' expectations. What will prove to be the decisive factor is the private wealth manager's ability to design an asset allocation solution that is a function of the kinds of particular risks to which the investor is exposed, as opposed to the market as a whole. Hence, an absolute return fund, often perceived as a natural choice in the context of private wealth management, would not be a satisfactory response to the needs of a client facing long-term inflation risk, where the concern is capital preservation in real, as opposed to nominal, terms. Similarly, a client whose objective would be related to the acquisition of a property would accept low and even negative returns in situations when real estate prices significantly decrease, but will not satisfy himself or herself with relatively high returns if such high returns are not sufficient to meet a dramatic increase in real estate prices. In such circumstances, a long-term investment in stocks and bonds with a performance weakly correlated with real estate prices would not be the right investment solution.

In other words, the success or failure of the satisfaction of the client's long-term objectives is fundamentally dependent on an ALM exercise that aims to determine the proper strategic inter-classes allocation as a function of the client's specific objectives and constraints. Asset management should only come next as a response to the implementation constraints of the ALM decisions. On the one hand, it is meant to deliver/enhance the risk and return parameters supporting the ALM analysis for each asset class. On the other hand, it can also allow for the management of short-term constraints, such as capital preservation at a given confidence level, which are not necessarily taken into account by an ALM optimization exercise, which by nature focuses on long-term objectives.

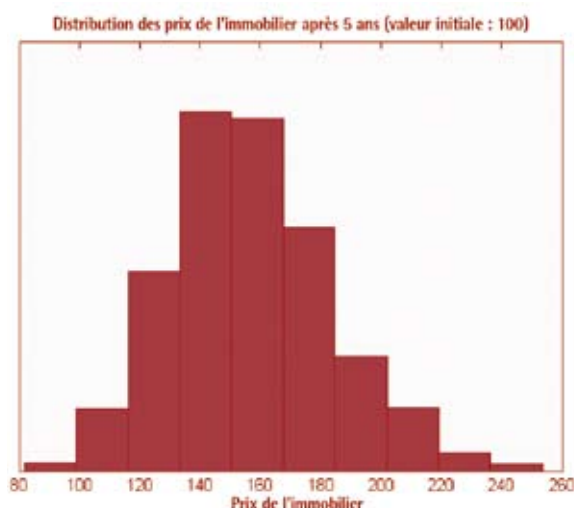
Résumé

Grâce à une croissance économique soutenue dans plusieurs régions du monde, l'industrie de la gestion privée s'est octroyée une place considérable dans le paysage financier mondial. Cette accélération sert actuellement de moteur dans un marché croissant de gestion de patrimoine, créant ainsi la possibilité pour les conseillers de ce domaine d'attirer de nouveaux clients et d'augmenter leurs bénéfices. En conséquence, la concurrence entre les sociétés de conseil en gestion de patrimoine est en constante progression dans le but de trouver des moyens d'améliorer les relations clients existantes et de se procurer de nouveaux outils afin d'améliorer l'efficacité de leurs conseils. Les expertises actuelles en gestion privée sont typiquement celles de la fiscalité et de la gestion des héritages propres à un pays particulier, ainsi que des progiciels de simulation financière, souvent basés sur une optimisation moyenne-variance d'un portefeuille d'actifs dans un cadre statique. Ces outils souffrent de limitations importantes et ne peuvent répondre aux besoins d'une clientèle sophistiquée.

Si quelques acteurs de l'industrie ont récemment développé des outils prévisionnels qui modélisent les actifs dans un cadre stochastique multi-périodes, la gestion actif-passif pour les particuliers reste un domaine à explorer. Ce document adapte les techniques de gestion actif-passif (GAP ou ALM en anglais), développées pour les investisseurs institutionnels, au contexte des clients privés. L'Asset-Liability Management (ALM) désigne l'adaptation du processus de gestion de portefeuille afin de prendre en compte la présence de diverses contraintes liées aux engagements que représente le passif d'un investisseur. Nous pensons qu'il est intéressant de transférer les techniques d'optimisation de portefeuille utilisées par les investisseurs institutionnels, par exemple les fonds de pension, au contexte de la gestion privée, parce que celles-ci ont précisément été conçues afin de permettre l'intégration des contraintes, des objectifs et des horizons de l'investisseur dans le processus de construction de portefeuille. En fait, la prise en compte des contraintes de passif et des objectifs

précis de l'investisseur a priori impacte de façon significative les décisions d'allocation d'actifs. Par exemple, les clients qui souhaitent garder un niveau donné de dépenses durant leurs années de retraite s'attendent à ce que le processus d'investissement appliqué à leur patrimoine actuel puisse générer des flux de trésorerie suffisants pour satisfaire à leurs besoins de consommation, ce qui justifie l'intégration d'une couverture par rapport à l'inflation, qui ne fait pas typiquement partie d'une solution de gestion d'actifs standard.

Afin d'illustrer ce concept, nous examinons la situation d'un investisseur qui souhaite allouer des contributions annuelles fixes ($x\text{€}$) pour une dépense future, par exemple l'achat d'une maison dans 5 ans, la valeur actuelle de celle-ci étant normalisée à 100€. Nous introduisons un modèle explicite pour la dynamique des prix de l'immobilier et le graphique ci-dessous montre l'impact de l'incertitude des prix de l'immobilier sur la valeur du versement de 100€ prévu pour dans 5 ans. Comme nous pouvons le constater, le risque de prix de l'immobilier est important, avec une valeur nominale de 156,59€ en moyenne à obtenir et un écart type de 27,18€.

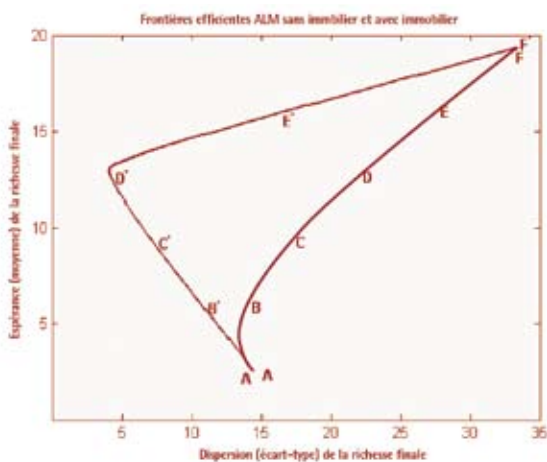


Distribution des prix de maison à la date finale ; valeur moyenne = 156,59 ; écart type = 27,18.

En termes pratiques, le but est de générer le versement d'une somme forfaitaire indexée au prix de l'immobilier à la date d'horizon (5 ans). Il n'est pas toujours possible de trouver un portefeuille parfaitement adossé au passif.

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En effet, dans cet exemple l'existence d'un portefeuille parfaitement adossé au passif dépendrait des deux conditions suivantes : l'investisseur peut emprunter sur la base des ses revenus futurs et peut investir la valeur actuelle de ses futures contributions à la date initiale ; et il existe un support d'investissement (par exemple REITS) avec un rendement directement lié à l'incertitude du prix de l'immobilier. Nous testons deux situations différentes, un exercice d'allocation avec un menu de classes d'actifs contenant des actions, des obligations et des obligations d'Etat indexées sur l'inflation (TIPS), et un exercice d'allocation avec un menu de classes d'actifs contenant des actions, des obligations, des TIPS et de l'immobilier (modélisé comme un investissement qui réalisera le rendement composé de l'immobilier). Afin de générer des portefeuilles comparables, nous avons regardé l'amélioration de la volatilité de l'excédent pour un niveau donné d'excédent escompté.



Frontières efficaces ALM sans immobilier (A, B, C, D, E, F) et avec immobilier (A', B', C', D', E', F')

Le graphique montre la frontière efficace dans les deux cas, et les indicateurs de risque et de rendement sont renseignés dans le tableau ci-contre. Comme on aurait pu s'y attendre, la présence d'actifs permettant aux investisseurs de couvrir l'incertitude des prix de l'immobilier est un élément clé dans l'amélioration des frontières efficaces obtenues dans une optique ALM. En regardant par exemple les portefeuilles D et D' dans le tableau, nous constatons que pour un même niveau d'excédent escompté (12,60 dans

les deux cas), la volatilité de l'excédent au niveau optimal atteint 21,95 quand l'immobilier ne fait pas partie du menu des classes d'actifs, alors qu'elle atteint 4,25, une réduction de risque très importante, quand l'actif immobilier est compris. Ceci témoigne à nouveau de la pertinence d'une approche ALM dans la gestion privée : ce n'est qu'en essayant de garantir l'adéquation des contraintes de passif du client que des solutions véritablement optimales peuvent être proposées.

Dans la même lignée, nous développons plusieurs autres expériences qui sont typiques des problématiques de gestion privée et nous montrons que les solutions optimales sont fortement impactées par la présence des contraintes de passif. Nous nous concentrons notamment sur différents objectifs liés à la retraite, et nous considérons le cas d'un individu qui est déjà retraité ou bien toujours salarié, et qui cherche à garantir un flux de versements fixes protégés contre l'inflation, à partir soit d'une contribution forfaitaire soit d'une série de contributions annuelles. Nous introduisons également divers objectifs relatifs à des legs.

En conclusion, nous avançons l'idée qu'il n'est pas tant la performance d'un fonds en particulier ni même d'une classe d'actifs donnée (y compris les matières premières ou les hedge funds) qui sera le facteur déterminant dans la capacité de la gestion privée à répondre aux attentes des investisseurs. Ce qui sera décisif est la capacité du gérant privé à concevoir une solution d'allocation d'actifs en fonction des risques précis auxquels l'investisseur, plutôt que le marché dans son ensemble, est exposé. Ainsi, un fonds de rendement absolu, souvent perçu comme un choix naturel dans le contexte de la gestion privée, ne fournira pas une réponse satisfaisante aux besoins d'un client qui doit faire face à un risque d'inflation sur le long terme, auquel cas le souci sera la préservation du capital en termes réels plutôt que nominaux. De même, un client dont l'objectif est lié à l'acquisition d'une propriété accepterait des rendements bas ou même négatifs dans des

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Portefeuille	Allocation Obligations d'état indexées sur l'inflation				Excédent escompté	Volatilité de l'excédent	Prob(S<0)	Déficit escompté		Contribution nominale nécessaire p.a.	Economie de contribution relative p.a.
	Actions	Obligations	(TIPS)	Immobilier							
A	0%	0%	100%	-	2,46	14,64	41,5%	11,40	(12,5%)	19,07	4,7%
B	10%	90%	0%	-	5,84	13,96	31,5%	10,32	(11,3%)	17,85	10,8%
C	33%	67%	0%	-	9,22	17,16	28,9%	11,03	(12,1%)	16,70	16,5%
D	55%	45%	0%	-	12,60	21,95	28,5%	12,93	(14,2%)	15,62	21,9%
E	78%	22%	0%	-	15,98	27,51	28,0%	15,92	(17,5%)	14,60	27,0%
F	100%	0%	0%	-	19,36	33,46	27,9%	19,09	(21,0%)	13,63	31,8%
A'	0%	0%	100%	0%	2,46	14,64	41,5%	11,40	(12,5%)	19,07	4,7%
B'	0%	0%	68%	32%	5,84	10,87	28,6%	7,44	(8,2%)	17,85	10,7%
C'	0%	0%	37%	63%	9,22	7,27	10,6%	4,57	(5,0%)	16,70	16,5%
D'	0%	0%	5%	95%	12,60	4,25	0,6%	2,07	(2,3%)	15,61	21,9%
E'	46%	0%	0%	54%	15,98	16,43	16,7%	7,36	(8,1%)	14,59	27,0%
F'	100%	0%	0%	0%	19,36	33,46	27,9%	19,09	(21,0%)	13,63	31,8%

Stratégies d'allocation et indicateurs de risque et de rendement ; toutes les valeurs sont présentées comme des valeurs actuelles à la date initiale à partir d'une contribution annuelle de 20€ pendant 5 ans. La valeur actuelle de celle-ci est égale à 90,91€ étant donné notre choix de valeurs de paramètres. Le déficit escompté est exprimé comme un pourcentage de cette valeur. L'économie de contribution relative correspond à l'augmentation (en pourcentage) de l'investissement initial qui aurait dû être appliquée afin de générer un excédent escompté égal à zéro.

situations où les prix de l'immobilier sont en nette diminution, mais ne se contentera pas de rendements relativement élevés si ceux-ci ne lui permettent pas de faire face à des hausses sensibles de prix de l'immobilier. Dans de telles circonstances, un investissement sur le long terme dans des actions et des obligations avec une performance faiblement corrélée avec les prix de l'immobilier ne serait pas la bonne solution d'investissement.

En d'autres termes, la capacité de répondre aux objectifs à long terme du client dépend fondamentalement de l'exercice d'ALM qui vise à déterminer la bonne allocation stratégique entre les classes en fonction des objectifs et contraintes spécifiques du client. La gestion d'actifs doit seulement suivre en réponse aux contraintes de mise en œuvre des décisions d'ALM. D'une part, cela doit permettre d'améliorer des paramètres de risque et de rendement soutenant l'analyse ALM pour chaque classe d'actifs. D'autre part, le processus de gestion d'actifs peut permettre la gestion de contraintes à court terme, telle que la préservation du capital à un seuil de confiance donné, qui ne sont pas forcément prises en compte par un exercice d'optimisation d'ALM, ce dernier se focalisant par nature sur les objectifs à long terme.

Asset-Liability Management Decisions in Private Banking



1. Introduction

The private wealth management industry has now become a very significant industry due to continuing strong economic growth in specific regions of the world. According to a recent survey, the wealth of high net worth individuals (HNWIs), people with net financial assets of at least US\$1 million excluding their primary residence and consumables, climbed to US\$33.3 trillion in 2005, which represents an annual rate of 8.0% over the last decade¹. According to the same survey, the number of HNWIs grew by 6.5 percent over 2004, to 8.7 million, and the number of Ultra-HNWIs – those who have financial assets of more than US\$30 million – grew by 10.2%, to 85,400 in 2005.

This increase is currently driving a larger wealth management market creating greater opportunities for wealth advisors to leverage new technology to acquire new clients and increase profits. As a result, competition among wealth advisory firms is increasing to find ways to improve existing client relationships and provide new tools to improve advisor effectiveness. Current private banking tools are typically tax and estate planning geared towards one specific country and financial simulation software, relying on single period mean-variance optimization of an asset portfolio. These tools suffer from significant limitations and cannot satisfy the needs of a sophisticated clientele.

Firstly, single country tax planning tools are of little relevance to high net worth individuals operating offshore or across multiple tax jurisdictions. Secondly, financial simulation software relying on single period mean-variance optimization of asset portfolios cannot yield a proper strategic allocation for at least two reasons. On the one hand, optimization parameters (especially expected returns) are defined as constants, a practice which is contradicted by empirical observation and does not allow for the length of the investment horizon. On the other hand, and most importantly perhaps, liability constraints and risk factors affecting them, such as inflation-risk on targeted spending, are neither modeled nor explicitly taken into account in the portfolio construction process.

The process involved in dealing with a private client typically leads to a detailed analysis of the client's objectives, constraints, as well as risk-aversion parameters (sometimes on the basis of rather sophisticated approaches). Yet it is striking that once this information has been collected, and sometimes formalized, very little is done in terms of customizing a portfolio solution to the benefit of the specific needs of the client. Typically, the approach consists in providing several profiles expressed in terms of volatility or drawdown levels, with in some instances a distinction in how the capital will eventually be accessed (annuities or lump sum payment), but the client's objectives, constraints and associated specific risk factors are simply not taken into account in the design of the optimal allocation.

While some industry players have recently developed planning tools that model assets in a multi-period stochastic framework, asset-liability matching for individuals remains an area for exploration. The objective of this paper is to adapt Asset-Liability Management (ALM) techniques developed for institutional investors to the context of private banking customers. Asset-Liability Management denotes the adaptation of the portfolio management process in order to handle the presence of various constraints relating to the commitments that represent the liabilities of an investor. It should be emphasized at this stage that the definition of "liabilities" we use in what follows is rather broad and encompasses any commitment, whether external or self-imposed, that a private investor is facing. For example, an investor committed to a real estate acquisition will perceive such an expense as a future commitment for which money shall be available when needed. Similarly, clients who desire to maintain a given level of expenses for their retirement years will expect the investment process performed on their current wealth to be able to generate sufficient cash-flows to meet their needs. In what follows, we argue that portfolio optimization techniques used by institutional investors, e.g., pension funds, could usefully be transposed to the context of private wealth management because

1 - Merrill Lynch Et
Caggemini World Wealth
Report 2006 available at
[www.us.caggemini.com/
worldwealthreport06](http://www.us.caggemini.com/worldwealthreport06)

1. Introduction

they have been engineered precisely to allow for the incorporation of an investor's specific constraints, objectives and horizon (all of which can be broadly summarized in terms of "liability constraints") in the portfolio construction process.

The rest of the paper is organized as follows. In section 2, we discuss the sources of added-value in private wealth management, and argue that asset-liability management is the natural approach for the design of truly client-driven services in private banking. In section 3, we provide a brief history of ALM techniques, with a specific emphasis on the benefits and weaknesses of competing approaches, both from a practical and a conceptual standpoint. In section 4, we present a series of illustrations of the usefulness of asset-liability management techniques in a private banking context. A conclusion can be found in section 5, while technical details are relegated to a dedicated appendix.

2. Asset-Liability Management as a Truly Client-Driven Approach to Private Banking

2.1. Sources of Added-Value in Wealth Management

It has often been argued that the proximity to clients is the main "raison d'être" and a key source of competitive advantage for private wealth management. Building on this proximity, private bankers should be ideally placed to better account for their clients' specific liability constraints when engineering an investment solution for them. In other words, asset-liability management is the true source of added-value in private wealth management.

Most private bankers actually implicitly promote an ALM approach to wealth management. In particular, they claim to account for the client's goals and constraints. The technical tools involved, however, are often non-existent or ill-adapted. As a result, current practice in addressing clients' needs is mostly a failure, with only a very limited fraction of private bankers actually designing portfolios consistent with the clients' needs. While the client is routinely asked all kinds of questions regarding current situation, goals, preferences, constraints, etc., the resulting service and product offering mostly boil down to a rather basic classification in terms of risk profiles.

In principle, several situations exist, corresponding to varying levels of sophistication and consideration of clients' needs.

The first case is when private bankers simply do not use any portfolio construction tool. Since the solutions they then offer do not take into account clients' objectives, risk-aversion or constraints, this is simply not acceptable. A slightly more satisfying situation involves private wealth management perceived as a pure asset-management exercise. The solution consists of the optimal design of different portfolios with different risk profiles, where the clients' goals and constraints are not taken into account. A third situation involves testing for the impact of asset allocation decisions in terms of compliance with respect to the clients' liability constraints.

For example, some private bankers use a model to test probability of a shortfall at horizon. The optimal asset portfolio, however, is designed independently of clients' needs. Finally, the last, fully satisfactory situation, involves the incorporation of the client's full profile in portfolio construction. Only this can ensure that clients' needs are properly addressed. This requires the development of proper portfolio construction tools similar to the ones used in institutional money management. Explaining how asset-liability management techniques used in the context of institutional money management can/should be transposed to private wealth management is precisely the focus of this paper.

2.2. A Typology of Clients' Profiles

Broadly speaking, there are at least four dimensions in a client profile.

- Objective profile
- Time-horizon profile
- Constraints and risk-aversion profiles
- Contribution profile

Each of these dimensions is related to the definition of a client's liabilities. The first dimension, the objective profile, is related to the particular type of liability a client is facing. Examples are pension needs, real estate acquisition, paying for children's education, etc. The second dimension, the time-horizon profile, is of significance since it can be shown that, unless under very specific assumptions, optimal portfolio allocations depend on the risk-horizon (see Merton (1971) for a general theory of dynamic asset allocation decisions). It is often the case that the actual horizon is long, sometimes with intermediate, short-term constraints or goals. The third dimension, the constraints and risk-aversion profiles, corresponds to a necessary enlargement of typical clientele segmentation, which often boils down to subjective classification in terms of risk tolerance. A better understanding can be obtained from the perspective in terms of risk constraints. The fourth dimension, the

2. Asset-Liability Management as a Truly Client-Driven Approach to Private Banking

contribution profile, is as important in private wealth management as it is in institutional money management. Just as a pension fund with a healthy sponsor company should not hold the same allocation as another pension fund with the same funding status but a more financially constrained sponsor company, two clients with the same objectives but different contribution schedules should not be proposed for the same portfolios. In other words, the risk level in a portfolio allocation does not solely depend on the (private or institutional) investor's risk-aversion but also on the margin for error (see for example Martellini (2006b) for an explicit model of asset-liability management where optimal portfolio allocation exhibits an explicit dependence in the margin for error measured in terms of distance with respect to a minimum funding ratio requirement). In particular, as we will see, a key distinction exists between fixed and flexible contribution schedules.

In what follows, we present an example of possible segmentation of a private wealth management clientele.

1) Objective profiles

- a/ Pension-related objectives
 - Client is retired and has already contributed
 - Client is not retired yet and will contribute in the future
- b/ Expenditure-related objectives
 - Accumulate for future expenditure based on a single contribution
 - Accumulate for future expenditure based on a schedule of contributions

2) Time-horizon profiles: 0-3 years, 3-5 years, 5-10 years, 10-20 years, 20-40 years

3) Constraints and risk-aversion profiles

- a/ Short-term constraints: no annual protection, annual capital protection up to a 100%, 95%, 90%, 85% confidence level
- b/ Long-term constraints: no protection at horizon, full protection at horizon in real or nominal terms, probability of not reaching the goal kept below 10%, 15%, 20%, 25%, 50%

4) Contribution profiles

- a/ Fixed contribution schedule
- b/ Uncertain contribution schedule
 - Presence of a schedule of minimum contributions
 - Absence of a schedule of minimum contributions

A simplified version could be proposed to a different, less affluent clientele with less room for flexibility.

1) Objective profiles

- a/ Pension-related objectives
 - Client is retired and has already contributed
 - Client is not retired yet and will contribute in the future
- b/ Expenditure-related objectives
 - Accumulate for future expenditure based on a single contribution
 - Accumulate for future expenditure based on a schedule of contributions

2) Time-horizon profiles: 0-3 years, 3-10 years, 10-30 years

3) Constraints and risk-aversion profiles

- a/ Short-term constraints: no annual protection, annual capital protection up to a 95% confidence level
- b/ Long-term constraints: no protection at horizon, full protection at horizon in real or nominal terms, probability of not reaching the goal kept below 10%

4) Contribution profiles

- a/ Fixed contribution schedule

In what follows, we provide an introduction to the sophisticated ALM techniques that have been used in institutional money management to design an offering of products and services that really account for investors' goals and constraints. We then provide illustrations of their usefulness in private wealth management.

3. A Brief History of ALM Techniques

Recent difficulties have drawn attention to the risk management practices of institutional investors in general and defined benefit pension plans in particular. What has been labeled “a perfect storm” of adverse market conditions at the turn of the millennium has devastated many corporate defined benefit pension plans. Negative equity market returns have eroded plan assets at the same time as declining interest rates have increased mark-to-market value of benefit obligations and contributions. In extreme cases, this has left corporate pension plans with funding gaps as large as or larger than the market capitalization of the plan sponsor. That institutional investors in general and pension funds in particular have been so dramatically affected by recent market downturns has emphasized the weakness of risk management practices. In particular, it has been argued that the kinds of asset allocation strategies implemented in practice, which used to be heavily skewed towards equities in the absence of any protection with respect to their downside risk, were not consistent with a sound liability risk management process.

In this context, a renewed interest in asset-liability management techniques has surfaced in institutional money management. New approaches that are referred to as liability driven investment (“LDI”) solutions have also been introduced following recent changes in accounting standards and regulations that have led to an increased focus on liability risk management. In what follows, we will provide a brief review of standard asset allocation techniques used in ALM, which can be classified into several categories.

3.1. Cash-Flow Matching and Immunization

A first approach called cash-flow matching involves ensuring a perfect static match between the cash flows from the portfolio of assets and the commitments in the liabilities. Let us assume for example that a pension fund has a commitment to pay out a monthly pension to

a retired person. Leaving aside the complexity relating to the uncertain life expectancy of the retiree, the structure of the liabilities is defined simply as a series of cash outflows to be paid, the real value of which is known today, but for which the nominal value is typically matched with an inflation index. It is possible in theory to construct a portfolio of assets whose future cash flows will be identical to this structure of commitments. To do so, assuming that securities of that kind exist on the market, would involve purchasing inflation-linked zero-coupon bonds with a maturity corresponding to the dates on which the monthly pension installments are paid out, with amounts that are proportional to the amount of real commitments. The technique can also be implemented in a synthetic way using interest rates and inflation swaps.

This technique, which provides the advantage of simplicity and allows, in theory, for perfect risk management, nevertheless presents a number of limitations. First of all, it will generally be impossible to find inflation-linked securities whose maturity corresponds exactly to the liability commitments. Moreover, most of those securities pay out coupons, which leads to the problem of reinvesting the coupons. To the extent that perfect matching is not possible, there is a technique called immunization, which allows the residual interest rate risk created by the imperfect match between the assets and liabilities to be managed in a dynamic way. This interest rate risk management technique can be extended beyond a simple duration-based approach to fairly general contexts, including for example hedging larger changes in interest rates (through the introduction of a convexity adjustment), hedging non-parallel shifts in the yield curve (see for example Fabozzi, Martellini and Priaulet (2005)), or simultaneous management of interest rate risk and inflation risk (Siegel and Waring (2004)). It should be noted, however, that this technique is difficult to adapt to hedging non-linear risks related to the presence of options hidden in the liability structures, and/or to hedging non-interest rate related risks in liability structures.

3. A Brief History of ALM Techniques

Another, probably more important, disadvantage of the cash-flow matching technique (or of the approximate matching version represented by the immunization approach) is that it represents a positioning that is extreme and not necessarily optimal for the investor in the risk/return space. In fact it can be said that the cash-flow matching approach in asset-liability management is the equivalent of investing in the risk-free asset in an asset management context. It allows for perfect management of the risks, namely a capital guarantee in the passive management framework, and a guarantee that the liability constraints are respected in the ALM framework. However, the lack of return, related to the absence of risk premia, makes this approach very costly, which leads to an unattractive level of contribution to the assets.

3.2. Surplus Optimization

In a concern to improve the profitability of the assets, and therefore reduce the level of contributions, it is necessary to introduce asset classes (stocks, government bonds and corporate bonds) that are not perfectly correlated with the liabilities into the strategic allocation. It will then involve finding the best possible compromise between the risk (relative to the liability constraints) thereby taken on, and the excess return that the investor can hope to obtain through the exposure to rewarded risk factors.

Different techniques are then used to optimize the surplus, i.e., the excess value of the assets compared to the liabilities, in a risk/return space. In particular, it is useful to turn to stochastic models that allow for a representation of the uncertainty relating to a set of risk factors that impact upon the liabilities. These can be financial risks (inflation, interest rate, stocks) or non-financial risks (demographic ones in particular).

Two key steps are involved in surplus optimization. The first step consists in using a mathematical model for generating stochastic scenarios for all risk factors affecting assets and liabilities

(typically, interest rates, inflation, stock prices, real estate, etc.). Models are chosen so as to represent actual as well as possible behaviors and parameters are chosen so as to be consistent with long-term estimates. The next step involves using an optimization technique to find the set of optimal portfolios.

In terms of stochastic scenario simulation, one typically distinguishes between three main risk factors affecting asset and liability values: interest rate risk (or, more accurately, interest rate risks, since there is more than one risk factor affecting changes in the shape of the yield curve), inflation risk, and stock price risk. When real estate is used as an ALM asset class, then an additional model for the dynamics of real estate prices should be added. In the illustrations that follow in a later section, we have used a set of standard stochastic models for these risk factors, including as key features a two-factor mean-reverting process for real interest rates, a one-factor mean-reverting process for inflation rates and a Markov regime switching model for excess return on equity (excess return)². Our model is borrowed from Ahlgrim, D'Arcy and Gorvett (2004) and can be written as³:

$$\begin{aligned} dr_t &= a_r (l_t - r_t) dt + \sigma_r dW_t^r \\ dl_t &= a_l (b_l - l_t) dt + \sigma_l dW_t^l \\ d\pi_t &= a_\pi (b_\pi - \pi_t) dt + \sigma_\pi dW_t^\pi \\ dS_t/S_t &= (r_t + \pi_t) dt + b_x^s dt + \sigma_x^s dW_t^x \end{aligned}$$

Here r_t (respectively, π_t) is the real short-term rate (respectively, inflation rate) at date t , a_r (respectively, a_π) is the speed of mean reversion of the short-term rate (respectively, inflation rate), l_t (respectively, b_π) is the long-term mean value of the short-term rate (respectively, inflation rate), and σ_r (respectively, σ_π) is the volatility of the short-rate (respectively, inflation rate). This model assumes a particular two-factor process for the real rate so as to account for the non-perfect correlation between bonds of different maturities. In particular, it assumes that the long-term mean value l_t of the short-term rate is also stochastically time-varying, with a speed of

2 - A mean-reverting model for real estate prices has also been used for the illustrations where real estate was introduced.

3 - Other competing models can of course be used in ALM simulations and optimization, but they are mostly consistent in spirit with this particular model, which we have chosen because it represents a standard example of a state-of-the-art ALM model which is made available for public use by the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA) (see reference list for exact references of the paper and a web site where the paper can be downloaded).

3. A Brief History of ALM Techniques

mean reversion denoted by a_r , a long-term mean value denoted by b_r and a volatility denoted by σ_r . By contrast the long-term mean value of the inflation rate is assumed to be a constant. Here W^r , W^l and W^π are three (correlated) standard Brownian motions representing uncertainty driving the three risk-factors. Besides, a Markov-regime switching model is assumed for equity returns, with b_x as the (state-dependent) excess expected return (over the nominal rate $(r_t + \pi_t)$) and σ_x as the (state-dependent) stock volatility. Here W^s is a standard Brownian motion representing uncertainty driving stock returns, and is correlated to W^r , W^l and W^π . The introduction of a Markov regime-switching model is motivated by the desire to fit important empirical characteristics of equity returns, such as the presence of fat-tails and stochastic volatility with volatility clustering effects. The basic idea is that returns are not drawn from a single normal distribution; rather there are two distributions at work generating the returns observed. The equity returns distribution is assumed to jump between two possible states, usually referred to as regimes, denoted by $x=1$ and $x=2$ and interpreted as a low and a high volatility regimes. A transition matrix controls the probability of moving between states.

In terms of optimization, the objective can be to minimize the volatility of the surplus/deficit; it can also involve other risk measures such as the expected shortfall (average value of a deficit conditional on a deficit), or the probability of an (extreme) deficit. The performance, on the other hand, is typically measured in terms of expected surplus, or necessary contributions. Different choices in terms of optimization model are also available, with possible options involving simple static optimization or dynamic optimization with time- and state-dependent solutions (see for example Ziemba and Mulvey (1998), as well as references therein for more details on optimization models used in ALM).

3.3. LDI Solutions

Surplus optimization typically allows for higher returns (on average), and hence lower contributions (on average), since it leads to the introduction of risky asset classes, with the access to associated risk premia. On the other hand, it introduces a significant source of risk since asset classes poorly correlated with liabilities are introduced.

In an attempt to mitigate these risks, and enhance liability risk management, a new approach (known as liability-driven investment, or LDI) has recently been proposed; it is based on the introduction of a liability-matching (or liability-hedging) portfolio in the menu of asset classes. It thus builds on the traditional approach of cash-flow matching and immunization, focused on risk management, to which it adds a component dedicated to performance.

It should be noted that when the liability matching portfolio is available in the menu of asset classes, the minimum risk solution of surplus optimization corresponds to the cash-flow matching strategy, which is thus recovered as a specific case. In principle, one should again distinguish between:

- Cash-flow matching: a perfect match is possible between asset and liability cash-flows, using cash instruments (nominal and real bonds) and possibly dedicated derivatives (interest rate and inflation swaps) (see Exhibit 1).

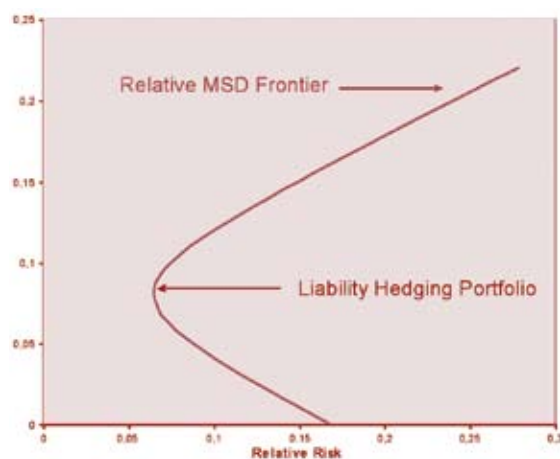


Exhibit 1: Surplus optimization without a liability-matching portfolio

3. A Brief History of ALM Techniques

- Cash-flow hedging (immunization): a perfect match is not possible and duration (or extended duration) hedging techniques are implemented so as to minimize mismatch risk (see Exhibit 2).

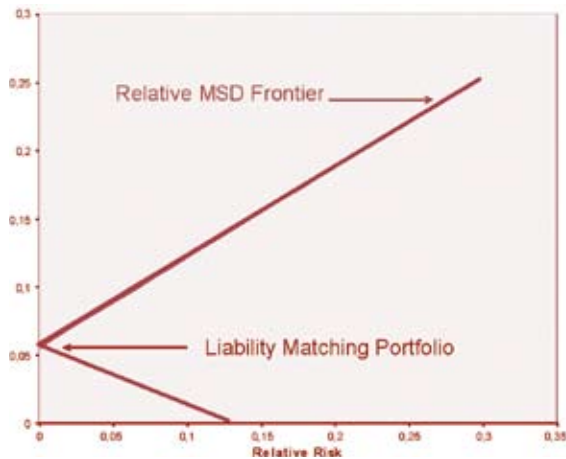


Exhibit 2: Surplus optimization with a liability-matching portfolio

From the previous comments, it might seem that so-called LDI solutions are merely a specific case of surplus optimization techniques, in a context where a liability-matching (or liability-hedging) portfolio is available in the menu of asset classes. There is a somewhat subtle difference, though, between LDI solutions and surplus optimization with a liability-matching portfolio. LDI solutions advocate an approach to ALM that is expressed in terms of allocation to three building blocks (cash, liability-matching portfolio, and performance portfolio), as opposed to allocation to standard asset classes, as done in the context of surplus optimization techniques. As such, it is consistent with an extended version of the standard fund separation theorem that is well-known in asset management (see next section and the appendix, or Martellini (2006ab)).

3.3.1. Static LDI Solutions

This is the standard approach that has rapidly gained interest from pension funds, insurance companies, and investment consultants alike. As recalled before, while they can vary significantly across providers, LDI solutions typically involve a hedge of the duration and convexity risks via several standard building blocks, while keeping some assets free for investing in higher yielding asset classes. These solutions may or

may not involve leverage, depending on the institutional investor's risk aversion. When no leverage is used, a fraction of the assets (known as the liability-matching portfolio) is allocated to risk management, while another fraction of the assets is allocated to performance generation. One may actually view this approach as a combination of two strategies, involving investing in immunization strategies (for risk management) as well as investing in standard asset management solutions (for performance generation). As explained above, this approach stands in contrast to more traditional surplus optimization methods (in particular when a dedicated liability-matching portfolio is not introduced), where both objectives (liability risk management and performance generation) are pursued simultaneously in an attempt to achieve the portfolio with the highest possible relative risk/relative return ratio.

3.3.2. Dynamic LDI Solutions

The implementation of LDI solutions crucially depends on the investor's risk aversion. High risk aversion leads to a predominant investment in the liability-hedging portfolio, which implies low extreme funding risk (zero risk in complete market case) as well as low performance (and therefore high necessary contributions), while low risk aversion leads to predominant investment in the performance-seeking portfolio, which implies high funding risk as well as higher expected performance, and hence lower contributions.

Another way to approach the trade-off between risk management on the one hand and performance generation on the other consists in implementing a dynamic, as opposed to static, allocation between the liability-matching portfolio and the performance-seeking portfolio. Such dynamic allocation methods, which attempt to deliver the best of both worlds (downside risk protection and access to upside potential), are inspired by the portfolio insurance techniques, which are extended to an ALM framework (see in particular Leibowitz and Weinberger (1982ab) for the contingent optimisation technique, as well as Amenc, Malaise and Martellini (2004) or

3. A Brief History of ALM Techniques

Martellini (2006b) for a generalisation in terms of a dynamic core-satellite management).

One interesting form of dynamic LDI strategies recommends that the fraction of wealth A_t allocated to the optimal growth portfolio is equal to a constant multiple m of the cushion, i.e., the difference between the asset value and the floor defined as $A_t - kL_t$, where k represents a regulatory or self-imposed minimum funding ratio requirement. This is reminiscent of constant proportion portfolio insurance strategies (CPPI strategies), which the present setup extends to a relative risk management context. While CPPI strategies are designed to prevent final terminal

wealth from falling below a specific threshold, extended CPPI strategies (a.k.a. contingent immunization strategies) are designed to protect asset value from falling below a pre-specified fraction of some benchmark value, here the liability value (see the Appendix for more details).

3.4. Overview

The following exhibit presents an overview of ALM techniques and the corresponding techniques in asset management.

Risk/Return Profile	Asset Management (absolute risk)	Asset-Liability Management (relative risk)
Zero risk – no access to risk premia	Investment in risk-free asset	Cash-flow matching and/or immunization
Optimal risk-return trade-off	Optimally diversified portfolio of risky assets	Optimisation of the surplus
Fund separation theorem	Capital market line (static mix of cash and optimal performance-seeking risky portfolio)	LDI solution (static mix of cash, liability matching portfolio and optimal risky portfolio)
Dynamic and skewed risk management (non-linear payoffs)	Portfolio insurance (dynamic mix of risk-free asset and optimal risky portfolio)	Dynamic LDI (also known as contingent immunization)

Exhibit 3: Overview of ALM techniques and the corresponding techniques in asset management.

4. Illustrations of the Usefulness of an ALM Approach to PWM

In what follows, we present a set of examples of the use of asset-liability management techniques in private banking. Our examples are drawn from the simplified typology of client profiles documented in section 2. We use the standard model introduced in section 3 for generating stochastic scenarios for risk factors affecting asset and liability values; and we generate a set of 1,000 scenarios for interest rates, inflation rate and equity prices, as well as real estate prices, when needed.

In order to alleviate a possible concern over the impact of arbitrary parameter values, we take parameter values that are identical to those in Ahlgrim, D'Arcy and Gorvett (2004), who calibrate the model with respect to long time-series. Other choices of parameter values can of course be adopted and their implementation would be straightforward.

The parameter values are given in Exhibit 4.

Real interest	
Mean reversion speed for short rate process	1
Volatility of short rate process	0.01
Mean reversion speed for long-term mean value	0.1
Volatility of long-term mean value	0.0165
Long-term mean reversion level for long-term mean value	0.028
Correlation between short-rate and long-term mean value	0.5
Inflation	
Mean reversion speed for inflation process	0.4
Volatility of inflation process	0.04
Long-term mean reversion level for inflation	0.048
Correlation between inflation and short-term interest rate	-0.3
Equity model – Regime switching	
(Monthly) mean equity excess return in state 1	0.008
(Monthly) volatility of equity return in state 1	0.039
(Monthly) mean equity excess return in state 2	-0.011
(Monthly) volatility of equity return in state 2	0.113
Equity model – Regime switching probabilities	
Probability of staying in state 1	0.989
Probability of switching from state 1 to state 2	0.011
Probability of staying in state 2	0.941
Probability of switching from state 2 to state 1	0.059
Real estate	
Real estate yield reversion speed	1.2
Real estate quarterly yield reversion level	0.023
Real estate yield volatility	0.013

Exhibit 4: Parameter values – borrowed from Ahlgrim, D'Arcy and Gorvett (2004)

4. Illustrations of the Usefulness of an ALM Approach to PWM

For simplicity of exposure, we have chosen to focus on static allocation strategies. While appealing from a conceptual standpoint, general time- and state-dependent portfolio strategies tend to generate a source of confusion for private clients, who may perceive such dynamic allocation strategies as attempts to implement tactical asset allocation decisions. In what follows, we have tested for the implementation of extended CPPI ALM strategies as a choice of pragmatic, rule-based techniques allowing us to better understand the benefits of introducing time-varying allocations. For the sake of brevity, the results related to the dynamic LDI strategies are only reported for a single illustration, the first one. The benefits to be expected from such strategies would be qualitatively equivalent in the context of the other illustrations discussed below.

In all cases, we report standard risk-return indicators such as expected surplus, volatility of the surplus, probability of a deficit, as well as expected shortfall (expected value of the deficit conditional on having a deficit).

4.1. Pension-Related Objective

As a first illustration, we focus on a pension objective and consider a 65-year-old individual who is already retired. His/her goal is to ensure a stream of inflation-protected fixed payments, which we normalized at €100, for the next 20 years (i.e., from age 65 to age 85)⁴. To achieve this goal the individual is prepared to invest a fixed amount of money.

We test three different strategies:

- Cash-flow matching strategies
- Surplus optimization strategies
- Dynamic LDI strategies

4.1.1. Cash-Flow Matching Strategy

One natural solution for meeting the client's objective consists in buying equal amounts of zero-coupon inflation-protected securities (TIPS) with maturities ranging from 1 year to

20 years, assuming they exist (alternatively, an OTC interest rate and inflation swaps can be used to complement existing cash instruments so as to generate a perfect match with liabilities, here a stream of 20 annual €100 payments). This equally-weighted portfolio of TIPS is the practical implementation of the liability matching portfolio introduced at a conceptual level in section 3.

Using the aforementioned stochastic model and associated parameter values, we generate random paths for the price of 20 zero-coupon TIPS with maturities matching expected payment dates. We find the present value of liability-matching portfolio, denoted as $L(0)$, and we obtain $L(0) = 1777.15$. As we can see, the performance is poor and the burden of contributions is very high: the amount of money needed to generate 20 annual €100 payments is not much smaller than 20×100 . This is due to the fact that rates are typically very low. The client needs a very high current contribution to sustain his/her future consumption needs.

On the other hand, one key advantage of this approach, which represents an extreme positioning in the risk-return space, is that the distribution of surplus at date 20 is trivially equal to 0. There is no possible deficit (nor surplus), because the present value of the future liability payments has been invested in a perfect replicating portfolio strategy.

In this context, it is reasonable, unless in the presence of an extremely (infinitely) high risk aversion, to add risky asset classes to enhance the return and decrease the pressure on contributions, at the costs of introducing a risk of mismatch between assets and liabilities. This is what we turn to next.

4.1.2. Surplus Optimization Strategies

We now generate stochastic scenarios for nominal bonds and stocks also. We then start with the same initial amount $L(0)$, and find the best fixed-mix strategy that consists of investment in stocks, bonds and a liability-matching portfolio

4 - We thus assume away the complexity related to mortality risk, which can be dealt with through an annuity contract provided for by an insurance company.

4. Illustrations of the Usefulness of an ALM Approach to PWM

(regarded as a whole) so as to generate an efficient frontier in a surplus space based on optimizing the trade-off between expected surplus and variance of the surplus (bold line in Exhibit 5). Of course, as highlighted in section 3, the minimum risk portfolio corresponds to 100% investment in the liability-matching portfolio (corresponding to point A in Exhibit 5). Formally, we assume that the asset portfolio is liquidated each year, a liability payment is made, and the remaining wealth is invested in an optimal portfolio; in scenarios such that the remaining wealth is not sufficient to make the promised liability payment, we assume that borrowing at the risk-free rate is performed so as to make up for the difference. We estimate probabilities of not meeting the objectives (probability of a deficit), which are reported in Exhibit 6, and also plot the distribution of the surplus at date 20 for a few points on the efficient frontier (see Exhibit 7). As can be seen in Exhibit 6, increasing the allocations to stocks and nominal bonds, which have a long-term performance higher than that of inflation-protected bonds but are not as good a match with respect to liabilities, leads to a higher value of the expected surplus, and therefore to average contribution savings, but also to an increased volatility of the surplus and an increased probability of the deficit.

For comparison purposes, we also perform the same exercise and design the efficient frontier when the liability-matching portfolio is not available in the menu of asset classes (see the fine line in Exhibit 5). The improvement induced by the introduction of a liability-matching portfolio is spectacular, as can be seen by a simple comparison between point A and A' or B and B'. Regarding point B and B' for instance, one can see that for the same level of expected surplus (€376.78), the volatility of the surplus is increased by more than 50% when the liability-matching portfolio is not available (€640.24 versus €423.65). The risk reduction benefits are also spectacular when risk is measured in terms of probability of a deficit or expected shortfall. Intuitively, such a dramatic improvement in investor's welfare is related to the fact that it

is only through the completion of the menu of asset classes that arises from the introduction of a dedicated liability-matching portfolio that the investor's specific objective and constraints as well as related risk exposures are fully taken into account.

Of course, the difference between optimal portfolios in the presence and in the absence of a liability-matching portfolio decreases with the investor's risk-aversion: risk-seeking investors do not seek to enjoy the benefits of liability protection and mostly invest in stocks and bonds anyway.

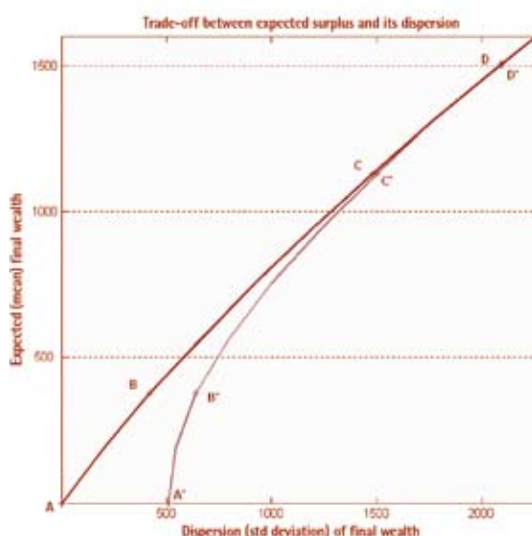


Exhibit 5: Efficient frontier in a mean-variance surplus space

This ALM optimization exercise consists in finding the portfolios that are optimal from the standpoint of protecting investors' liabilities. A pure asset management (AM) exercise, on the other hand, focuses on designing the portfolios with the optimal risk-return trade-off. Of course, nothing guarantees that AM efficient portfolios will be efficient from an ALM perspective (and vice-versa); in particular, the focus is on nominal return from an AM perspective, while it is on real return from an ALM perspective. To test for the ALM performance of AM efficient portfolios, we have conducted the following experiment. We first find the standard (Markowitz efficient) frontier based on horizon returns, i.e., the portfolios that achieve the lowest level of

4. Illustrations of the Usefulness of an ALM Approach to PWM

	Weights			Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
	Stocks	Bonds	Liab-PF							
A	0%	0%	100%	0.00	0.00	0.00%	0.00	(0.00%)	1777.15	0.00%
B	19%	22%	59%	376.78	423.65	15.50%	204.45	(11.50%)	1556.99	12.39%
C	43%	41%	16%	1130.33	1478.91	18.70%	427.92	(24.08%)	1314.07	26.06%
D	52%	45%	3%	1507.11	2093.77	19.60%	502.64	(28.28%)	1233.58	30.59%
A'	10%	90%	0%	0.00	508.92	36.60%	432.34	(24.33%)	1777.15	0.00%
B'	20%	80%	0%	376.78	640.24	25.20%	385.97	(21.72%)	1556.99	12.39%
C'	43%	57%	0%	1130.33	1500.11	19.30%	457.69	(25.75%)	1314.07	26.06%
D'	51%	49%	0%	1507.11	2094.78	20.00%	499.21	(28.09%)	1233.58	30.59%

Exhibit 6: Allocation strategies and risk-return indicators; all values are given as present values at initial date (based on a $L(0)=1777.15$ initial investment); losses relative to $L(0)$ are reported in parentheses for expected shortfall; the relative contribution saving corresponds to the increase (in percentage) in initial investment that should have taken place with a given strategy so as to generate an expected surplus equal to zero.

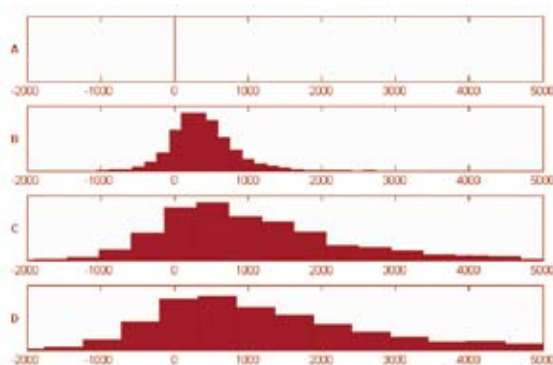


Exhibit 7: Distribution of final surplus/deficit

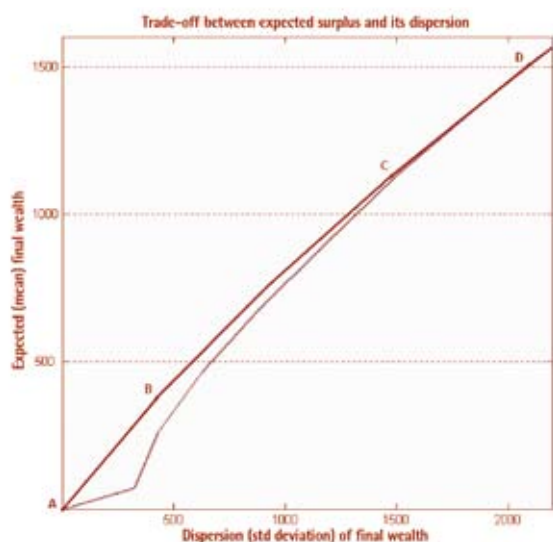


Exhibit 8: AM and ALM efficient frontiers in a mean-variance surplus space

volatility (across scenarios at horizon) for a given expected return (across scenarios at horizon). We then plot these portfolios (fine line) in the (expected surplus-volatility of the surplus) ALM space (see Exhibit 8).

From Exhibit 8, we can see that a portfolio efficient in an AM sense is indeed not necessarily efficient in an ALM sense, and vice-versa. Hence, not taking into account liability constraints leads to potentially severe inefficiencies from the investor's standpoint.

We now turn to dynamic portfolio strategies.

4.1.3. Dynamic LDI Strategies

In testing the implementation of the dynamic LDI strategies, the performance portfolio is taken to be the stock-bond portfolio with the highest Sharpe ratio (with our choice of parameter values, and a 4% risk-free rate, we obtain the following portfolio: 28.5% in stocks and 71.5% in bonds), while the liability-matching portfolio is the aforementioned portfolio invested in the 20 zero-coupon TIPS with maturities matching expected payment dates.

We consider the extended CPPI strategy introduced in section 3. We consider 6 variants of the strategy, with the level of protection $k=90%$, or $k=95%$, and the multiplier value $m=2$, 3 and 4. The results are reported in exhibits 9 to 12, where we present the performance of the various dynamic strategies and compare them to

4. Illustrations of the Usefulness of an ALM Approach to PWM

the performance of their static counterpart. The static counterpart of a given dynamic portfolio strategy is defined as the strategy involving constant (fixed-mix) allocation to the portfolio with the highest Sharpe ratio and liability-matching portfolio that matches the initial allocation of the corresponding dynamic strategy. For example, when $k=95%$ and $m=4$, the initial allocation to the liability-matching portfolio (respectively the highest Sharpe ratio portfolio) is given by $1-(1-k)m=80%$ (respectively, 20%). The static counterpart of the extended CPPI strategy with parameters $k=95%$ and $m=4$ is therefore a fixed-mix strategy with a constant 80%-20% allocation to liability matching portfolio and performance-seeking portfolio.

As can be seen in Exhibit 9 and Exhibit 10, most dynamic strategies allow for significantly lower expected shortfall numbers as well as higher expected surplus (and hence higher contribution savings) when compared to their static counterparts. On the other hand, they tend to generate higher volatility. Also, the probability of a deficit is rather large with dynamic strategies, which aim to avoid all deficit beyond the minimum threshold (90% or 95%), as opposed to minimizing the probability of facing such a relatively low deficit. In essence, dynamic ALM strategies generate asymmetric surplus distributions, as confirmed by Exhibits 11 and 12, where the various surplus distributions are presented. We also note, as expected, that

Dynamic CPPI	Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
m=2 k=0.90	121.97	188.42	25.20%	66.45	(3.74%)	1694.19	4.67%
m=3 k=0.90	184.75	326.33	30.20%	97.21	(5.47%)	1658.70	6.66%
m=4 k=0.90	203.97	388.70	36.60%	119.11	(6.70%)	1646.97	7.33%
Static CPPI	Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
m=2 k=0.90	99.39	110.72	14.90%	80.80	(4.55%)	1706.70	3.96%
m=3 k=0.90	153.12	174.95	15.90%	119.84	(6.74%)	1673.88	5.81%
m=4 k=0.90	209.74	245.63	16.80%	158.93	(8.94%)	1642.37	7.58%

Exhibit 9: risk-return indicators for extended CPPI strategies for a level of guarantee $k=90%$, as well as for their static counterpart; all values are given as present values at initial date (based on a $L(0)=1777.15$ initial investment); losses relative to $L(0)$ are reported in parentheses for expected shortfall; the relative contribution saving corresponds to the increase (in percentage) in initial investment that should have taken place with a given strategy so as to generate an expected surplus equal to zero.

Dynamic CPPI	Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
m=2 k=0.95	58.48	88.18	25.10%	32.75	(1.84%)	1734.51	2.40%
m=3 k=0.95	94.38	175.82	29.90%	48.12	(2.71%)	1711.43	3.70%
m=4 k=0.95	115.40	240.19	36.80%	58.48	(3.29%)	1698.24	4.44%
Static CPPI	Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
m=2 k=0.95	48.38	52.58	14.10%	40.43	(2.27%)	1741.04	2.03%
m=3 k=0.95	73.55	80.93	14.30%	61.46	(3.46%)	1723.66	3.01%
m=4 k=0.95	99.39	110.72	14.90%	80.80	(4.55%)	1706.70	3.96%

Exhibit 10: risk-return indicators for extended CPPI strategies for a level of guarantee $k=95%$, as well as for their static counterpart; all values are given as present values at initial date (based on a $L(0)=1777.15$ initial investment); losses relative to $L(0)$ are reported in parentheses for expected shortfall; the relative contribution saving corresponds to the increase (in percentage) in initial investment that should have taken place with a given strategy so as to generate an expected surplus equal to zero.

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increasing the guaranteed level k and decreasing the multiplier value m lead to more conservative strategies, with less potential for surplus performance and lower risk.

Overall, the results reported in exhibits 7 to 10 show the very significant risk management benefits that arise from dynamic strategies.

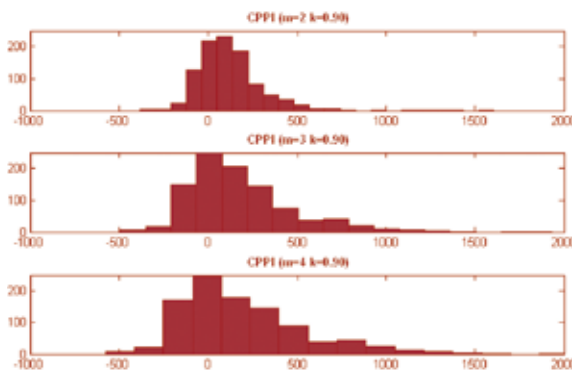


Exhibit 11: Distribution of final surplus/deficit for extended CPPI strategies for a 90% guarantee level

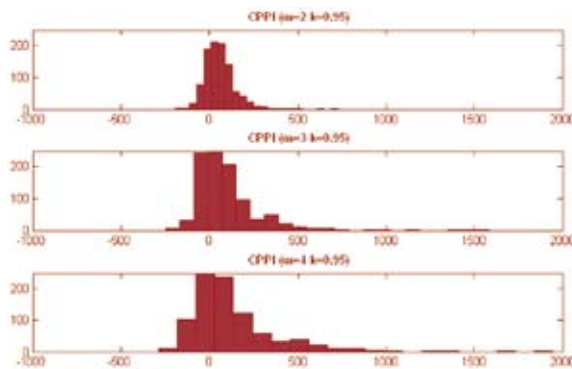


Exhibit 12: Distribution of final surplus/deficit for extended CPPI strategies for a 95% guarantee level

4.1.4. A Variant

We now consider a slight variant of the pension related objective, where the client is assumed to be a 45-year-old individual who is not retired yet and plans to retire at age 65. The goal is to ensure at age 65 a single lump-sum payment normalized at €100 plus inflation for retirement. To achieve this goal the individual is prepared to contribute an amount x (out of his yearly salary) for the remaining 20 years of his working life.

Exhibit 13 shows the impact of inflation risk on the value of the €100 payment scheduled

to be paid in 20 years from now. As we can see, inflation risk is significant, with a nominal amount to be secured for retirement equal to €247.39 on average and a 94.50 standard deviation.

The main difference with the previous case is that the investor may not be able to implement

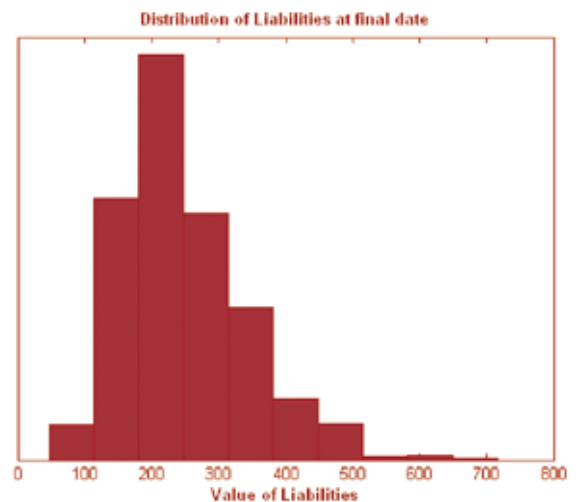


Exhibit 13: Distribution of liabilities at final date; mean value = 247.39, standard deviation = 94.50.

a perfect liability-matching portfolio unless he/she is allowed to borrow against his/her future income.

Where borrowing is possible, the strategy is as follows:

- Borrow $xB(0,1)+xB(0,2)+\dots+xB(0,20)$, where $B(s,t)$ is the price at date s of a unit face value pure discount nominal bond that matures at time t .
- Invest this amount in a zero-coupon inflation protected bond with a 20-year maturity.

The optimal value for x is given by: $x = 100P(0,20)/(B(0,1)+B(0,2)+\dots+B(0,20))$, where $P(s,t)$ is the price at date s of a unit face value pure discount real bond that matures at time t . With our choice of parameter values, x turns out to be equal to 6.07. This is the amount needed to allow for a perfect ALM match. In practice, it is however generally not feasible/practical to borrow against future income, and it is therefore impossible to generate a perfect ALM match due to uncertainty over investment conditions for future contributions.

4. Illustrations of the Usefulness of an ALM Approach to PWM

As an attempt to estimate the optimal allocation strategies in this context, we perform the following numerical exercise. We first generate random paths for stock, bond and TIPS prices with parameters consistent with long-term estimates, where bond and TIPS are regarded as indices (modelled as constant maturity zero-coupon securities). We then take $x = 100P(0,20)/(B(0,1)+B(0,2)+\dots+B(0,20)) = 6.07$, as explained before, and find the set of optimal portfolios that will minimize the volatility of a deficit/surplus, defined as asset value at date 20 minus liability value on retirement date (i.e., 100 plus

20 years worth of inflation), for a given level of surplus expected value. For each portfolio on the efficient frontier, we then find the value $x' < x$ that is needed to generate a zero expected surplus at retirement date. The relative contribution saving is given by (one minus) the ratio of the present value of the 20 annual x' payments over the 20 annual x payments. The optimal portfolio allocations and risk-return indicators are given in Exhibit 14, while the efficient frontier in the ALM space of expected value and surplus volatility appears in Exhibit 15.

Portfolio	Weights			Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
	Stocks	TIPS	Bonds							
LMP	-	-	-	0.00	0.00	0.0%	0	(00.0%)	6.07	-
A	0%	0%	100%	9.18	27.10	34.8%	17.65	(22.7%)	5.44	10.4%
B	9%	0%	91%	23.61	39.11	24.0%	17.62	(22.6%)	4.75	21.8%
C	19%	0%	81%	39.64	59.93	19.6%	18.64	(23.9%)	4.16	31.5%
D	30%	0%	70%	55.67	83.03	18.2%	19.42	(24.9%)	3.70	39.1%
E	40%	0%	60%	71.70	106.94	17.9%	20.39	(26.2%)	3.33	45.2%
F	50%	11%	39%	87.73	131.20	18.1%	22.38	(28.7%)	3.03	50.1%
G	60%	24%	16%	103.76	155.64	17.8%	25.92	(33.3%)	2.78	54.3%
H	70%	30%	0%	119.80	180.17	17.3%	29.69	(38.1%)	2.57	57.8%
I	80%	20%	0%	135.83	204.84	17.3%	31.60	(40.6%)	2.38	60.8%
K	90%	10%	0%	151.86	229.60	16.9%	34.79	(44.7%)	2.22	63.4%
L	100%	0%	0%	167.89	254.43	17.5%	36.31	(46.6%)	2.09	65.7%

Exhibit 14: Allocation strategies and risk-return indicators; all values are given as present values at initial date based on a €6.07 annual contribution for 20 years, whose present value $6.07x(B(0,1)+B(0,2)+\dots+B(0,20))$ amounts to €77.87 given our choice of parameter values. Expected shortfall is expressed as a percentage of this value. The relative contribution saving corresponds to the increase (in percentage) in initial investment that should have taken place with a given strategy so as to generate an expected surplus equal to zero.

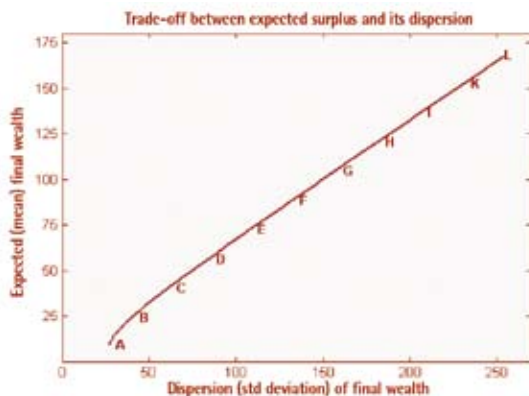


Exhibit 15: Efficient frontier in a mean-variance surplus space

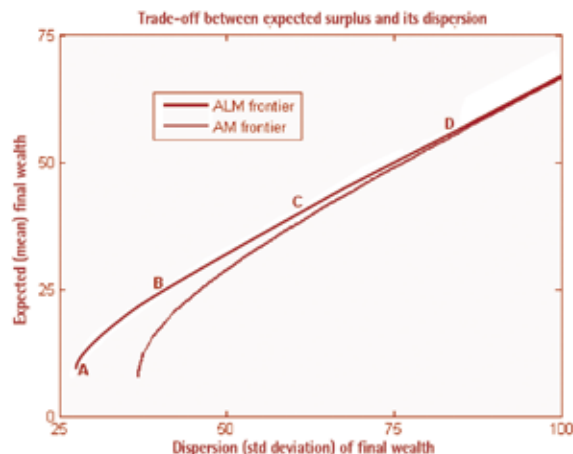


Exhibit 16: AM and ALM efficient frontiers in a mean-variance surplus space

4. Illustrations of the Usefulness of an ALM Approach to PWM

As before, we can see that a portfolio efficient in an AM sense is indeed not necessarily efficient in an ALM sense, and vice-versa (see exhibit 16), which suggests that omitting to take liability constraints into account in the design of optimal portfolio solutions leads to potentially severe efficiency losses from the investor's standpoint.

4.2. Expenditure-Related Objective: the Case of Real Estate

We now consider an investor who wishes to invest fixed annual contributions (€x) for future expenditure, e.g., to buy a house in 5 years, the current value of which is normalized at €100 (we may alternatively interpret this as the required down payment). For simplicity, one could assume that house prices increase with inflation and use the stochastic model for inflation to generate a distribution of future house prices. Of course, because real estate prices are only imperfectly correlated with a broad-based consumer price index, it is more accurate to introduce an explicit model for the dynamics of real estate prices, which is what we do here.

Exhibit 17 shows the impact of real estate price uncertainty on the value of the €100 payment scheduled to be paid in 5 years from now. As we can see, real estate price risk is significant, with a nominal amount to be secured equal to €156.59 on average and a €27.18 standard deviation.

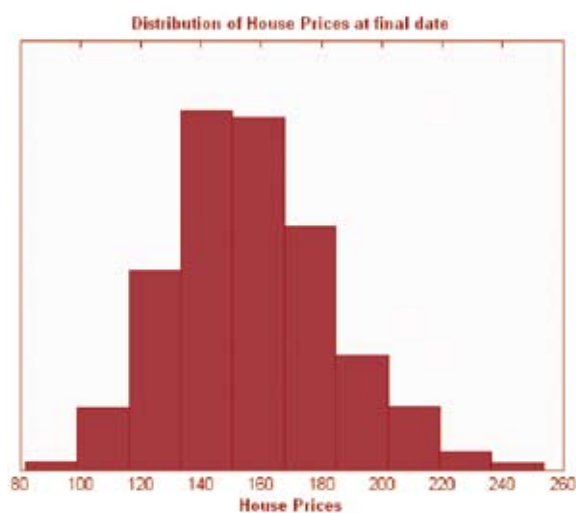


Exhibit 17: Distribution of house prices at final date; mean value = 156.59, standard deviation = 27.18.

In practical terms, the goal is to generate a lump sum payment at horizon date (5 years). As in the previous example, it is not possible in general to find a perfect liability-matching portfolio. The existence of a perfect liability-matching portfolio is actually only ensured on the following two conditions:

- Investors can borrow against future income and can invest at the initial date the present value of the future contributions.

- There exists an investment vehicle (e.g., REITS) whose payoff is directly related to real estate price uncertainty.

In what follows, we test two different situations:

- The opportunity set contains stocks, bonds and TIPS
- The opportunity set contains stocks, bonds, TIPS, plus real estate (modelled as an investment that will pay the compounded return on real estate)

To generate comparable portfolios, we have looked at the improvement in surplus volatility for a given level of expected surplus.

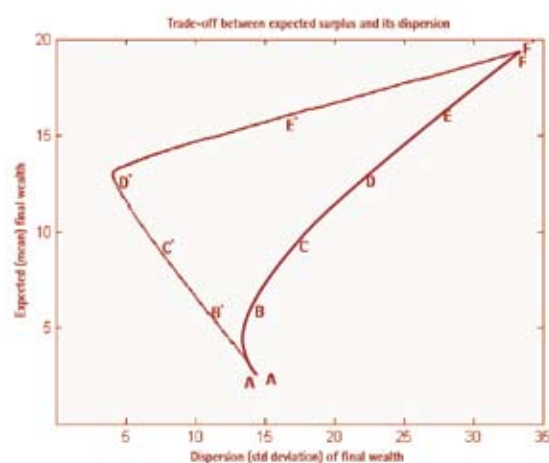


Exhibit 18: ALM Efficient Frontiers without Real Estate (A, B, C, D, E, F) and with Real Estate (A', B', C', D', E', F')

4. Illustrations of the Usefulness of an ALM Approach to PWM

Portfolio	Weights				Expected surplus	Volatility of surplus	Prob(S<0)	Expected shortfall		Necessary nominal contribution p.a.	Relative contribution saving p.a.
	Stocks	Bonds	TIPS	Real Estate							
A	0%	0%	100%	-	2.46	14.64	41.5%	11.40	(12.5%)	19.07	4.7%
B	10%	90%	0%	-	5.84	13.96	31.5%	10.32	(11.3%)	17.85	10.8%
C	33%	67%	0%	-	9.22	17.16	28.9%	11.03	(12.1%)	16.70	16.5%
D	55%	45%	0%	-	12.60	21.95	28.5%	12.93	(14.2%)	15.62	21.9%
E	78%	22%	0%	-	15.98	27.51	28.0%	15.92	(17.5%)	14.60	27.0%
F	100%	0%	0%	-	19.36	33.46	27.9%	19.09	(21.0%)	13.63	31.8%
A'	0%	0%	100%	0%	2.46	14.64	41.5%	11.40	(12.5%)	19.07	4.7%
B'	0%	0%	68%	32%	5.84	10.87	28.6%	7.44	(8.2%)	17.85	10.7%
C'	0%	0%	37%	63%	9.22	7.27	10.6%	4.57	(5.0%)	16.70	16.5%
D'	0%	0%	5%	95%	12.60	4.25	0.6%	2.07	(2.3%)	15.61	21.9%
E'	46%	0%	0%	54%	15.98	16.43	16.7%	7.36	(8.1%)	14.59	27.0%
F'	100%	0%	0%	0%	19.36	33.46	27.9%	19.09	(21.0%)	13.63	31.8%

Exhibit 19: Allocation strategies and risk-return indicators; all values are given as present values at initial date based on a €20 annual contribution for 5 years, the present value of which amounts to €90.91 given our choice of parameter values. Expected shortfall is expressed as a percentage of this value. The relative contribution saving corresponds to the increase (in percentage) in initial investment that should have taken place with a given strategy so as to generate an expected surplus equal to zero.

Exhibit 18 shows the efficient frontier in both cases, while risk-return indicators are reported in Exhibit 19. As was expected, the presence of assets allowing investors to span real estate price uncertainty proves to be a key element in improving the efficient frontiers obtained from an ALM perspective. Looking for example at portfolio D and D' from Exhibit 19, we see that for the same level of expected surplus (12.60 in both cases), the surplus volatility at the optimal level reaches 21.95 when the opportunity set does not contain a real estate asset, while it merely amounts to 4.25, a dramatic risk reduction, when the real estate asset is included. Again this signals the relevance of an ALM approach to private wealth management: it is only by trying to fit the client liability constraints that truly optimal solutions can be proposed.

4.3. Bequest-Related Objective

We now consider a wealthy 65-year-old individual who is already retired. He/she has significant wealth (say 100 million euros) and wishes to maintain a standard of living (annual expenses say at 2 million euros plus inflation) with an additional bequest motive in 20 years⁵.

The analysis aims to find the optimal policy so as to generate the highest possible bequest level with a given probability denoted by α . We first discuss this situation as a base case, and subsequently turn to different variants.

4.3.1. The Base Case

Exhibit 20 shows the optimal allocation strategy, as well as related risk-return indicators, for various values of the confidence level α , while Exhibit 21 shows the distribution of the discounted value of final bequest also for these different values.

	Target percentile	Weights			Expected bequest	Volatility of bequest	Bequest percentiles					
		Stocks	Bonds	LMP			5	10	20	Median	75	95
No constraints	alpha= 5%	11%	49%	40%	90.58	30.36	48.98	56.01	64.79	86.75	107.56	145.93
	alpha=10%	25%	38%	37%	118.43	58.20	46.78	59.52	71.76	105.84	147.39	226.50
	alpha=20%	53%	23%	24%	194.64	163.44	34.58	51.15	76.35	149.39	254.62	518.50

Exhibit 20: Allocation strategies and risk-return indicators as a function of the confidence level α

5 - We again assume away the complexity related to mortality risk, which can be dealt with through an annuity contract provided for by an insurance company.

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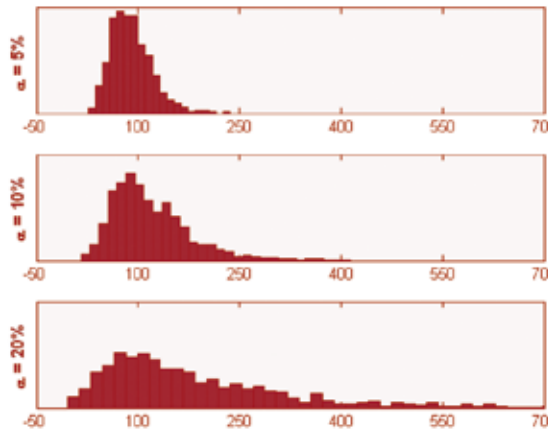


Exhibit 21: Distribution of the discounted value of final bequest as a function of the confidence level α

4.3.2. Introducing Constraints

We also consider two variants in which:

- Half of the client wealth (100 million) is held as stock in his/her own private company, which will be sold in 5 years from now; in this case, we impose a 50% lower constraint on equity allocation)⁶.
- The value of existing property is accounted for (e.g., the client has a €10 million worth of property value in addition to the €100 million).

These results can be found in Exhibits 22 and 23.

	Target Percentile	Weights			Expected bequest	Volatility of bequest	Bequest percentiles					
		Stocks	Bonds	LMP			5	10	20	Median	75	95
Min 50% in stocks	alpha= 5%	50%	40%	10%	181.35	142.68	37.23	52.05	73.55	142.62	239.44	462.09
	alpha=10%	50%	15%	35%	187.21	152.59	35.82	53.11	75.00	147.21	242.09	490.45
	alpha=20%	53%	23%	24%	194.20	162.69	34.59	51.10	76.33	149.15	254.25	516.30
Additional 10m in real estate property at T0	alpha= 5%	22%	36%	42%	152.14	59.32	79.21	88.47	103.63	141.31	182.29	260.71
	alpha=10%	24%	44%	32%	155.60	62.90	78.76	90.32	104.51	143.97	186.50	277.50
	alpha=20%	52%	23%	25%	241.94	179.88	62.03	83.67	111.22	192.13	309.43	594.38

Exhibit 22: Allocation strategies and risk-return indicators as a function of the confidence level α , including allocation constraints (real estate or equity)

As we can see through a comparison with Exhibit 20, the presence of constraints related to the

client's situation will impact upon the optimal portfolio strategy.

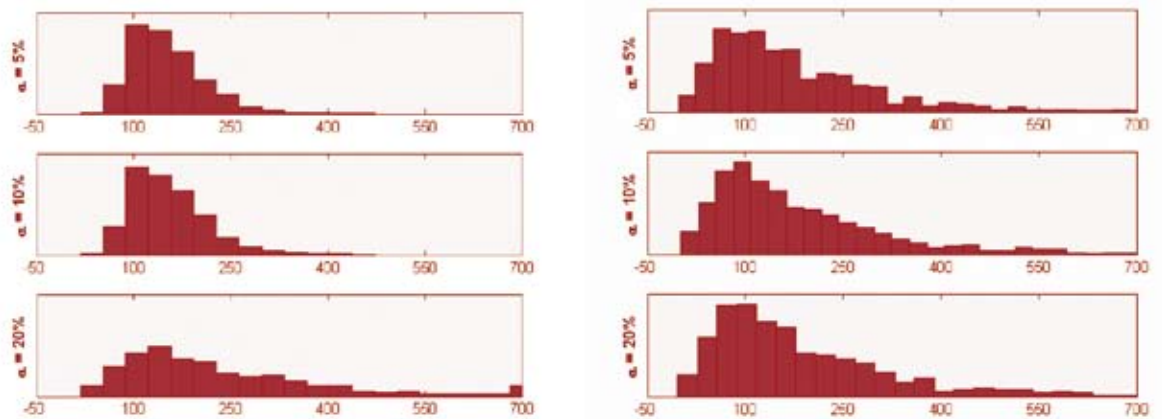


Exhibit 23: Distribution of the discounted value of final bequest as a function of the confidence level α , with additional €10 million initially held in real estate (left) and with a minimum of 50% invested in equity (right)

4.3.3. A Variant with Significant Lump-Sum Payments Expected

We finally consider a 65-year-old individual who is already retired. He/she has significant wealth (say €100 million) and wishes to maintain a

standard of living (annual expenses say at 2 million euros plus inflation), plus two significant expenses (10 million in 5 years and 10 million in 10 years), e.g., to buy a private jet or a yacht, with an additional bequest motive in 20 years.

6 - In other words, we assume away specific risk in private equity return when optimizing the portfolio, and model the €100 million as if it was invested in the equity index.

4. Illustrations of the Usefulness of an ALM Approach to PWM

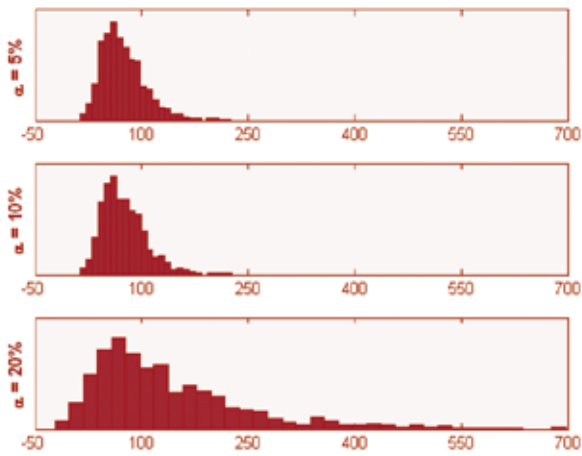


Exhibit 24: Distribution of the discounted value of final bequest as a function of the confidence level α .

The analysis aims at finding the optimal policy so as to:

- Generate the optimal distribution of bequest for a given level of annual expenses (Exhibits 24 and 25).
- Generate the optimal distribution of level of annual expenses for a given level of bequest (Exhibits 26 and 27).

Target Percentile	Weights			Expected Bequest	Volatility of Bequest	Bequest Percentiles					
	Stocks	Bonds	LMP			5	10	20	median	75	95
alpha= 5%	17%	47%	36%	74.57	31.11	35.30	41.26	48.70	69.52	91.33	133.35
alpha=10%	17%	40%	43%	75.88	32.27	34.43	41.84	49.69	70.01	93.46	135.48
alpha=20%	48%	31%	21%	141.03	118.82	19.73	32.68	54.10	110.95	187.23	363.20

Exhibit 25: Allocation strategies and risk-return indicators as a function of the confidence level α .

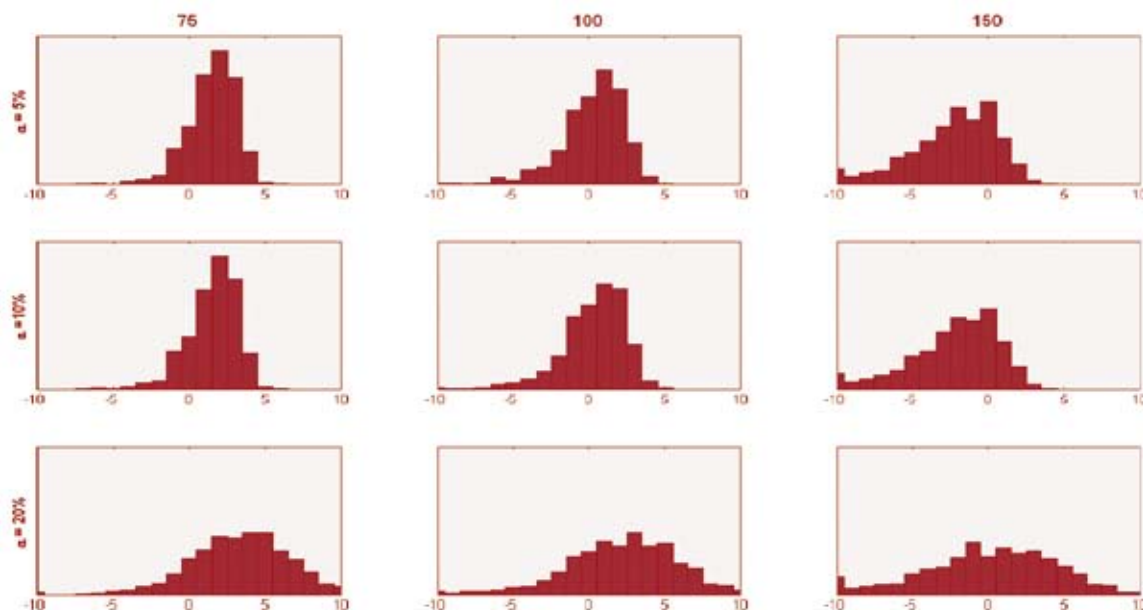


Exhibit 26: Distribution of the discounted value of final bequest as a function of the confidence level α and expected bequest level

4. Illustrations of the Usefulness of an ALM Approach to PWM

Bequest level	Target percentile	Weights			Expected annual expenses	Volatility of annual expenses	Annual expenses percentiles					
		Stocks	Bonds	LMP			5	10	20	Median	75	95
75	alpha= 5%	17%	47%	36%	1.53	1.58	-1.20	-0.60	0.33	1.73	2.64	3.72
	alpha=10%	17%	40%	43%	1.55	1.62	-1.11	-0.67	0.31	1.76	2.67	3.81
	alpha=20%	48%	31%	21%	3.28	3.23	-2.06	-0.63	0.81	3.44	5.43	8.01
100	alpha= 5%	17%	47%	36%	0.26	2.04	-3.62	-2.30	-1.10	0.56	1.70	2.98
	alpha=10%	17%	40%	43%	0.30	2.08	-3.63	-2.34	-1.02	0.65	1.74	3.08
	alpha=20%	48%	31%	21%	2.15	3.70	-3.98	-2.17	-0.61	2.47	4.66	7.38
150	alpha= 5%	17%	47%	36%	-2.35	3.07	-8.18	-6.26	-4.51	-1.86	-0.18	1.50
	alpha=10%	17%	40%	43%	-2.31	3.14	-8.29	-6.38	-4.54	-1.78	-0.08	1.59
	alpha=20%	48%	31%	21%	-0.13	4.80	-8.33	-5.63	-3.44	0.37	3.10	6.40

Exhibit 27: Allocation strategies and risk-return indicators as a function of the confidence level α and expected bequest level

5. Conclusion

This paper has provided ample evidence that asset-liability management is an essential improvement in private wealth management that allows private bankers to provide their clients with investment solutions and asset allocation advice that truly meet their needs. We have also provided a series of illustrations that show that some of the most sophisticated ALM techniques used in institutional money management can satisfactorily be implemented in private wealth management.

Ultimately, we argue that it is not the performance of a particular fund nor that of a given asset class (including commodities or hedge funds) that will be the determining factor in the ability of private wealth management to meet investors' expectations. What will prove to be the decisive factor is the private wealth manager's ability to design an asset allocation solution that is a function of the kinds of particular risks to which the investor is exposed, as opposed to the market as a whole. Hence, an absolute return fund, often perceived as a natural choice in the context of private wealth management, shall not be a satisfactory response to the needs of a client facing long-term inflation risk, where the concern is capital preservation in real, as opposed to nominal, terms. Similarly, a client whose objective would be related to the acquisition of a property would accept low and even negative returns in situations when real estate prices significantly decrease, but will not satisfy himself or herself with relatively high returns if such high returns are not sufficient to meet a dramatic increase in real estate prices. In such circumstances, a long-term investment in stocks and bonds with a performance weakly correlated with real estate prices would not be the right investment solution.

In other words, the success or failure of the satisfaction of the client's long-term objectives is fundamentally dependent on an ALM exercise that aims to determine the proper strategic inter-classes allocation as a function of the client's specific objectives and constraints. Asset management should only come next as a

response to the implementation constraints of the ALM decisions. On the one hand, it is meant to deliver/enhance the risk and return parameters supporting the ALM analysis for each asset class. On the other hand, it can also allow for the management of short-term constraints, such as capital preservation at a given confidence level, which are not necessarily taken into account by an ALM optimization exercise, which by nature focuses on long-term objectives.

6. Mathematical Appendix

In this appendix, we present a general continuous-time model of asset allocation decisions in the presence of liability constraints. This material is borrowed from Martellini (2006ab). From an academic standpoint, several authors have attempted to cast the ALM problem in a continuous-time framework, and extend Merton's intertemporal selection analysis (see Merton (1969, 1971)) to account for the presence of liability constraints in the asset allocation policy. A first step in the application of optimal portfolio selection theory to the problem of pension funds has been taken by Merton (1990) himself, who studies the allocation decision of a university that manages an endowment fund. In a similar vein, Boulier et al. (1995) have formulated a continuous-time dynamic programming model of pension fund management. It contains all of the basic elements for modeling dynamic pension fund behavior, and can be solved by means of analytical methods⁷. Rudolf and Ziemba (1994) extend these results to the case of a time-varying opportunity set, where state variables are interpreted as currency rates that affect the value of the pension's asset portfolio. Also related is a paper by Sundaresan and Zapatero (1997), which is specifically aimed at asset allocation and retirement decisions in the case of a pension fund.

This continuous-time stochastic control approach to ALM is appealing because it enjoys the desirable property of tractability and simplicity, allowing one to fully and explicitly understand the various mechanisms affecting the optimal allocation strategy⁸. Our paper falls within this strand of the literature, which it complements in a variety of ways. In particular, we argue that the value of the liability portfolio is a natural numeraire in the investor's objective function, and we relate the solution to this problem, which involves a three fund separation theorem, to some of the recent liability-driven investment solutions offered by several investment banks and asset management firms.

6.1. Stochastic Model for the Value of Asset and Liabilities

Let $[0, T]$ denote the (finite) time span of the economy, where uncertainty is described through a standard probability space (Ω, \mathcal{A}, P) and endowed with a filtration $\{\mathcal{F}_t; t \geq 0\}$, where $\mathcal{F}_\infty \subset \mathcal{A}$ and \mathcal{F}_0 is trivial, representing the P -augmentation of the filtration generated by the n -dimensional Brownian motion (W^1, \dots, W^n) . We consider n risky assets (or asset classes), the prices of which are given by :

$$dP_t^i = P_t^i \left(\mu_i dt + \sum_{j=1}^n \sigma_{ij} dW_t^j \right), i = 1, \dots, n$$

We shall sometimes use the shorthand vector notation for the expected return (column) vector $\mu = (\mu_i)_{i=1, \dots, n}$ and matrix notation $\sigma = (\sigma_{ij})_{i, j=1, \dots, n}$ for the asset return variance-covariance matrix, assumed to be non-singular. $\mathbf{1} = (1, \dots, 1)'$ denotes an n -dimensional vector of ones and $W = (W^j)_{j=1, \dots, n}$ denotes the vector of Brownian motions. A risk-free asset, the 0^{th} asset, is also traded in the economy. The return on that asset, typically a default free bond, is given by $dP_t^0 = P_t^0 r dt$, where r is the risk-free rate in the economy. We assume throughout the paper that all parameter values are constant⁹.

We also introduce a separate process that represents in a reduced-form manner the dynamics of the present value of the liabilities:

$$dL_t = L_t \left(\mu_L dt + \sum_{j=1}^n \sigma_{L,j} dW_t^j + \sigma_{L,\varepsilon} dW_t^\varepsilon \right)$$

where (W_t^ε) is a standard Brownian motion, uncorrelated with W , that can be regarded as the projection residual of liability risk onto asset price risk and represents the source of uncertainty that is specific to liability risk, emanating from various factors such as uncertainty in the growth of the work force, uncertainty in mortality and retirement rates, etc. μ_L represents the expected return on the liability portfolio; $\sigma_{L,\varepsilon}$

7 - A related reference is Siegmann and Lucas (2002) who extend the approach taken by Boulier et al. (1995) by considering CARA and CRRA preferences, as opposed to a simple quadratic loss function.

8 - A second strand of the literature has therefore focused on developing more comprehensive models of uncertainty in an ALM context. This has led to the development of a stochastic programming approach to ALM, including Kallberg et al. (1982), Kusy and Ziemba (1986), or Mulvey and Vladimirov (1992). A good number of applications in asset-liability management are provided in Ziemba and Mulvey (1998) and Ziemba (2003).

9 - More generally, one can make expected return and volatilities of the risky assets, as well as the risk-free rate, depend upon a multi-dimensional state variable X . These state variables can be thought of as various sources of uncertainty impacting upon the value of assets and liabilities. In particular, one may consider the impact of stochastic interest rate on the optimal policy.

6. Mathematical Appendix

represents pure liability volatility, while the vector $\sigma_L = (\sigma_{L,j})_{j=1,\dots,n}$ can be regarded as measuring the magnitude of financial risks in liability streams.

The integration of the above stochastic differential equation gives: $L_T = L_t \eta(t, T) \eta_L(t, T)$, with:

$$\eta(t, T) = \exp \left\{ \int_t^T \left(\mu_L(s) - \frac{1}{2} \sigma_L'(s) \sigma_L(s) \right) ds + \int_t^T \sigma_L'(s) dW_s \right\}$$

$$\eta_L(t, T) = \exp \left\{ - \int_t^T \frac{1}{2} \sigma_{L,\varepsilon}^2(s) ds + \int_t^T \sigma_{L,\varepsilon}(s) dW_s^\varepsilon \right\}$$

When $\sigma_{L,\varepsilon} = 0$, then we are in a complete market situation where all liability uncertainty is spanned by existing securities. Because of the presence of non-financial risks (e.g., actuarial risks), such a situation never occurs in practice, and the correlation between the liability and the liability-hedging portfolio (i.e., the portfolio with the highest correlation with liability values) is always strictly lower than one. In general therefore, $\sigma_{L,\varepsilon} \neq 0$ and the presence of liability risk that is not spanned by asset prices induces a specific form of market incompleteness.

6.2. Objective and Investment Policy

The investment policy is a (column) predictable process vector $(w_t' = (w_{1t}, \dots, w_{nt}))_{t \geq 0}$ that represents allocations to risky assets, with the remainder invested in the risk-free asset. We define by A_t^w the asset process, i.e., the wealth at time t of an investor following the strategy w starting with an initial wealth A_0 .

We have that:

$$dA_t^w = A_t^w \left[(1 - w_t) \frac{dB_t}{B_t} + w_t' \frac{dP_t}{P_t} \right]$$

or:

$$dA_t^w = A_t^w [(r + w_t'(\mu - r1))dt + w_t' \sigma dW_t]$$

We now introduce one variable of interest, which will be used as a state variable in this model – the funding ratio – defined as the ratio of assets to liabilities: $F_t = A_t^w / L_t$. A pension trust is said to have a surplus when the funding ratio is greater than 100%, fully funded when it is equal to 100%, and under funded when it is less than 100%.

In an asset-liability management context, what matters is not the value of the assets per se, but how the asset value compares to the value of liabilities. This is also the reason why it is natural to assume that the (institutional) investor's objective is written in terms of relative wealth (relative to liabilities), as opposed to absolute wealth: $\max_w E_0 [U(F_T)]^{10}$.

Using Itô's lemma, we can also derive the stochastic process followed by the funding ratio under the assumption of a strategy w :

$$dF_t = d \left(\frac{A_t^w}{L_t} \right) = \frac{1}{L_t} dA_t^w - \frac{A_t^w}{L_t^2} dL_t - \frac{1}{L_t^2} dA_t^w dL_t + \frac{A_t^w}{L_t^3} (dL_t)^2$$

which yields:

$$\frac{dF_t}{F_t} = ((r + w_t'(\mu - r1))dt + w_t' \sigma dW_t) - (\mu_L dt + \sigma_L' dW_t + \sigma_{L,\varepsilon} dW_t^\varepsilon) - (w_t' \sigma \sigma_L dt) + ((\sigma_L' \sigma_L dt + \sigma_{L,\varepsilon}^2 dt)$$

or

$$\frac{dF_t}{F_t} = (r - \mu_L + \sigma_L' \sigma_L + \sigma_{L,\varepsilon}^2) dt + w_t' ((\mu - r1) - \sigma \sigma_L) dt + w_t' ((\mu - r1) - \sigma \sigma_L) dt + (w_t' \sigma - \sigma_L') dW_t - \sigma_{L,\varepsilon} dW_t^\varepsilon$$

For later use, let us define the following quantities as the mean return and volatility of the funding ratio portfolio, subject to a portfolio strategy w :

$$\mu_F^w = (r - \mu_L + \sigma_L' \sigma_L + \sigma_{L,\varepsilon}^2) + w_t' ((\mu - r1) - \sigma \sigma_L)$$

$$\sigma_F^w = \left((w_t' \sigma - \sigma_L') (w_t' \sigma - \sigma_L') + \sigma_{L,\varepsilon}^2 \right)^{1/2}$$

10 - Another relevant variable is the surplus, defined as the difference, as opposed to the ratio, of assets and liabilities.

6. Mathematical Appendix

6.3. Solution using the Dynamic Programming Approach

Define the indirect or derived utility process at time t : $J_t = \max E_t[U(F_T)]$, where $E_t[\cdot]$ denotes the expectation conditional on information available at time t , such as described by the filtration generated by the asset prices that are driven by n Brownian motion and the $(n+1)^{th}$ Brownian motion driving pure liability uncertainty.

6.3.1. General Solution

For a Markovian control process $(w_t)_{t \geq 0}$ and a function $\varphi(t, F_t) \in C^{1,2}$ the infinitesimal generator of the funding ratio process is:

$$A^w \varphi(t, F_t) = \varphi_t + F \varphi_F \mu_F^w + \frac{1}{2} F^2 \varphi_{FF} (\sigma_F^w)^2$$

where the derivative of a function f with respect to variable x is denoted as f_x .

Given the objective function, the appropriate Hamilton-Jacobi-Bellman equation associated with this problem is: $\sup_w \{A^w J(t, F_t)\} = 0$,

subject to $J(T, F_T) = U(F_T)$.

Optimizing with respect to w yields:

$$F \varphi_F \frac{\partial \mu_F^w}{\partial w} (w^*) + \frac{1}{2} F^2 \varphi_{FF} \frac{\partial (\sigma_F^w)^2}{\partial w} (w^*) = 0$$

or:

$$F \varphi_F ((\mu - r1) - \sigma \sigma_L) + F^2 \varphi_{FF} (w^* \sigma \sigma' - \sigma \sigma_L) = 0$$

with solution:

$$w^* = w^*(t, F_t) = -(\sigma \sigma')^{-1} \left((\mu - r1) - \sigma \sigma_L \right) \frac{\varphi_F(t, F_t)}{F \varphi_{FF}(t, F_t)} + (\sigma \sigma')^{-1} \sigma \sigma_L$$

or:

$$w^* = w^*(t, F_t) = -(\sigma \sigma')^{-1} \left((\mu - r1) \frac{\varphi_F(t, F_t)}{F \varphi_{FF}(t, F_t)} + \left(1 + \frac{\varphi_F(t, F_t)}{F \varphi_{FF}(t, F_t)} \right) (\sigma \sigma')^{-1} \sigma \sigma_L \right)$$

We thus obtain a three fund separation theorem, where the optimal portfolio strategy consists of holding two funds, one with weights:

$$w_M = \frac{(\sigma \sigma')^{-1} (\mu - r1)}{1' (\sigma \sigma')^{-1} (\mu - r1)}$$

and another one with weights:

$$w_L = \frac{(\sigma \sigma')^{-1} \sigma_L}{1' (\sigma \sigma')^{-1} \sigma_L}$$

the rest being invested in the risk-free asset.

The first portfolio is the standard mean-variance efficient portfolio. Note that the amount invested in that portfolio is directly proportional to the investor's Arrow-Pratt coefficient of risk-tolerance $-\frac{\varphi_F}{F \varphi_{FF}}$ (the inverse of the relative

risk aversion). This makes sense: the higher the investor's (funding) risk tolerance, the higher the allocation to that portfolio will be.

In order to better understand the nature of the second portfolio, it is useful to remark that it is a portfolio that minimizes the local volatility σ_F^w of the funding ratio. To see this, recall that the expression for the local variance is given by $\sigma_F^w = \left((w' \sigma - \sigma_L') (w' \sigma - \sigma_L) + \sigma_{L,\varepsilon}^2 \right)^{1/2}$, which reaches a minimum for $w^* = (\sigma \sigma')^{-1} \sigma_L$, with the minimum being $\sigma_{L,\varepsilon}^2$. As such, it appears as the equivalent of the minimum variance portfolio in a relative return-relative risk space, and also as the equivalent of the risk-free asset in a complete market situation where liability risk is entirely spanned by existing securities ($\sigma_{L,\varepsilon}^2 = 0$). Alternatively, this portfolio can be shown to have the highest correlation with the liabilities. As such, it can be called a liability-hedging portfolio, in the spirit of Merton (1971) intertemporal hedging demands. Indeed, if we want to maximize the covariance $w' \sigma \sigma_L$ between the asset portfolio and the liability portfolio L , under the constraint that $\sigma_A^2 = w' \sigma \sigma' w$, we obtain the following Lagrangian:

$$L = w' \sigma \sigma_L - \lambda (w' \sigma \sigma' w - \sigma_A^2)$$

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Differentiating with respect to w yields:

$$\frac{\partial L}{\partial w} = \sigma \sigma_L - 2\lambda \sigma' \sigma w,$$

with a strictly negative second derivative function. Setting the first derivative equal to zero for the highest covariance portfolio leads to the following portfolio, which is indeed proportional to the liability hedging portfolio:

$$w = \frac{1}{2\lambda} (\sigma \sigma')^{-1} \sigma \sigma_L = \frac{1}{2\lambda} (\sigma')^{-1} \sigma_L.$$

6.3.2. Specific Solution in Case of CRRA Utility and Constant Parameter Values

Let us now consider a specific utility function of the CRRA type:

$$U(F_T) = \frac{(F_T)^{1-\gamma}}{1-\gamma}$$

We try a solution to the non-linear Cauchy problem:

$$\varphi_t + F \varphi_F \mu_F^{w^*} + \frac{1}{2} F^2 \varphi_{FF} (\sigma_F^{w^*})^2 = 0$$

which is separable in F and can be written as:

$$\varphi(t, F_t) = \frac{g(t, T) (F_t)^{1-\gamma}}{1-\gamma}$$

with:

$$g(t, T) = \exp \left[(T-t) \left(-\frac{1}{2} \left(1 - \frac{1}{\gamma} \right) (\theta - \sigma_L) (\theta - \sigma_L) - \gamma (\sigma_{L,e}^2 - \theta_L \sigma_{L,e}) + \frac{(1-\gamma)(2-\gamma)}{2} \sigma_{L,e}^2 \right) \right]$$

where $\sigma - \sigma_L$ is defined as the matrix the general term of which is equal to that of σ outside the diagonal and is equal to $\sigma_{ii} - \sigma_L$, also written as $\sigma_i^2 - \sigma_L$, on the diagonal.

Given that $-\frac{J_F}{F J_{FF}} = \frac{1}{\gamma}$, we finally obtain:

$$w^* = w^*(t, F_t) = \frac{1}{\gamma} (\sigma \sigma')^{-1} (\mu - r1) + \left(1 - \frac{1}{\gamma} \right) (\sigma')^{-1} \sigma_L$$

As is well known, it should be noted that when $\gamma = 1$, i.e., in the case of the log investor, the intertemporal hedging demand is zero (myopic investor). In general, again, the optimal strategy consists of holding two funds, in addition to the risk-free asset, the standard mean-variance portfolio and the liability hedging portfolio, and the proportions invested in these two funds are constant in time.

6.4. From Static to Dynamic Portfolio Management

It can be desirable from an investor's standpoint to set a strict constraint on the potential underperformance of the portfolio with respect to the liability benchmark. There can be two types of constraints, explicit or implicit. In a program with explicit constraints, marginal indirect utility from wealth discontinuously jumps to infinity:

$$\text{Max}_{w_t, t \leq s \leq T} E_t \left[\frac{\left(\frac{A_T}{L_T} \right)^{1-\gamma}}{1-\gamma} \right]$$

such that $A_T \geq k L_T$ almost certainly. On the other hand, in a program with implicit constraints, marginal utility goes smoothly to infinity:

$$\text{Max}_{w_t, t \leq s \leq T} E_t \left[\frac{\left(\frac{A_T}{L_T} - k \right)^{1-\gamma}}{1-\gamma} \right]$$

It can be shown (Martellini (2006b)) in a complete market setting that the solution to a program with implicit constraints yields the following time-dependent solution¹¹:

$$w^* = w^*(F_t) = \frac{1}{\gamma} \left(1 - \frac{k}{F_t} \right) (\sigma \sigma')^{-1} (\mu - r1) + \left(1 - \frac{1}{\gamma} \left(1 - \frac{k}{F_t} \right) \right) (\sigma')^{-1} \sigma_B$$

¹¹ - See Martellini (2006b), who uses in a complete market setting the convex duality (or martingale) technique for solving the optimal portfolio allocation problem in the presence of minimum funding ratio constraints.

6. Mathematical Appendix

Consider the fraction of wealth A_t allocated to the performance-seeking portfolio. It is given by :

$$\frac{1'(\sigma\sigma')^{-1}(\mu - r1)}{\gamma} \left(A_t - \frac{A_t k}{F_t} \right) = \frac{1'(\sigma\sigma')^{-1}(\mu - r1)}{\gamma} (A_t - kL_t)$$

Hence it appears that the fraction of wealth allocated to the satellite is equal to a constant multiple m of the cushion, i.e., the difference between the asset value and the floor defined as $A_t - kL_t$. This strategy is reminiscent of constant proportion portfolio insurance (CPPI) strategies, which it extends to a relative risk management context. While CPPI strategies are designed to prevent final terminal wealth from falling below a specific threshold, extended CPPI strategies (or dynamic core-satellite strategies) are designed to protect asset value from falling below a pre-specified fraction of the benchmark value, here given by the liability portfolio.

References

- Ahlgrim, K., S. D'Arcy and R. Gorrivett (2004), "Modeling of Economic Series coordinated with Interest Rate Scenarios", working paper sponsored by the Casualty Actuarial Society and the Society of Actuaries, available on the web at www.casact.org/research/econ.
- Amenc, N., L. Martellini and P. Malaise (2004), "Revisiting Core-Satellite Investing - A Dynamic Model of Relative Risk Management", *Journal of Portfolio Management*, 31, 1, 64-75.
- Beltratti, A., A. Consiglio and S. Zenios (1999), "Scenario Modeling for the Management of International Bond Portfolios", *Annals of Operations Research*, 85, 227-247.
- Boulier, J-F, E. Trussant and D. Florens (1995), "A Dynamic Model for Pension Funds Management", in *Proceedings of the 5th AFIR International Colloquium*, 361-384.
- Cariño, D., T. Kent, D. Myers, C. Stacy, M. Sylvanus, A. Turner, K. Watanabe and W. Ziemba (1994), "The Russell-Yasuda Kasai model: An Asset/Liability Model for a Japanese Insurance Company using Multistage Stochastic Programming", *Interfaces*, 24, 29-49.
- Cariño, D., D. Myers and W. Ziemba (1998), "Concepts, Technical Issues, and uses of the Russell-Yasuda Kasai Financial Planning Model", *Operations Research*, 46, 450-462.
- Cariño, D. and W. Ziemba (1998), "Formulation of the Russell-Yasuda Kasai Financial Planning Model", *Operations Research*, 46, 433-449.
- Fabozzi, F., L. Martellini and P. Priaulet (2005), "Hedging Interest Rate Risk with Term Structure Factor Models", in *The handbook of fixed-income securities*, 7th edition, edited by Frank Fabozzi, John Wiley.
- Geyer, A., W. Herold, K. Kontriner and W. Ziemba (2001), "The Innovest Austrian Pension Fund Financial Planning Model InnoALM", working paper, University of British Columbia, Vancouver, BC.
- Kallberg, J., R. White and W. Ziemba (1982), "Short-Term Financial Planning under Uncertainty", *Management Science*, 28, 670-682.
- Kusy, M. and W. Ziemba (1986), "A Bank Asset and Liability Management Model", *Operations Research*, 34, 356-376.
- Leibowitz, M. and A. Weinberger (1982), "Contingent Immunization—Part I: Risk Control Procedures", *Financial Analysts Journal* (November-December), 17-31.
- Leibowitz, M. and A. Weinberger (1982), "Contingent Immunization—Part II: Problem Areas", *Financial Analysts Journal* (January-February), 35-50.
- Martellini, L. (2006), "The Theory of Liability driven Investments", *Life & Pensions Magazine*, 2, 5, 39-44.
- Martellini, L. (2006), "Dynamic Allocation Decisions in the Presence of Liability Constraints", working paper, EDHEC Risk and Asset Management Research Centre.
- Merton, R. (1969), "Lifetime Portfolio Selection under Uncertainty: the Continuous Time Case", *Review of Economics and Statistics*, 51, 247-257.
- Merton, R. (1971), "Optimum Consumption and Portfolio rules in a Continuous Time Model", *Journal of Economic Theory*, 3, 373-413.
- Merton, R. (1990), *Continuous-Time Finance*, Padstow, UK: Basil Blackwell Inc..
- Mulvey, J., Fabozzi, F., Pauling, W., Simsek, K. and Z. Zhang (2005), "Modernizing the Defined-Benefit Pension System", *Journal of Portfolio Management*, 31, 2, 73-82.

References

- Mulvey, J., G. Gould and C. Morgan (2000), "The Asset and Liability Management System for Towers Perrin-Tillinghast", *Interfaces*, 30, 96–114.
- Mulvey, J. and H. Vladimirou (1992), "Stochastic Network Programming for Financial Planning Problems", *Management Science*, 38, 1642–1664.
- Rudolf, M. and W. Ziemba (2004), *Journal of Economic Dynamics and Control*, 28, 975–990.
- Siegel, L. and B. Waring (2004), "TIPS, the Dual Duration and the Pension Plan", *Financial Analysts Journal*, 60, 5, 52–64.
- Siegmann, A. and A. Lucas (2002), "Continuous-Time Dynamic Programming for ALM with Risk-Averse loss Functions", working paper, Tinbergen Institute.
- Standard Life Investments (2003), "Bridging the pensions gap", Global Bytes, http://ukstandardlifeinvestments.com/content/strategy/strategy_index.html.
- Sundaresan, S. and F. Zapatero (1997), "Valuation, Optimal Asset Allocation and Retirement Incentives of Pension Plans", *The Review of Financial Studies*, 10, 3, 631–660.
- Watson Wyatt (2003), Global Asset Study (ongoing); as cited by "Finanz und Wirtschaft" (28/01/2004), <http://www.finanzinfo.ch>
- Zenios, S. (1995), "Asset/Liability Management under uncertainty for Fixed-Income Securities", *Annals of Operations Research*, 59, 77–97.
- Ziemba, W. (2003), *The Stochastic Programming Approach to Asset-Liability and Wealth Management*, AIMR-Blackwell.
- Ziemba, W. and J. Mulvey (Eds.) (1998), *Worldwide Asset and Liability Modeling*, Cambridge, UK: Cambridge University Press.

About the EDHEC Risk and Asset Management Research Centre

EDHEC is one of the top five business schools in France and was ranked 7th in the Financial Times Masters in Management Rankings 2006 owing to the high quality of its academic staff (over 100 permanent lecturers from France and abroad) and its privileged relationship with professionals that the school has been developing since it was established in 1906. EDHEC Business School has decided to draw on its extensive knowledge of the professional environment and has therefore concentrated its research on themes that satisfy the needs of professionals. EDHEC is one of the few business schools in Europe to have received the triple international accreditation: AACSB (USGlobal), Equis (Europe-Global) and AMBA (UK-Global). EDHEC pursues an active research policy in the field of finance. Its "Risk and Asset Management Research Centre" carries out numerous research programmes in the areas of asset allocation and risk management in both the traditional and alternative investment universes.

The choice of asset allocation

The EDHEC Risk and Asset Management Research Centre structures all of its research work around asset allocation. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of active management based solely on stock picking as a source of performance. On the other, the appearance of new asset classes (hedge funds, private equity), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity in both conceptual and operational terms. This strategic choice is applied to all of the centre's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; measuring the performance of funds while taking the tactical allocation dimension of the alphas into account; taking extreme risks into account in the allocation; or studying the usefulness of derivatives in constructing the portfolio.



Source: EDHEC (2002) and Ibbotson, Kaplan (2000)

An applied research approach

In a desire to ensure that the research it carries out is truly applicable in practice, EDHEC has implemented a dual validation system for the work of the EDHEC Risk and Asset Management Research Centre. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the centre's advisory board, which is made up of both internationally recognised researchers and the centre's business partners.

To date, the centre has implemented six research programmes:

- Multi-style/multi-class allocation
- Performance and style analysis
- Indices and benchmarking
- Asset allocation and extreme risks
- Asset allocation and derivative instruments
- ALM and asset management

Research for business

To optimise exchanges between the academic and business worlds, the research centre maintains a website devoted to asset management research for the industry: www.edhec-risk.com, circulates a monthly newsletter to over 75,000 practitioners, conducts regular industry surveys and consultations, and organises annual conferences for the benefit of institutional investors and asset managers. The centre's activities have also given rise to the business offshoots EDHEC Investment

About the EDHEC Risk and Asset Management Research Centre

Research, which supports institutional investors and asset managers in the implementation of the centre's research results and proposes asset allocation services in the context of a 'core-satellite' approach encompassing alternative investments and EDHEC Asset Management Education, which helps investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.

About Pictet & Cie

Founded in 1805 in Geneva, Pictet & Cie is today Switzerland's leading private bankers and one of the leading independent asset managers in Europe. Headquartered in Geneva, it has offices in: Florence, Frankfurt, Hong-Kong, Lausanne, London, Luxembourg, Madrid, Milan, Montreal, Nassau, Paris, Rome, Singapore, Tokyo, Turin and Zurich. With 2,300 employees, including 500 investment professionals, as well as 100 financial analysts and economists, Pictet's investments spread over more than 80 countries. The Pictet Group has CHF 370bn under administration (over CHF 230bn under management) (31 December 2006 figures).

The Bank is a partnership owned and managed by eight Partner-Managers who pledge their entire personal assets against the bank's liabilities. Since its inception, Pictet has specialised in wealth management services and related services: private banking, institutional asset management (Pictet Asset Management), management and administration of investment funds (Pictet Funds), Global Custody and family office services to a discerning private and institutional clientele worldwide.

Pictet Asset Management (PAM) includes all the operating subsidiaries and divisions of the Group that carry out institutional asset management. Through PAM, Pictet is Switzerland's third biggest institutional asset manager after UBS and Credit Suisse with CHF 121bn under management. The foundations of this area of business were laid in 1967, and in the early 1980s a separate institutional entity was set up, making the Bank one of the pioneers of institutional management in Switzerland.

Pictet & Cie has always been committed to the maxim «Innovation is our tradition», and, in keeping with this, the Bank was one of the first to invest in newly industrialising countries through an Emerging Markets fund, which it set up in 1990. The development of several indices that allow investors to compare the performance of occupational pension funds (including CHF bond indices in 1983 and the BVG Index in 1985) is further evidence of this spirit of innovation.

Pictet is also Switzerland's number three for fund distribution, with a volume of CHF 53 billion,

and was a trailblazer for the biotech, water and generics sectors, being the first to market these as separate investment products. Today, the product range includes over 70 funds. In addition to emerging markets and themed portfolios, Pictet is an acknowledged specialist in global and European equities and small caps, and is also well known for its top-class fixed income products. Furthermore, Pictet now has some CHF 15 billion invested in hedge funds, and can call on nearly 20 years of expertise in the selection of funds of hedge funds.

Pictet was the first bank in Continental Europe to launch a Family Office in 1998 and has been consistently ranked as one of the best Global Custodians worldwide for the quality of the service over the past 10 years.

This specialization and independence enable Pictet to avoid conflicts of interest and to make recommendations with optimal objectivity, free from pressures that can occur in major financial groups. Pictet's diversified client base includes private clients, (e.g. individuals, family trusts and foundations and independent asset managers), as well as institutional clients (e.g. pension funds, companies, insurance firms and state entities, distribution partners for Pictet's investment funds).

The partnership status ensures that a long-term view is taken since the bank is passed down from one generation to the next. The bank's legal structure, still operating on a human scale, places person-to-person relations at the centre of the dealings. Moreover, the very low staff turnover of Pictet enables the bank to foster a unique, long-term relationship with each and every client.

Pictet remains true to its traditional values and intends to continue to expand in a natural way, preserving both its independence and its legal status as a partnership, whereby the partners stand entirely and personally liable for the Bank's commitments.

Ivan Pictet, who today represents the eighth generation of the Pictet family, has been senior partner since 1 July 2005 and recently reiterated that "the bedrock of our business has always been wealth management, and this will remain so in the future as well.»

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